Piecewise Functions

Updated: 05/15/10

Objectives:
Students will review linear functions and their properties and be introduced to piecewise functions.

Connections to Previous Learning:
Students should be able to write the equation of a line and to use rate of change to create a story to model a linear function.

Connections to AP*:
AP Calculus Topics: Analysis of Functions; Rate of Change; Position/Velocity/Acceleration

Materials:
Student Activity pages

Teacher Notes:
Before beginning the student activity pages, model the path of the bicycle by having a student walk along a horizontal line across the front of the room while you describe the information from the graph of the bicycle trip. Scale both distance and time to fit into the classroom.

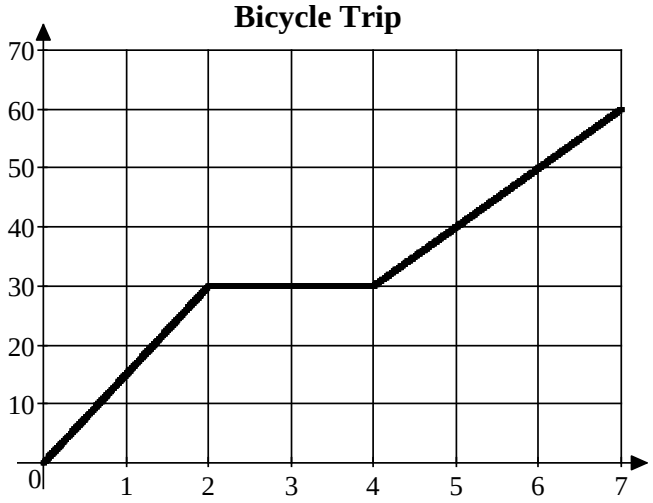
- Walk forward 30 "miles" at a constant rate during a 2 "hour" time period.
- Stop and rest for 2 "hours".
- Walk forward 30 "miles" at a constant rate during a three "hour" time period.

When the walk is complete, ask the class to draw a picture of the path. The path will be a horizontal line. Discuss that the distance-time graph of the bicycle trip is not a replica of the path followed by the bicycles.

Question 5 leads the students through using the notation for writing a piecewise function as one equation.

Piecewise Functions

The Free Wheelers Bicycle Club went on a Saturday bicycle trip. The graph shows the relationship between time in hours and distance in miles from the starting point for one female club member. She traveled away from her start point in a straight line. When the timing started, she was already traveling at the given rate. Use the graph, which is $g(t)$, and the table to answer the following questions.



t	$g(t)$
0	0
2	30
4	30
7	60

1. For this situation, _____ is a function of _____.

2. Write these labels on the appropriate axes with the units, where distance is measured in miles and time is measured in hours.

3. During which interval is she moving the fastest? Explain your answer.

4. Describe what is happening on the bicycle trip for the first 2 hours.

5. The graph consists of three distinct pieces of lines. Write an equation for each part: one for the interval $t = 0$ to $t = 2$, one for the interval $t = 2$ to $t = 4$, and one for the interval $t = 4$ to $t = 7$. This graph is called a piecewise function and would be written as

$$g(t) = \begin{cases} 15t & \text{for } 0 \le t \le 2 \\ \_\_\_\_\_ & \text{for } \_\_\_\_\_ \\ \_\_\_\_\_ & \text{for } \_\_\_\_\_ \end{cases}$$

6. When is $g(t) = 22$ miles? Show the work that leads to your conclusion and interpret the meaning of what you found.

7. What is the average speed for the interval $t = 1$ to $t = 3$?

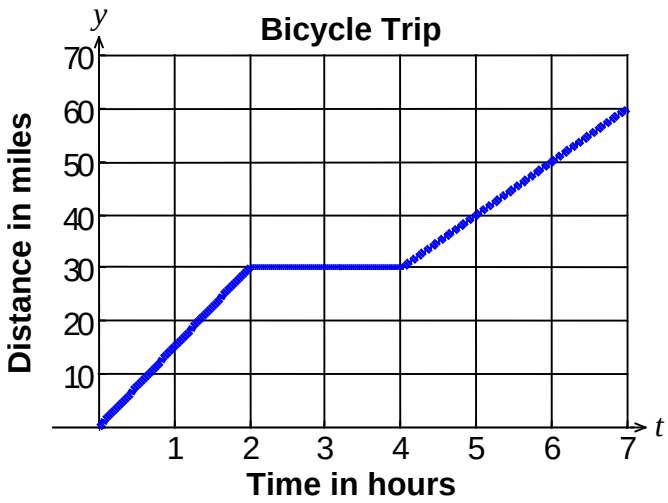
8. Suppose that at the end of the fourth hour, the bicyclist decides to go back to her starting position. Illustrate what the graph might look like for the return trip.

9. Create a story for the graph of the function $g(t)$ for the interval $t = 0$ to $t = 7$.

Piecewise Functions

Answers:

1. **distance** is a function of **time**

2. 

3. Between 0 and 2 hours because that is where the line is the steepest which says that is where her speed is the greatest

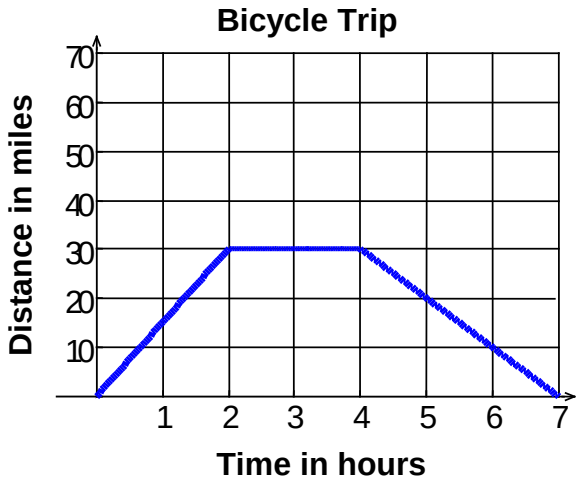
4. For the first two hours she is traveling at a constant rate of change going away from her starting position. By 2 hours, she has biked a distance of 30 miles and at a rate of 15 miles per hour.

5. $g(t) = \begin{cases} 15t, \text{ for } 0 \le t \le 2 \\ 30, \text{ for } 2 \le t \le 4 \\ 10(t-4)+30, \text{ for } 4 \le t \le 7 \end{cases}$

6. Since the line passes through (0,0) and has a slope of $15\frac{\text{miles}}{\text{hour}}$, the equation for this portion of the graph is $g(t) = 15t$. To determine $g(t) = 22$, set $22 = 15t$. Solving for t results in $\frac{22}{15} = 1.467 \text{ hours}$ hours which means at 1.467 hours, the person had biked 22 miles.

7. 7.5 miles per hour

8. Graphs may vary. Graphs must decrease beginning at $t = 4$ and return to the time axis. One example of a correct graph is below.



9. At the second hour she stops for 2 hours to eat lunch and take a nap under a tree in a park that is 30 miles away from where she started. At the 4th hour, she decides to continue on her bike journey still traveling away from her starting position. She is going at a constant rate of 10 miles per hour for 3 more hours. At the end of 7 hours she is 60 miles away from her starting position.