Lesson Plan Title	Introduction to Inverse Functions (Possible Sentences, p. 69, *Beyond the Blueprint*)
Lesson Plan Created by	Paul Edelen; David Gross; Marlene Lovanio, CSDE Educational Consultant for Secondary Mathematics
Grades	10-12
Subject	Algebra 2
Content/Content Standards Addressed	Algebraic Reasoning: Patterns and Functions Patterns and functional relationships can be represented and analyzed using a variety of strategies, tools and technologies. 1.2 Represent and analyze quantitative relationships in a variety of ways. a. Relate the behavior of functions and relations to specific parameters and determine functions to model real-world situations.
Time	One–two class periods
Objective(s) of Lesson	Students will apply their everyday understanding of working backward to set the stage for writing and operating with inverse functions. Students will explore the relationship between a function and its inverse and then investigate the relationship between a radical function and its inverse.
Required Materials for Lesson/Technology	Graph paper, calculator as needed
Initiation (prior knowledge; connections; vocabulary)	Activate prior knowledge with a modified version of Possible Sentences (*Beyond the Blueprint*, p. 69). As a warm-up, have students in their small groups write sentences using one–two of the following words in each sentence and put them on the board. The teacher also writes sentences. **Direction, algorithm, quantity, function, operation, inverse, radical** After a brief discussion of the sentences, have small groups revisit their sentences briefly to confirm, revise or extend.
Learning Procedures	Students collaborate to connect how to write directions backward with “undoing” an algorithm and lead to using functional notation to represent the original algorithm and the functional algorithm. Questions are somewhat open ended to allow students to draw their own conclusions in the beginning. Students then look at a function and the graphs and tables of inverse functions. Through the process of “undoing,” they are expected to determine inverse functions. Encourage them to check their own work with a graphing calculator (does their answer produce the intended graph) or through checking the inverse with the table of values. Note: Inverse functional notation is used.

Guided Practice	Have students complete the practice examples at the end of the attached document and additional examples as needed.
Instructional Strategies	Use the "possible sentences" vocabulary strategy to activate prior knowledge. Students can work in pairs or in small groups to come up with their own connections to inverses. Teacher circulates and scaffolds questions to assist groups. Students report results. Whole class discussion.
Closure	Ask students to describe a situation or process and its inverse and how it relates to inverse functions.
Independent Practice	Have students find inverse functions (if possible) and verify whether two functions are inverses of each other. Include various representations and contextual examples but focus on radical functions.
Assessment based on Objectives (informal, formal, formative, summative – essential question)	Pre-assess their understanding through the possible sentences activity. Circulate during group work and scaffold questions for groups so that they don't get completely hung up. Have students self-assess using a graphing calculator or plugging in values from the tables. Assess their understanding through the closure activity and key practice problems and provide feedback to them (do not grade as this is an introductory activity).
Interventions (for struggling students)	Begin with a review of solving equations (undoing) and simple linear functions. Work up to radical functions. Keep the functions to one or two steps. Provide more scaffolding with additional questions to lead them to the same conclusions in the activity.
Enrichment (for gifted students)	Have students find a contextual problem that is associated with radical functions, write the inverse function (if possible) and explain the inverse as it relates to the context. Post these around the room. Have students generalize when a function has an inverse and when it does not for the functions they've studied so far.
Connections to Other Subjects	Economics, social studies, business, marketing

Suppose you are given the following directions:

- From home, go north on Rt 23 for 5 miles
- Turn east (right) onto Orchard Street
- Go to the 3rd traffic light and turn north (left) onto Avon Drive
- Tracy's house is the 5th house on the right.

If you start from Tracy's house, write down the directions to get home.

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How did you come up with the directions to get home from Tracy's?

Suppose you are given the following algorithm:

- Starting with a number, add 5 to it
- Divide the result by 3
- Subtract 4 from that quantity
- Double your result

The final result is 10. Working backwards knowing this result, find the original number. Show your work.

Write a function $f(x)$, which when given a number x (the original number) will model the operations given above.

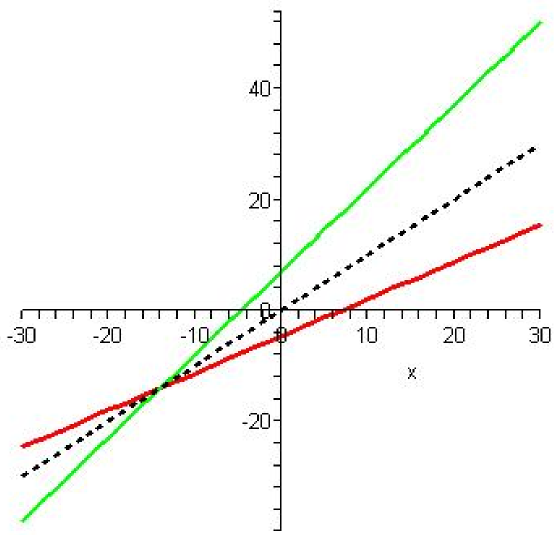
Write a function $g(x)$, which when given a number x (the final result), will model the backward algorithm that you came up with above.

Fill out the following table:

x	*y=f(x)*	*z=g(y)*	*f(z)*
22			
1			
7			
-20			

What patterns have you noticed in the columns (outputs)?

The plots of $f(x)$ and $g(x)$ are shown below. Notice the symmetry of the two functions when plotted together.

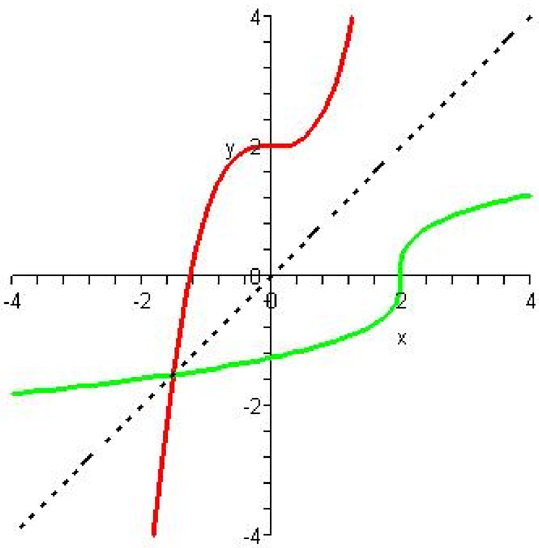


In this scenario, *f*(*x*) and *g*(*x*) are inverses of each other because *g*(*x*) will undo the actions of *f*(*x*). Thus, we could write *g*(*x*) as $f^{-1}(y)$ described as "*f* inverse."

Describe the symmetry in the graph above between the function and its inverse.

What radical function $f^{-1}(y)$ would undo the actions of the function $y = f(x) = x^3 + 2$ as shown in the table and plot below:

x	$y = f(x)$	$f^{-1}(y)$
-1	1	-1
0	2	0
1	3	1
2	10	2



Solution:
Using the table: $f^{-1}(f(x)) =$ ____________

For the following radical functions $y = f(x)$ find the inverse function $f^{-1}(y)$ if an inverse function exists.

1.) $f(x) = \sqrt{4x-4}$

2.) $f(x) = \sqrt[3]{8x-8}$

3.) $f(x) = \sqrt[5]{x+1}$