F-IF, A-REI Springboard Dive

Task

Suppose $h(t) = -5t^2 + 10t + 3$ is the height of a diver above the water (in meters), t seconds after the diver leaves the springboard.

a. How high above the water is the springboard? Explain how you know.

b. When does the diver hit the water?

c. At what time on the diver's descent toward the water is the diver again at the same height as the springboard?

d. When does the diver reach the peak of the dive?

IM Commentary

The purpose of this task is to gives students experience translating questions about a real world context into mathematical questions about a quadratic function and then answering those questions using methods such as graphing, completing the square, and applying the quadratic formula. Which methods are used depends on the goals of the teacher: the task could be approached using graphical methods early on in the study of quadratic functions to motivate finding general algebraic methods, or it could be used as a culminating task in order to practice those methods.

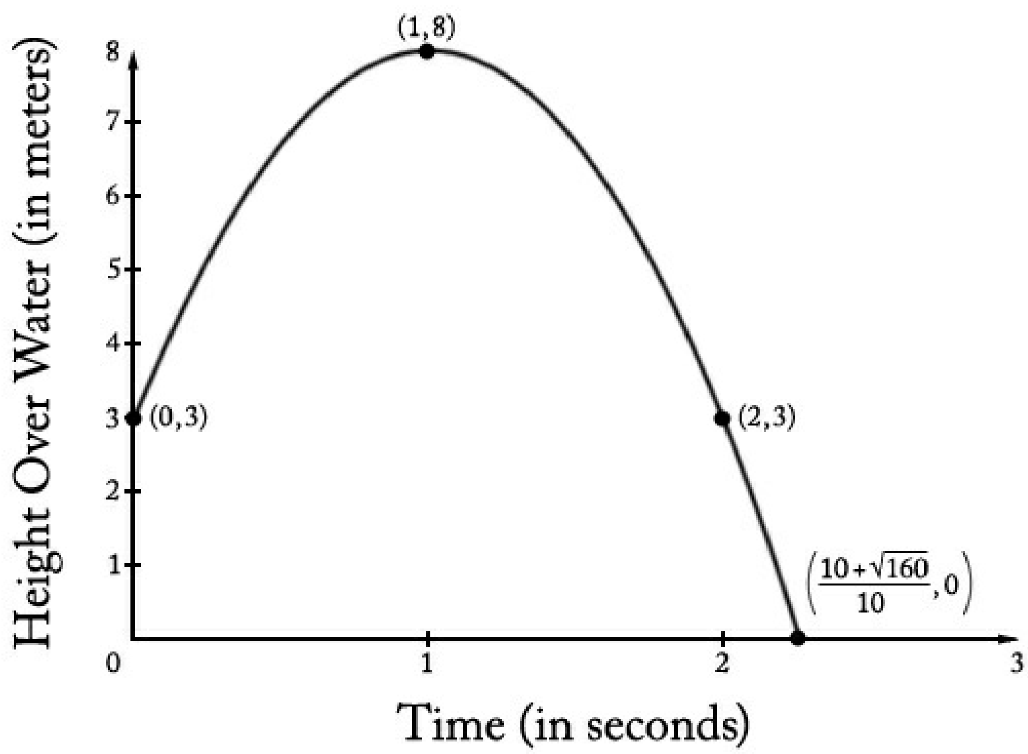
Solutions

Solution: 3. Graphing Technology

A calculator can be used, either to generate a graph of $h(t)$ to help visualize the situation, or to find and label the important points on the graph:

- The point $(0, 3)$ when the dive begins
- The point $(1, 8)$ when the diver reaches the highest elevation
- The point $(2, 3)$ when the diver returns to the level of the springboard
- The point $\left(\frac{10+\sqrt{160}}{10}, 0\right)$ when the diver reaches the water

Below is a graph of $h(t) = -5t^2 + 10x + 3$ with the appropriate points labelled:



Solution: 1.

a. The height of the springboard is found by evaluating h at 0, the instant the diver leaves the springboard:

$$h(0) = 3$$

so the springboard is 3 meters above the water.

b. The diver will hit the water when $h(t) = 0$. Using the quadratic formula, the solutions to $h(t) = 0$ are

$$t = \frac{-10 \pm \sqrt{160}}{-10}.$$

One of these values of t, namely $t = \frac{-10+\sqrt{160}}{-10}$, is negative and so has no significance relative to the dive. So the diver hits the water after

$$t = \frac{10 + \sqrt{160}}{10}$$

seconds or about $2\frac{1}{4}$ seconds.

c. Since we have already determined that the springboard is three meters above the water, we are looking for a second time t for which above the water when

$$h(t) = 3.$$

Substituting in the definition of $h(t)$ and re-writing, we are left trying to solve $-5t^2 + 10t = 0$. Factoring this give $5t(-t + 2) = 0$, which occurs only when $t = 0$ and when $t = 2$. The value $t = 0$ is the moment the dive begins and so it is two seconds after the dive, $t = 2$, that the diver returns again to the level of the springboard.

d. The graph of $h(t)$ is a parabola. Parabolas are symmetric about the vertical line through the vertex of the parabola. Since $h(0) = h(2) = 3$ the vertex must be at $t = 1$ as this is the only vertical reflection which will interchange the points $(0, 3)$ and $(2, 3)$ on the graph of $h(t)$.

Solution: 2 Finding when diver reaches peak by completing the square

The last part of the problem can be solved by rewriting the function $h(t)$ and manipulating as follows:

$$\begin{aligned} h(t) &= -5t^2 + 10t + 3 \\ &= -5\left(t^2 - 2t - \frac{3}{5}\right) \\ &= -5\left((t-1)^2 - \frac{8}{5}\right) \\ &= 8 - 5(t-1)^2 \end{aligned}$$

The structure of the expression $8-(t-1)^2$ shows that $h(t)$ is largest when $t=1$ and that the maximal value is 8. Thinking back to the first solution, the expression $8-5(t-1)^2$ for $h(t)$ also makes it clear that the graph of $y=h(t)$ will be symmetric about $t=1$, that is $h(1+d)$ will take the same value as $h(1-d)$ namely $8-5d^2$.

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