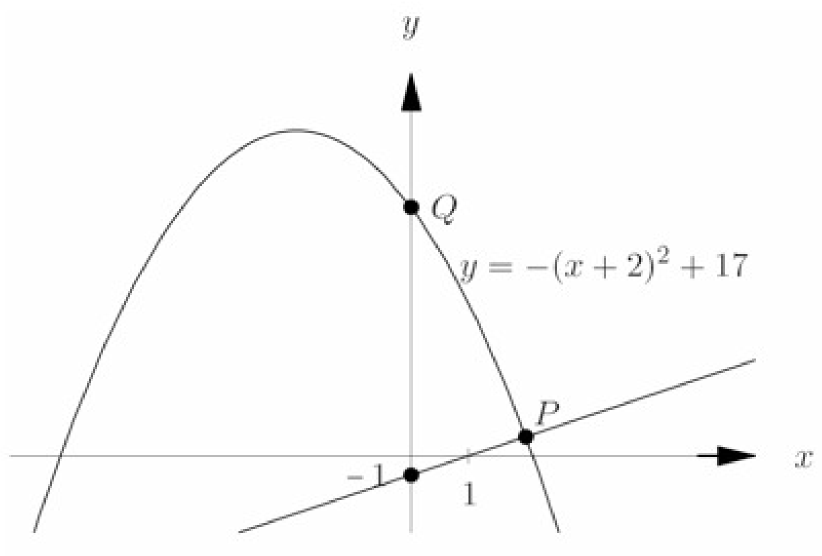
A-REI A Linear and Quadratic System

Task

The figure shows graphs of a linear and a quadratic function.



a. What are the coordinates of the point Q?

b. What are the coordinates of the point P?

IM Commentary

The purpose of this task is to give students the opportunity to make connections between equations and the geometry of their graphs. They must read information from the graph (such as the vertical intercept of the quadratic graph or the slope of the linear one), use that information to construct and solve an equation, then interpret their solution in terms of the graph. The task also requires the basic understanding that the coordinates of the points of intersection of the graphs are the pairs of values of the variables that solve the system.

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Solution

a. Point Q is located at the intersection of the parabola defined by the quadratic function $y = -(x+2)^2 + 17$ with the y-axis. Thus, in order to find the coordinates of the point Q, we need to find the point at which the parabola and the y-axis intersect. We know that all of the points along the y-axis have an x-coordinate of $x = 0$ by definition. Thus, the x-coordinate of point Q is $x = 0$. In order to find the y-coordinate of point Q we now need to substitute $x = 0$ into our equation for the parabola. If $x = 0$, then, by substituting in this value of x, we find that $y = -(0+2)^2 + 17 = -(2)^2 + 17 = -4 + 17 = 13$. Thus, $y = 13$ when $x = 0$, meaning that point Q has coordinates $(0, 13)$.

b. Point P is the point at which the function $y = -(x+2)^2 + 17$ intersects the line passing through the points $(0, -1)$ and $(1, 0)$. In order to find this intersection point we must first find an equation for this line. The general equation of a straight line is given by $y = mx + b$ where m is the slope of the line and b is the y-intercept. We know that the slope, m, of this straight line is in general given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

where (x_1, y_1) and (x_2, y_2) are two of the points on the line. We know that $(0, -1)$ and $(1, 0)$ are two points on this line. Thus, the slope for this line can be found by substituting these points into our general formula:

$$m = \frac{0 - (-1)}{1 - 0}$$
$$m = \frac{1}{1}$$
$$m = 1$$

The slope of our line is $m = 1$. We already know that the coordinate of the y-intercept for this line is $(0, -1)$ because this is one of our given points, giving $b = -1$. We have $m = 1$ and $b = -1$ so, by substituting these values into our general equation for a straight line, we find that the equation for the straight line passing through point P is given by $y = 1 \cdot x - 1$ or $y = x - 1$.

Now that we know the equation of the straight line passing through point P we can find the coordinates of point P by finding the intersection of this line with the parabola defined by $y = -(x+2)^2 + 17$. In order to find the coordinates of the intersection point we must set the quadratic equation of the parabola equal to the linear equation of the straight line:

$$\begin{aligned}
-(x+2)^2 + 17 &= x - 1 \\
-(x^2 + 2x + 2x + 4) + 17 &= x - 1 \\
-(x^2 + 4x + 4) + 17 &= x - 1 \\
-x^2 - 4x - 4 + 17 &= x - 1 \\
-x^2 - 4x - x + 13 + 1 &= 0 \\
-x^2 - 5x + 14 &= 0 \\
x^2 + 5x - 14 &= 0 \\
(x-2)(x+7) &= 0
\end{aligned}$$

Thus $x = 2$ or $x = -7$. We know that point P is in the first quadrant of the xy-plane so the x-coordinate of point P cannot be $x = -7$, giving that the x-coordinate of point P is $x = 2$. We can then substitute $x = 2$ into the equation for the straight line passing through point P, giving: $y = 2 - 1 = 1$. Therefore, point P has coordinates $(2, 1)$.