Unit 9: Square & Cube Roots; Irrational Numbers

Clusters: Work with radicals and integer exponents.
Know that there are numbers that are not rational, and approximate them by rational numbers.

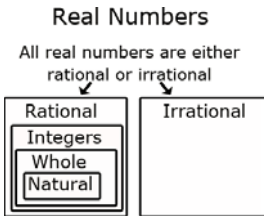
Nevada Academic Content Standard	What does this standard mean that a student will know and be able to do? (adapted from North Carolina *8th Grade Standards, Unpacked Content*)
8.EE.A.2 Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.	**8.EE.2** Students recognize perfect squares and cubes, understanding that non-perfect squares and non-perfect cubes are irrational. Students recognize that squaring a number and taking the square root $\sqrt{\ }$ of a number are inverse operations; likewise, cubing a number and taking the cube root $\sqrt[3]{\ }$ are inverse operations. *Example 1:* $4^2 = 16$ and $\sqrt{16} = \pm 4$ NOTE: $(-4)^2 = 16$ while $-4^2 = -16$ since the negative is not being squared. This difference is often problematic for students, especially with calculator use. *Example 2:* $\left(\frac{1}{3}\right)^3 = \left(\frac{1^3}{3^3}\right) = \frac{1}{27}$ and $\sqrt[3]{\frac{1}{27}} = \frac{\sqrt[3]{1}}{\sqrt[3]{27}} = \frac{1}{3}$ NOTE: there is no negative cube root since multiplying 3 negatives would give a negative. This understanding is used to solve equations containing square or cube numbers. Rational numbers would have perfect squares or perfect cubes for the numerator and denominator. In the standard, the value of p for square root and cube root equations must be positive. *Example 3:* Solve: $x^2 = 25$ *Solution:* $\sqrt{x^2} = \pm\sqrt{25}$ $x = \pm 5$ NOTE: There are two solutions because 5 • 5 and -5 • -5 will both equal 25. *Example 4:* Solve: $x^2 = \frac{4}{9}$ *Solution:* $\sqrt{x^2} = \pm\sqrt{\frac{4}{9}}$ $x = \pm\frac{2}{3}$ *Example 5*: Solve: $x^3 = 27$ *Solution:* $\sqrt[3]{x^3} = \sqrt[3]{27}$ $x = 3$ *Example 6:* Solve: $x^3 = \frac{1}{8}$ *Solution:* $\sqrt[3]{x^3} = \sqrt[3]{\frac{1}{8}}$ $x = \frac{1}{2}$

Students understand that in geometry the square root of the area is the length of the side of a square and a cube root of the volume is the length of the side of a cube. Students use this information to solve problems, such as finding the perimeter.

Example 7:
What is the side length of a square with an area of 49 ft2?
Solution:
$\sqrt{49}$ = 7 ft. The length of one side is 7 ft..

8.NS.A.1
Know that numbers that are not rational are called irrational. ~~Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.~~

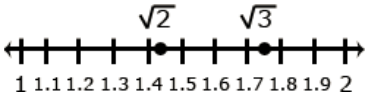
8.NS.1
Students understand that Real numbers are either rational or irrational. They distinguish between rational and irrational numbers, recognizing that any number that can be expressed as a fraction is a rational number. The diagram illustrates the relationship between the subgroups of the real number system.



8.NS.A.2
Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). *For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.*

8.NS.2 Students locate rational and irrational numbers on the number line. Students compare and order rational and irrational numbers. Students also recognize that square roots may be negative and written as $-\sqrt{28}$.

Example 1:
Compare $\sqrt{2}$ and $\sqrt{3}$.



Solution: Statements for the comparison could include:
- $\sqrt{2}$ and $\sqrt{3}$ are between the whole numbers 1 and 2
- $\sqrt{3}$ is between 1.7 and 1.8
- $\sqrt{2}$ is less than $\sqrt{3}$

Additionally, students understand that the value of a square root can be approximated between integers and that nonperfect square roots are irrational.

Example 2:
Find an approximation of $\sqrt{28}$.
- Determine the perfect squares $\sqrt{28}$ is between, which would be 25 and 36.
- The square roots of 25 and 36 are 5 and 6 respectively, so we know that $\sqrt{28}$ is between 5 and 6.
- Since $\sqrt{28}$ is closer to 25, an estimate of the square root would be closer to 5. One method to get an estimate is to divide 3 (the distance between 25 and 28) by 11 (the distance between the perfect squares of 25 and 36) to get 0.27.
- The estimate of $\sqrt{28}$would be 5.27 (the actual is 5.29).

Approximate Time Frame: *1-2 weeks*

Terms:

- ✓ rational numbers
- ✓ irrational numbers
- ✓ exponent
- ✓ power
- ✓ base
- ✓ radical
- ✓ perfect squares (≤ 400)
- ✓ perfect cubes (≤ 1000)
- ✓ principal square root
- ✓ square root
- ✓ cube root
- ✓ pi (π)

Resources

MGH – McGraw Hill, Glencoe Math (2015)
ML – McDougal Littell, Pre-Algebra Book; Larson, 2005
EX – Explorations in Core Math (Holt McDougal)
NY – *Engage* New York
LZ – Learn Zillion Website
IL – Illinois *Model Math Curriculum*
MAP – Math Assessment Project (MARS)

	Suggested Topics for Lessons	***Suggested Resources***
8.EE.A.2	**Evaluate square & cube roots.** **Solve equations of the form $x^2 = p$ and $x^3 = p$.** SBAC Evidence: ➢ The student represents solutions to equations of the form $x^2 = p$ using square root symbols. ➢ The student represents solutions to equations of the form $x^3 = p$ using cube root symbols.	➢ MGH 1-8 *Roots* (page 71) ➢ ML 9.1 *Square Roots* (page 453) ➢ ML 9.1 *Simplifying Square Roots* (page 458) ➢ EX 3-5 *Squares and Square Roots*; *Extension: Cube Roots* (page 105) ➢ VA Sample Lesson Plans: *Perfect Squares and Square Roots* ➢ NY Module 7, Topic A, Lesson 2: *Square Roots* ➢ GA Lesson: *Find Square Roots and Cube Roots* ➢ LZ Video Lesson: *Use squares to find side length of a square* ➢ LZ Video Lesson: *Use visuals to understand and generate perfect squares* ➢ LZ Video Lesson: *Find the square root of a perfect square* ➢ LZ Video: *Understand the relationship between squaring and taking the square root* ➢ LZ Video Lesson: *Solve equations with squares and square roots* ➢ LZ Video Lesson: *Identify perfect cubes and find cube roots* ➢ NY Module 7, Topic A, Lesson 5: *Solving Radical Equations* ➢ LZ Video Lesson: *Solve equations with cubes and cube roots*
8.NS.A.1 **8.NS.A.2**	**Identify real numbers as rational and irrational numbers.** **Approximate and compare irrational numbers.** SBAC Evidence: ➢ The student classifies real numbers as rational or irrational. ➢ The student writes approximations of irrational numbers as rational numbers.	➢ MGH 1-9 Inquiry Lab: *Roots of Non-Perfect Squares* (pg 79) ➢ MGH 1-9 *Estimate Roots* (pg 81) ➢ MGH 1-10 *Compare Real Numbers* (pg 89) ➢ ML 9.4 Real Numbers (page 470) ➢ EX 3-6 *Estimating Square Roots* (page 111) ➢ EX 3-7 *The Real Numbers* (page 117) ➢ IL Lesson-Choose Lesson 1 of 3: *Real Numbers* ➢ VA Sample Lesson Plans: *Organizing Real Numbers* ➢ Khan Academy Video & Practice Problems: *Rational and Irrational Numbers* ➢ NY Regents Prep: *Rational and Irrational Numbers* ➢ NY Regents Prep: *Review Activity for Ordering Rational & Irrational Numbers* ➢ Math Rap (YouTube): *Rational and Irrational Numbers* ➢ Foldable: *Classifying Real Numbers* ➢ Stations: *Real Numbers*

	➢ The student compares the sizes of irrational numbers by using rational approximations of irrational numbers. ➢ The student approximates the locations of irrational numbers on the number line by using rational approximations of irrational numbers.	➢ LZ Video Lesson: *Understand & apply the definition of irrational numbers* ➢ LZ Video Lesson: *Distinguish between rational and irrational numbers* ➢ LZ Video Lesson: *Compare irrational and rational numbers* ➢ LZ Video Lesson: *Place nonperfect square roots between 2 integers* ➢ PBS Learning Media: *Approximating Square Roots of Non-Squares* ➢ NC Lessons for Learning: *Real Number Race & The Laundry Problem* ➢ NCTM Illuminations Resource: *Square Roots Go Rational* ➢ MAP: *Rational and Irrational Numbers 1*