

# Let's Go to the Furniture Market

Group Member Names \_\_\_\_\_

## Background Information:

**Linear programming** is used to identify conditions that maximize profit or minimize cost. Today we will investigate how much profit a furniture Company will make in one week given a limited amount of resources. Using LEGOS® to model building tables and chairs, you will use linear programming to relate mathematics to the real world by maximizing furniture profits in preparation of Georgia's famous, semi-annual "Furniture Market."

## Each group will need a weekly allotment of materials:

- A total of  $6 - 4 \times 2$  toy LEGOS to represent table tops.
- A total of  $10 - 1 \times 1$  toy LEGOS to represent chair backs.
- A total of  $16 - 2 \times 1$  toy LEGOS to represent bases needed for both tabletops and chair backs.

## AND

- TI-84 Plus graphing calculator with Inequality Graphing APPlication

## Investigation:

1. A chair can be built using 1 chair back ( $1 \times 1$  LEGO) and 1 base ( $2 \times 1$  LEGO).  
A table can be built using 1 tabletop ( $4 \times 2$  LEGO) and 2 bases.  
Take an inventory and confirm that you have your weekly allotment of tabletops, chair backs, and bases to build the furniture.
2. The furniture company makes a profit of \$30 for every table it produces and \$10 for every chair it produces. Let  $x$  represent the number of tables produced in a week and let  $y$  represent the number of chairs produced in a week. Using **algebra**, write an algebraic expression to represent a profit function for the expected total amount of weekly profit.

$$P(x, y) = \underline{\hspace{10em}}$$

3. If your goal is to maximize profit, begin building possible combinations of tables and chairs. Complete the **numerical** data table with possible combinations of tables and chairs along with their respective profit totals calculated using the above profit function.

(Tables, Chairs) ( $x, y$ )	Profit Function	Total Profit $f(x, y)$

4. Because of the limited amount of weekly resources in chair backs, tabletops, and bases, use **algebra** to develop a system of linear inequalities to represent these constraints.

Tabletop constraint: \_\_\_\_\_

Chair back constraint: \_\_\_\_\_

Bases constraint: \_\_\_\_\_

Are there any non-negativity constraints? \_\_\_\_\_

5. You will now use your graphing calculator to obtain a **graphical** picture of your constraints.

What are the vertices?

Vertices	Substitute	Answer

What is the equation for maximum profit? \_\_\_\_\_

**Summarize your results:**

1. When building tables and chairs using toy Legos©, what combination of tables and chairs built will produce a maximum profit? What is the maximum profit?

---

2. Why did the bases constraint involve both variables,  $x$  and  $y$ ?

---

3. When developing algebraic inequalities to represent the constraints for the limited resources, why should non-negativity be considered?

---

4. When developing the polygonal feasible region, why were there five corner points?

---

5. If you were the Chief Executive Officer of a furniture company that produced similar tables and chairs and you wanted to invest more money into one of the three limited resources, which resource would you choose and why?

---

# Let's Go to the Furniture Market

## Background Information:

**Linear programming** is used to identify conditions that maximize profit or minimize cost. Today we will investigate how much profit a furniture Company will make in one week given a limited amount of resources. Using LEGOS® to model building tables and chairs, you will use linear programming to relate mathematics to the real world by maximizing furniture profits in preparation of North Carolina's famous, semi-annual "Furniture Market."

## Each group will need a weekly allotment of materials:

- A total of  $6 - 4 \times 2$  toy LEGOS to represent table tops.
- A total of  $10 - 1 \times 1$  toy LEGOS to represent chair backs.
- A total of  $16 - 2 \times 1$  toy LEGOS to represent bases needed for both tabletops and chair backs.

## Each group member will need:

- A TI-83 Plus or TI-84 Plus graphing calculator with Inequality Graphing APPLICATION

## Investigation:

1. A chair can be built using 1 chair back ( $1 \times 1$  LEGO) and 1 base ( $2 \times 1$  LEGO).  
A table can be built using 1 tabletop ( $4 \times 2$  LEGO) and 2 bases.  
Take an inventory and confirm that you have your weekly allotment of tabletops, chair backs, and bases to build the furniture.
2. The furniture company makes a profit of \$30 for every table it produces and \$10 for every chair it produces. Let  $x$  represent the number of tables produced in a week and let  $y$  represent the number of chairs produced in a week. Using **algebra**, write an algebraic expression to represent a profit function for the expected total amount of weekly profit.

$$f(x, y) = \underline{\hspace{10em}}$$

3. If your goal is to maximize profit, begin building possible combinations of tables and chairs. Complete the **numerical** data table with possible combinations of tables and chairs along with their respective profit totals calculated using the above profit function.

(Tables, Chairs) ( $x, y$ )	Profit Function	Total Profit $f(x, y)$

4. Because of the limited amount of weekly resources in chair backs, tabletops, and bases, use **algebra** to develop a system of linear inequalities to represent these constraints.

Tabletop constraint: \_\_\_\_\_

Chair back constraint: \_\_\_\_\_

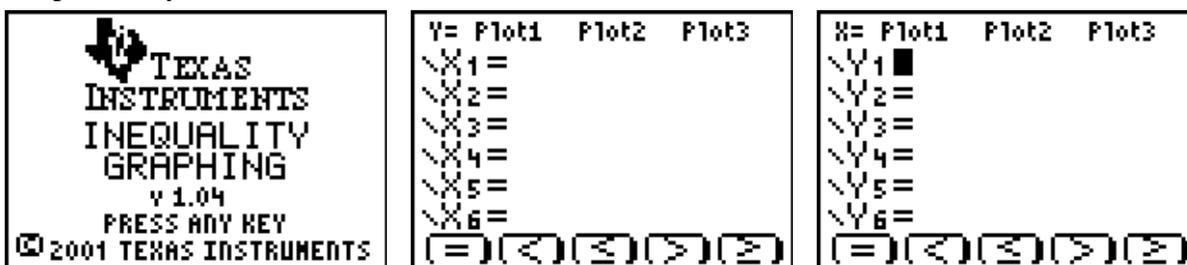
Bases constraint: \_\_\_\_\_

Are there any non-negativity constraints? \_\_\_\_\_

5. You will now use your graphing calculator to obtain a **graphical** picture of your constraints.

Press APPS and begin the Inequality Graphing APPLICATION on your graphing calculator. Put all constraints in the  $x=$  and  $o$  menus. *Don't forget your non-negativity constraints.* Press  $\square$  and the appropriate soft key ( $o$ ,  $\pi$ ,  $\theta$ ,  $\rho$ , or  $\sigma$ ) to obtain the appropriate inequality symbol needed for your constraint.

Adjust your  $\pi$  accordingly to obtain a complete graph of the feasible region that represents your constraints.



Press  $\sigma$ ;  $\square$ ; Shades (soft key  $o$  or  $\pi$ ); and 1:Ineq Intersection. The calculator will produce the feasible region. It should be in the shape of a polygon.

6. It is now time to “walk” the polygonal feasible region collecting corner points as you go. Begin this journey by pressing  $\square$  and POI-TRACE (soft key  $\theta$  or  $\rho$ ). To automatically store the corner point  $(x, y)$  in a **numerical** table, press  $\downarrow$ . The calculator will automatically store this point in the statistical editor as name lists (LINEQX and LINEQY). Assign a formula to L1 that will calculate profits using your original profit function,  $L1 = "30^{TM} \text{LINEQX} + 10^{TM} \text{LINEQY}"$

LINEQX	LINEQY	-----	6
████████	-----		
LINEQX(1)=			

L1	L2	L3	1
-----	-----	-----	
L1 = "30*LINEQX+10*LINEQY"			

## Summarize your results:

1. When building tables and chairs using toy Legos<sup>©</sup>, what combination of tables and chairs built will produce a maximum profit? What is the maximum profit?  

---
2. Why did the bases constraint involve both variables,  $x$  and  $y$ ?  

---
3. When developing algebraic inequalities to represent the constraints for the limited resources, why should non-negativity be considered?  

---
4. When developing the polygonal feasible region, why were there five corner points?  

---
5. Was there any correlation between the numerical table developed by hand and the table the graphing calculator developed using corner points? If so, what was that correlation?  

---
6. If you were the Chief Executive Officer of a furniture company that produced similar tables and chairs and you wanted to invest more money into one of the three limited resources, which resource would you choose and why?  

---

## More Independent Practice:

During the summer break, Nathan works as many as 35 hours per week. On Saturdays he spends between two and six hours delivering furniture. On weekdays he can work between 10 and 40 hours at the recreation center as a counselor for the children's day camp. Delivering furniture pays \$10 per hour while the summer counselor's job pays \$6.25 per hour. How many hours should Nathan work at each job to produce the most income during his summer break?

### Algebraic Representation:

1. If  $x$  represents delivering furniture and  $y$  represents working as a counselor, develop an algebraic expression that represents Nathan's maximum summer earnings as a function of hours worked.
2. Develop a system of linear inequalities that represents the constraints (or limitations) to the number of hours Nathan can work. *Don't forget non-negativity!*

### Graphical Representation:

3. Using the Inequality Graphing APPlication, create a visual picture of the constraints.

### Numerical Representation:

4. "Walk" the polygonal feasible region collecting corner points on your journey. Store these points in the statistical editor.
5. Confirm the collection of corner points from the feasible region in the statistical editor. They should be stored in (LINEQX, LINEQY).
6. Assign a formula to L1 that will calculate the maximum amount of income during Nathan's summer.

**Summarize Your Results** – Complete the following table:

(Furniture, Counselor) $(x, y)$	Maximum Earnings Function	Total Earnings $f(x, y)$

7. Does the table above agree with the table of earnings from the statistical editor?
8. How many hours at each job should Nathan work to maximize his summer earnings?
9. If Nathan could work more than 35 hours per week, at which job should he work more hours?

# Let's Go to the Furniture Market - Teacher Notes

## Objectives:

This lesson is designed to have students use linear programming to relate this mathematical topic to the business world. Students calculate maximum profits for a furniture business to prepare for the famous, semi-annual "Furniture Market" in High Point, North Carolina. Upon completion of this lesson, students will:

- Use manipulatives to determine maximum profit in context of a real life application.
- Develop a model for profit as a function of two variables.
- Develop inequalities in two variables that represent constraints.
- Use a system of linear inequalities to locate corner points.
- Use a graphing calculator to create a visual, graphic image of the feasible region.
- Use a table to evaluate a function in two variables.

## Materials:

- A total of  $6 - 4 \times 2$  toy LEGOS® to represent table tops.
- A total of  $10 - 1 \times 1$  toy LEGOS to represent chair backs.
- A total of  $16 - 2 \times 1$  toy LEGOS to represent bases needed for both tabletops and chair backs.
- "Let's Go to the Furniture Market" Student Activity
- A TI-83 Plus or TI-84 Plus graphing calculator with Inequality Graphing Application

**Time Allowed:** One 90-minute block period or two 55-minute regular periods

## NCTM Standards Addressed:

### *Algebra:*

- Understand relations and functions and select, convert flexibly among, and use various representations for them.
- Interpret representations of functions in two variables.
- Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency-mentally or with paper and pencil in simple cases using technology in all cases.
- Draw reasonable conclusions about a situation being modeled.

### *Geometry:*

- Use Cartesian coordinates to analyze geometric situations (corner points).

### *Number Operations:*

- Judge the reasonableness of numerical computations and their results.

### *Problem Solving:*

- Build new mathematical knowledge through problem solving.
- Solve problems that arise in mathematics and other contexts.
- Apply and adapt a variety of appropriate strategies to solve problems.

**Investigation Answers:**

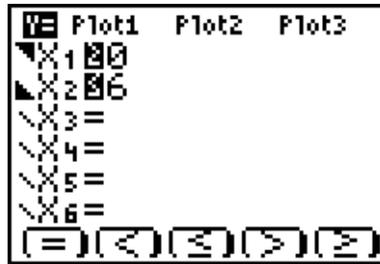
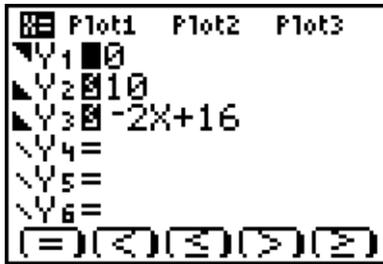
2.  $f(x, y) = 30x + 10y$

3.

(Tables, Chairs) (x, y)	Profit Function	Total Profit f(x, y)
(0, 0)	$30(0) + 10(0)$	0
(0, 10)	$30(0) + 10(10)$	100
(3, 10)	$30(3) + 10(10)$	190
(6, 4)	$30(6) + 10(4)$	220
(6, 0)	$30(6) + 10(0)$	180

4. Tabletop constraint:  $x \leq 6$   
 Chair back constraint:  $y \leq 10$   
 Bases constraint:  $2x + y \leq 16$   
 Non-negativity constraints:  $x \geq 0$   
 $y \geq 0$

5.



6.

L1	INEQX	INEQY	1
180	6	0	
0	0	0	
220	6	4	
100	0	10	
190	3	10	
-----	-----	-----	
L1(G)=			

**Summarize your results:**

- 6 tables and 4 chairs for a maximum profit of \$220.
- Bases are needed for both tables (x) and chairs (y).
- Because you would not produce a negative table (x) or a negative chair (y).
- The five corner points correlate with the five constraints.
- In the table developed by hand, possible combinations of tables and chairs were listed along with their respective profit totals. The values obtained on the calculator from corner points were the same values as developed by hand.
- Because bases are needed to build both a table and a chair, an investment in bases would produce more profit.

## Independent Practice Answers:

1.  $f(x, y) = 10x + 6.25y$

2. Constraints:

35 hours total per week  $\Rightarrow x + y \leq 35$

Delivering furniture  $\Rightarrow 2 \leq x \leq 6$

Counseling  $\Rightarrow 10 \leq y \leq 40$

Non-negativity constraints:  $\Rightarrow x \geq 0$

$\Rightarrow y \geq 0$



4. Corner points are (6, 29), (2, 33), (6, 10), and (2, 10).

5. 

INEQX	INEQY	-----	3
2	33		
2	10		
6	10		
6	29		
-----			
INEQY(5) =			

6. 

L1	#	INEQX	INEQY	1
226.25	2	33		
82.5	2	10		
122.5	6	10		
241.25	6	29		
-----		-----		
L1(5) =				

(Furniture, Counselor) (x, y)	Maximum Earnings Function	Total Earnings f(x, y)
(6, 29)	$10(6) + 6.25(29)$	\$241.25
(6, 10)	$10(6) + 6.25(10)$	\$122.50
(2, 33)	$10(2) + 6.25(33)$	\$226.25
(2, 10)	$10(2) + 6.25(10)$	\$82.50

7. Yes

8. Nathan should work 6 hours delivering furniture and 29 hours at his counseling job for total earnings of \$241.25.

9. If Nathan could work more than 35 hours per week, he should invest more hours in counseling (y).