

# Sneaking up on Slope

After the warm-up, the lesson starts with a handout where the students will be graphing points and looking at the pattern it creates to find the next point in the line. The handout is set up to do a “think-pair-share” activity. When you debrief as a class, have the students explain their partner’s method to finding the next point. This makes them responsible to listening to their partner. The first page of the lesson is the teacher’s note for the student’s handout. After that, the lesson continues. It is necessary to the lesson to do the beginning activity.

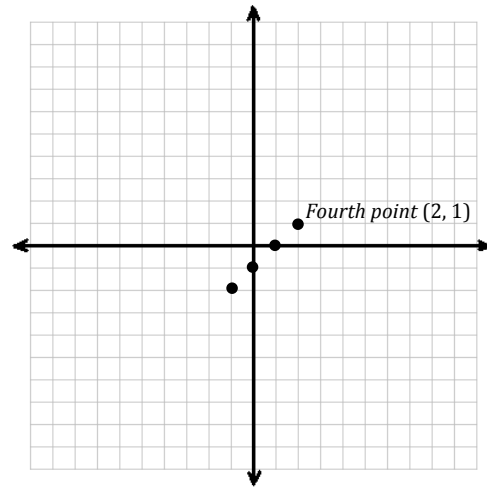
**Materials:** Student Handout, graph paper

**Time:** 1 hour to 1.5 hours, depending on the level of your class

**Teacher:** Remember, graphs are read from the left to the right. Graph the given points on the coordinate plane. Look at the pattern of the point in the coordinate plane. Find the next point in the pattern, plot it, and write it in space provided.

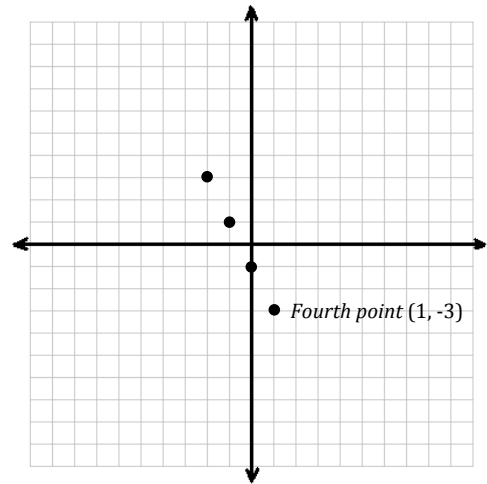
1) Graph the points:  $(-1, -2), (0, -1), (1, 0)$

- What is the fourth point?  $(2, 1)$
- What was your method to finding the point?  
*Students will say things like, "I went up and over", "I went over and up", "I went diagonal"...anyway is ok as long as it gives them the correct answer.*
- What was a method of someone else in the class? *Give students a chance to talk to their elbow partner before sharing out. When you call on students for this question you could ask them to explain how their elbow partner found the fourth point.*



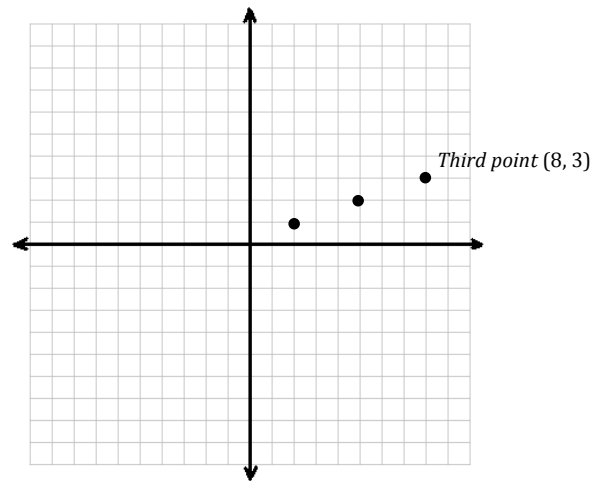
2) Graph the points:  $(-2, 3), (-1, 1), (0, -1)$

- What is the fourth point?  $(1, -3)$
- What was your method to finding the point?  
*Students will say things like, "I went up and over", "I went over and up", "I went diagonal"...anyway is ok as long as it gives them the correct answer.*
- What was a method of someone else in the class? *Give students a chance to talk to their elbow partner before sharing out. When you call on students for this question you could ask them to explain how their elbow partner found the fourth point.*



3) Using only two points, graph the points and find the third:  $(2, 1), (5, 2)$

- What is the third point?  $(8, 3)$
- What was your method to finding the point?  
*Hopefully, by this time the students are finding the next point by going up and over or visa versa.*
- What was a method of someone else in the class?



Let's look at our points in a table, what patterns do you see with the x-values? With the y-values?

x	y	x	y	x	y
-1	-2	-2	3	2	1
0	-1	-1	1	5	2
1	0	0	-1	8	3
2	1	1	-3		

“In mathematics we see this concept of values changing. We use a symbol to signify the ‘change-in’ some value. The symbol is the Greek letter ‘delta’  $\Delta$ . When we see that our  $x$ -value is changing, we call that our change in  $x$  or  $\Delta x$ . The same goes for  $y$ , the change in  $y$  can be written  $\Delta y$ . In all of our groups of numbers we can find a constant change in both the  $x$ - and  $y$ -values. When there is a constant change in a linear group of points, we refer to that change as the slope”

“Write these definitions in your notes”

$\Delta$ : The change in

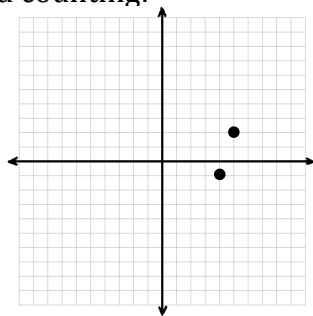
Definition of Slope: The change in  $y$  values over the change in  $x$  values or  $\frac{\Delta y}{\Delta x}$ .

“Let’s look at our sets of points and the changes we saw in both the  $x$ -values and the  $y$ -values. We will then write the changes as the slope”

<table border="1"> <thead> <tr> <th></th> <th><math>x</math></th> <th><math>y</math></th> <th></th> </tr> </thead> <tbody> <tr> <td>1+ ↪</td> <td>-1</td> <td>-2</td> <td>↪ +1</td> </tr> <tr> <td>1+ ↪</td> <td>0</td> <td>-1</td> <td>↪ +1</td> </tr> <tr> <td>1+ ↪</td> <td>1</td> <td>0</td> <td>↪ +1</td> </tr> <tr> <td></td> <td>2</td> <td>1</td> <td></td> </tr> </tbody> </table>		$x$	$y$		1+ ↪	-1	-2	↪ +1	1+ ↪	0	-1	↪ +1	1+ ↪	1	0	↪ +1		2	1		<table border="1"> <thead> <tr> <th></th> <th><math>x</math></th> <th><math>y</math></th> <th></th> </tr> </thead> <tbody> <tr> <td>1+ ↪</td> <td>-2</td> <td>3</td> <td>↪ -2</td> </tr> <tr> <td>1+ ↪</td> <td>-1</td> <td>1</td> <td>↪ -2</td> </tr> <tr> <td>1+ ↪</td> <td>0</td> <td>-1</td> <td>↪ -2</td> </tr> <tr> <td></td> <td>1</td> <td>-3</td> <td></td> </tr> </tbody> </table>		$x$	$y$		1+ ↪	-2	3	↪ -2	1+ ↪	-1	1	↪ -2	1+ ↪	0	-1	↪ -2		1	-3		<table border="1"> <thead> <tr> <th></th> <th><math>x</math></th> <th><math>y</math></th> <th></th> </tr> </thead> <tbody> <tr> <td>3+ ↪</td> <td>2</td> <td>1</td> <td>↪ +1</td> </tr> <tr> <td>3+ ↪</td> <td>5</td> <td>2</td> <td>↪ +1</td> </tr> <tr> <td>3+ ↪</td> <td>8</td> <td>3</td> <td>↪ +1</td> </tr> <tr> <td></td> <td>11</td> <td>4</td> <td></td> </tr> </tbody> </table>		$x$	$y$		3+ ↪	2	1	↪ +1	3+ ↪	5	2	↪ +1	3+ ↪	8	3	↪ +1		11	4	
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$\Delta y = 1$ and $\Delta x = 1$ $\frac{\Delta y}{\Delta x} = \frac{1}{1}$	$\Delta y = -2$ and $\Delta x = +1$ $\frac{\Delta y}{\Delta x} = \frac{-2}{1}$	$\Delta y = 1$ and $\Delta x = 1$ $\frac{\Delta y}{\Delta x} = \frac{1}{3}$																																																												

Example 1: Find the slope that passes through  $(5,2)$  and  $(4,-1)$ .

By graphing and counting:



Slope:  $\frac{\Delta y}{\Delta x} = \frac{3}{1}$

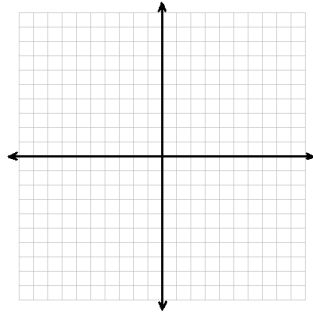
By finding the change in the values:

	$x$	$y$	
1- ↪	5	2	↪ -3
	4	-1	

Slope:  $\frac{\Delta y}{\Delta x} = \frac{-3}{-1} = \frac{3}{1}$

You Try: Find the slope that passes through  $(-3,1)$  and  $(4,5)$ .

By graphing and counting:



$$\text{Slope: } \frac{\Delta y}{\Delta x} = \frac{4}{7}$$

By finding the change in the values:

x	y
-3	1
4	5

$$\text{Slope: } \frac{\Delta y}{\Delta x} = \frac{4}{7}$$

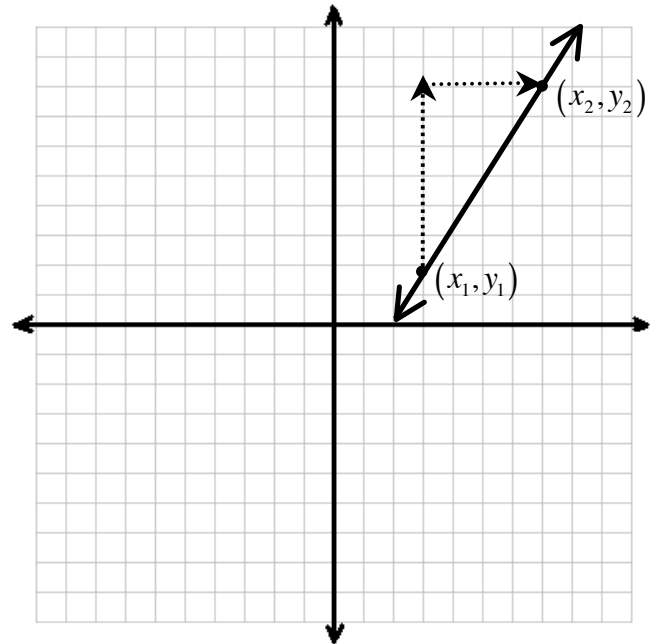
“Slope is an important concept and because of the patterning we see, there is a formula that can be used to find slope. We are now going to ‘derive’ the slope formula knowing what we know about the change in the values of  $x$  and  $y$ ”

Start with any two points  
 $(x_1, y_1)$  and  $(x_2, y_2)$

$$\Delta y = y_2 - y_1$$

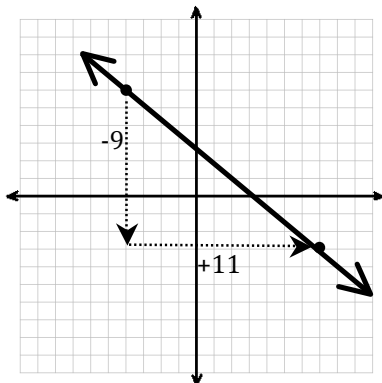
$$\Delta x = x_2 - x_1$$

Slope then is defined as  $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$



Ex.2 Find the slope of the line through the two points  $(-4,6)$  and  $(7,-3)$  three different ways.

Graphing:



$$\text{Slope is } \frac{\Delta y}{\Delta x} = \frac{-9}{11}$$

Change in values:

+11	<table style="border-collapse: collapse; text-align: center;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"><math>x</math></td> <td style="padding: 0 5px;"><math>y</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">-4</td> <td style="padding: 0 5px;">6</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">7</td> <td style="padding: 0 5px;">-3</td> </tr> </table>	$x$	$y$	-4	6	7	-3	-9
$x$	$y$							
-4	6							
7	-3							

$$\text{Slope is } \frac{\Delta y}{\Delta x} = \frac{-9}{11}$$

Using the slope formula

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

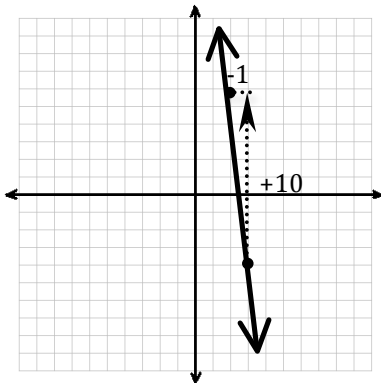
Let  $(x_1, y_1) = (-4, 6)$  and

Let  $(x_2, y_2) = (7, -3)$

$$\begin{aligned} \frac{\Delta y}{\Delta x} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(-3 - 6)}{(7 - (-4))} \\ &= \frac{-9}{11} \end{aligned}$$

You Try: Find the slope of the line through the two points  $(3,-4)$  and  $(2,6)$  three different ways.

Graphing:



$$\begin{aligned} \text{Slope is } \frac{\Delta y}{\Delta x} &= \frac{10}{-1} \\ &= -10 \end{aligned}$$

Change in values:

-1	<table style="border-collapse: collapse; text-align: center;"> <tr> <td style="border-right: 1px solid black; padding: 0 5px;"><math>x</math></td> <td style="padding: 0 5px;"><math>y</math></td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">3</td> <td style="padding: 0 5px;">-4</td> </tr> <tr> <td style="border-right: 1px solid black; padding: 0 5px;">2</td> <td style="padding: 0 5px;">6</td> </tr> </table>	$x$	$y$	3	-4	2	6	+10
$x$	$y$							
3	-4							
2	6							

$$\begin{aligned} \text{Slope is } \frac{\Delta y}{\Delta x} &= \frac{10}{-1} \\ &= -10 \end{aligned}$$

Using the slope formula

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Let  $(x_1, y_1) = (3, -4)$  and

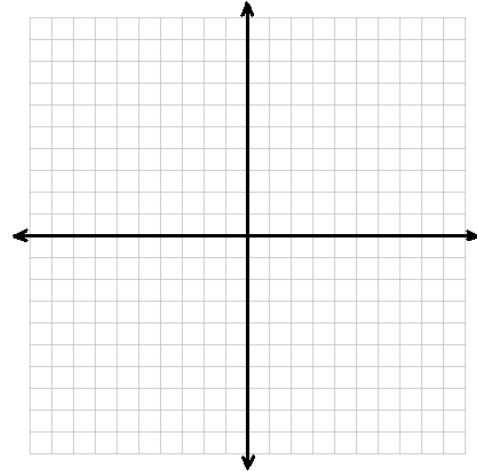
Let  $(x_2, y_2) = (2, 6)$

$$\begin{aligned} \frac{\Delta y}{\Delta x} &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{(6 - (-4))}{(2 - 3)} \\ &= \frac{10}{-1} \\ &= -10 \end{aligned}$$

**Student Handout:** Remember, graphs are read from the left to the right. Graph the given points on the coordinate plane. Looking at the pattern of given points, find the next point in the pattern.

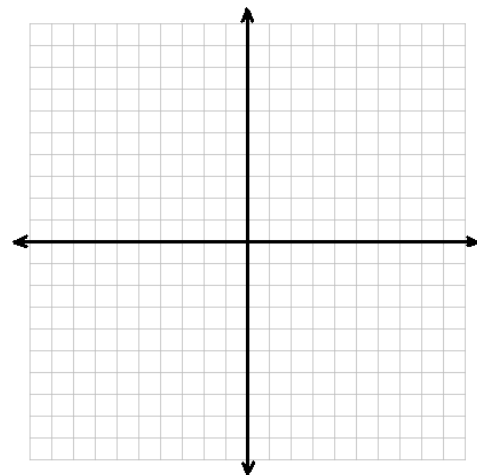
1) Graph the points:  $(-1,-2), (0,-1), (1,0)$

- What is the fourth point?
- What was your method to finding the point?
- What was a method of someone else in the class?



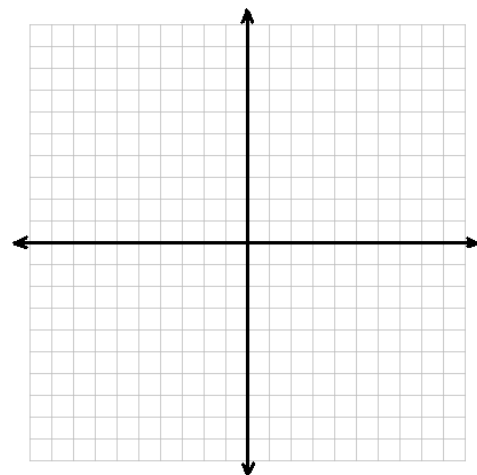
2) Graph the points:  $(-2,3), (-1,1), (0,-1)$

- What is the fourth point?
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3) Using only two points, graph the points and find the third:  $(2,1), (5,2)$

- What is the third point?
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- What was a method of someone else in the class?



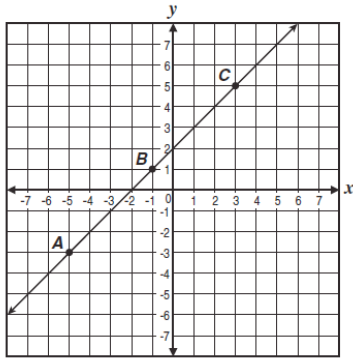
Let's look at our points in a table, what patterns do you see with the x-values? With the y-values?

	$x$	$y$			$x$	$y$			$x$	$y$	

# Warm-Up

<b>CST 7AF3.3</b>	<b>Review:</b>
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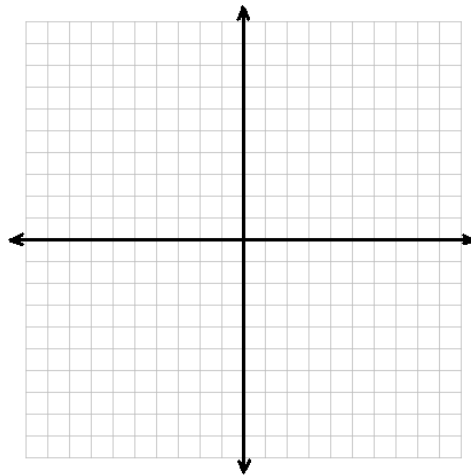
Which statement is true about the slope of line  $\overline{AC}$ ?



- A. The slope is the ratio of the  $x$ - and  $y$ -intercepts.
  - B. The slope is the same between any two points on the line
  - C. The slope between point  $A$  and point  $B$  is greater than the slope between point  $B$  and point  $C$ ?
  - D. The slope between point  $A$  and point  $C$  is greater than the slope between point  $A$  and point  $B$ .
- Why would a student choose answer C or D?

Graph the following points on the coordinate plane.

$(-1,3), (0,-4), (4,5), (-2,-6), (6,-3)$



<b>Current:</b>	<b>Other:</b>
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Evaluate and simplify  $\frac{a-b}{c-d}$  for the following values:

- a)  $a = 5, b = 4, c = 8, d = 3$
- b)  $a = -2, b = 4, c = 9, d = 0$
- c)  $a = 6, b = 9, c = 4, d = -3$

Find the pattern of each set of numbers below and find the next number in the pattern.

- a) 4, 6, 8,.....
- b) -4, 5, 14,....
- c) 3, -1, -5,....

**Today's Objective/Standards: 6MR2.4, 7AF3.3, Algebra 6.0**