UNIT 5 – LINEAR FUNCTIONS AND LINEAR INEQUALITIES IN TWO VARIABLES

PREREQUISITE SKILLS:

- students must know how to graph points on the coordinate plane
- students must understand ratios, rates and unit rate

VOCABULARY:

- relation: a set of ordered pairs
- function: a special relation that has a rule that establishes a mathematical relationship between two quantities, called the input and the output. For each input, there is exactly one output
- domain: the collection of all input values
- range: the collection of all output values
- independent variable: the variable in a function with a value that is subject to choice
- dependent variable: the variable in a relation with a value that depends on the value of the independent variable (input)
- function notation: a way to name a function that is defined by an equation. In function notation, the y in the equation is replaced with $f(x)$
- rate of change: a ratio that compares the amount of change in a dependent variable to the amount of change in an independent variable
- rise: the difference in the y-values of two points on a line
- run: the difference in the x-values of two points on a line
- slope: the ratio of rise to run for any two points on a line
- x-intercept: the x-coordinate of the point where the graph intersects the x-axis.
- y-intercept: the y-coordinate of the point where the graph intersects the y-axis.

SKILLS:

- find rates of change and slopes
- relate a constant rate of change to the slope of a line
- Graph linear equations that are in standard or slope-intercept form
- Graph linear inequalities in two variables
- Write linear equations in all three forms

STANDARDS:

F.IF.A.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. *(Modeling Standard)

A.REI.D.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).

F.IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima. *(Modeling Standard)

F.LE.A.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *(Modeling Standard)

F.IF.B.4-1 For a linear, exponential, or quadratic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. *(Modeling Standard)

F.IF.C.7a Graph linear and quadratic functions and show intercepts, maxima, and minima. *(Modeling Standard)

N.Q.A.1 Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.

F.IF.B.6-1 Calculate and interpret the average rate of change of a linear, exponential, or quadratic function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph of a function over a specified interval. *(Modeling Standard)

A.CED.A.2-1 Create linear, exponential, and quadratic equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. Limit exponentials to have integer inputs only. *(Modeling Standard)

A-CED.A.3-1 Represent constraints by linear equations or inequalities, and by systems of linear equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. *(Modeling Standard)

A.REI.D.12-1 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

LEARNING TARGETS:

- 5.1 To identify a linear function from a table, graph, or equation.
- 5.2 To use intercepts to graph linear functions in standard form.
- 5.3 To relate constant rate of change and slope in linear relationships.
- 5.4 To graph linear equations using slope intercept form.
- 5.5 To graph linear inequalities in two variables.
- 5.6 To create a linear equation in slope-intercept form.
- 5.7 To create a linear equation in point-slope form.
- 5.8 To create a linear equation in standard form.

BIG IDEA:

The concept of slope is important because it is used to measure the rate at which changes are taking place. In real-life problems, we often need to explore and understand how things change and about how one item changes in response to a change in another item.

Describe the similarities and differences between equations and inequalities – including solutions and graphs. Linear functions can be created from various forms of information. The solutions to real-word problems can be found by modeling them with equations and graphs.

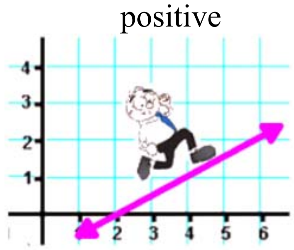
Notes, Examples and Exam Questions

<u>Units 5.1, 5.3 To identify a linear function from a table, graph, or equation and To relate constant rate of change and slope in linear relationships.</u>

The <u>slope</u> of a line is the constant rate of change occurring as you move along the line from left to right (the steepness of the line).

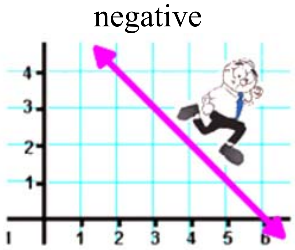
There are four types of slope:

positive



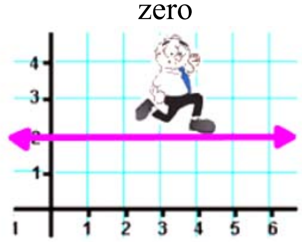
as x increases, y increases

negative



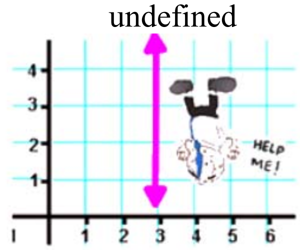
as x increases, y decreases

zero



as x increases, y is constant

undefined



x is constant

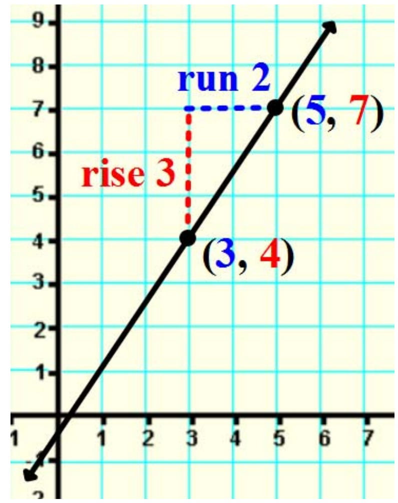
<u>Slope of a Line</u>: the number of units the line "rises" or "falls" for each unit of horizontal change.

$$m=\frac{\text{horizontal change}}{\text{vertical change}}=\frac{\text{rise}}{\text{run}}=\frac{\text{change in } y}{\text{change in } x}=\frac{y_1-y_2}{x_1-x_2}=\frac{y_2-y_1}{x_2-x_1}$$ where (x_1,y_1) and (x_2,y_2) are two points on the line. Note: Slope is a *rate of change*. It determines the steepness of a line.

Finding slope given a graph:
Ex 1: Find the slope of the given line.

Step One: find two points that fall on the line.

Step Two: find the ratio of vertical change to horizontal change (from left to right) between the two points.



$$m = \frac{rise}{run} = \frac{vertical\ change}{horizontal\ change}$$

$$m = \frac{rise}{run} = \frac{3}{2} = \frac{3}{2}$$

Find the slope of a line using Slope Formula.
If you don't have a graph of the line, you can find the slope of the line using the slope formula when given two points; (x_1, y_1) and (x_2, y_2):

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Ex 2: Find the slope of a line that goes through the points (**3**, **4**) and (**5**, **7**).

Substitute in the given values in the ordered pairs into the slope formula and simplify.

$$m = \frac{change\ in\ y}{change\ in\ x} = \frac{7-(4)}{5-(3)} = \frac{3}{2} = \frac{3}{2}$$

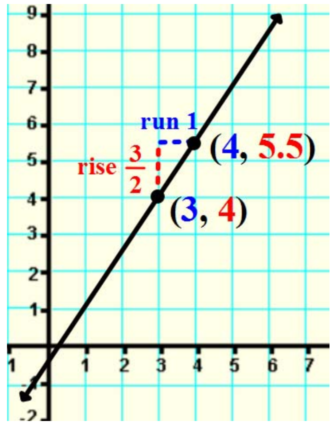
Note: Do not express slope as a mixed number. Leave it in simplified fraction form.

***How is the slope like unit rate?**

Any slope is a ratio comparing the change in *y* to the change in *x*. A rate is a unit rate if it has a denominator of 1. In the examples above, the slope is $\frac{3}{2}$; which tells us that as *y* increases by three units, *x* increases by two units. But, another way to look at is that as *y* increases by $\frac{3}{2}$ units, *x* increases by one unit.

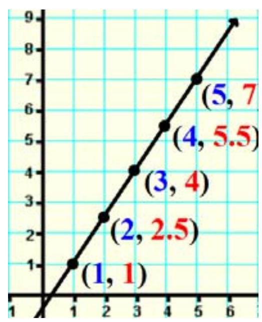
Let's look at the line in the first example.

As *x* increases by 1 unit (from 3 to 4), *y* increases by 1.5 units (from 4 to 5.5)



Let's look at a table of values for some points that fall on the graph of that linear function. Notice that each time *x* increases by 1 unit, *y* increases by 1.5 units.

x	*y*
1	1
2	2.5
3	4
4	5.5
5	7



The slope is the unit rate of the function. It shows the rate of change vertically, as the graph moves one unit to the right.

Ex 3: Use the table of values to find the rate of change and then explain its meaning.

number of video games	**total cost ($)**
x	***y***
2	**78**
4	**156**
6	**234**

Step One: select any two points to use in the slope formula.

$\Rightarrow$ (2, 78) and (4, 156)

Step Two: calculate the slope $\Rightarrow$ $m = \dfrac{156-(78)}{4-(2)} = \dfrac{78}{2} = 39$

Step Three: explain what it means in the context of the problem.

It means that $39 is the cost per game or the unit rate.

Ex 4: Determine if the function is linear. Explain your answer.

x	y
−3	10
−1	12
1	16
3	18

Step One: Pick two points and find the slope between them.

$\Rightarrow$ (−3, 10) and (−1, 12)

$$\Rightarrow \quad m=\frac{12-(10)}{-1-(-3)}=\frac{2}{2}=1$$

Step Two: Pick two other points and find the slope between them.

$\Rightarrow$ (−1, 12) and (1, 16)

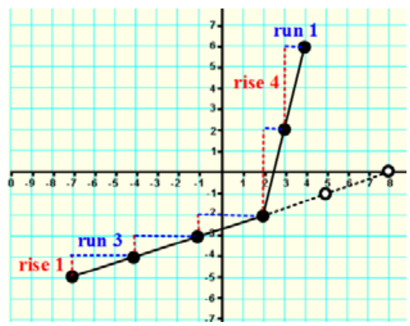
$$\Rightarrow \quad m=\frac{16-(12)}{1-(-1)}=\frac{4}{2}=2$$

Step Three: Determine if it's linear. It is not linear because the slope is not constant.

Why does the slope have to be constant in a linear equation?

Because, if the slope is not constant than it is not a linear function. Let's look at that on a graph.

Notice that if we start at the point (−7, −5) and follow a slope of $\frac{1}{3}$, it takes us to the point (−4, −4). If we follow the same slope again, we get to the point (−1, −3) and then to the point (2, −2). All of these points fall on a straight line. But, if we change the slope (pattern) to $\frac{4}{1}$, we do not get to a point that falls on the same line as all of the other points.



Comparing slopes:

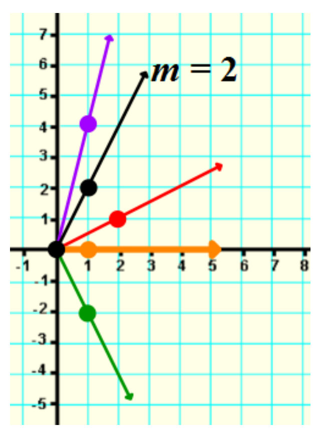
Ex 5: Look at the graph below. Using the line with a slope of 2 as the original line, describe how the steepness of the line changes as the slope changes.

$m=\frac{4}{1}$ *The line is much steeper than the original line. It is rising 4 times quicker while running the same amount.

$m=\frac{1}{2}$ *The line is less steep than the original line. It is rising half as quickly, and running twice as far.

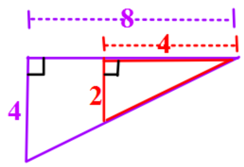
$m=0$ *The line is flat. It is not rising at all as it runs to the right.

$m=-\frac{2}{1}$ *The line has the same steepness as the original line, except it is going downward.



Relating Slope to Similar Triangles:

Similar triangles are proportional. *All corresponding ratios are proportional.

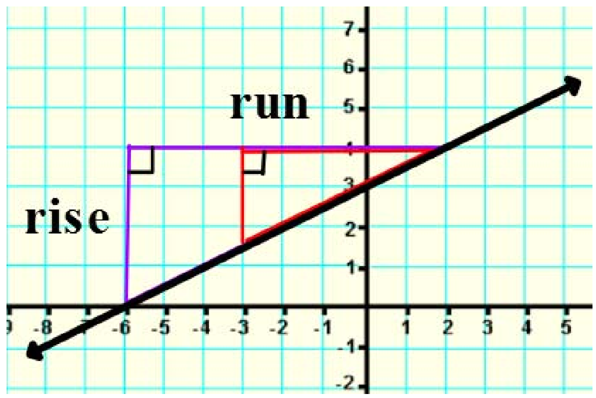


4 is to 8 as 2 is to 4

$$\frac{4}{8} = \frac{2}{4}$$

Let's put the similar triangles on a coordinate plane. Notice that both triangles' hypotenuses fall on the same straight line. The legs' of each triangle are like the rise and the run of the slope of the line. The purple triangle's rise is 4 and its run is 8. The red triangle's rise is 2 and its run is 4. Which means that both slopes reduce to an equivalent slope of $\frac{1}{2}$.

*Using similar triangles we can prove that no matter what two points we choose on a line, we will always find the same slope between those points.



Ex 6: Find the value of y so that the line passing through the points $(-2,1)$ and $(4, y)$ has a slope of $\frac{2}{3}$.

Step One: write the slope formula; $m = \frac{y_2 - y_1}{x_2 - x_1}$

Step Two: plug in the given values into the formula; $\frac{2}{3} = \frac{y-(1)}{4-(-2)} \Rightarrow \frac{2}{3} = \frac{y-1}{6}$

Step Three: solve for y; $6 \cdot \frac{2}{3} = \frac{y-1}{6} \cdot 6 \Rightarrow 4 = y - 1 \Rightarrow y = 5$

Ex 7: Find the slope of the line that passes through the points $(-2,3)$ and $(-1,-4)$.

We will use the slope formula $m=\dfrac{y_1-y_2}{x_1-x_2}$, where $(x_1,y_1)=(-2,3)$ and $(x_2,y_2)=(-1,-4)$.

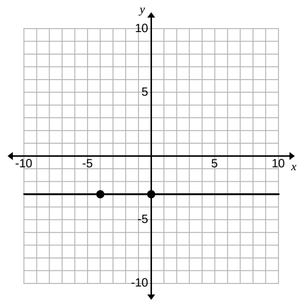
Substituting into the formula: $m=\dfrac{3-(-4)}{-2-(-1)}=\dfrac{3+4}{-2+1}=\dfrac{7}{-1}=\boxed{-7}$

On Your Own: If you change the order of the points, will you get the same slope? Try it!

Review: Find the quotients. $\dfrac{0}{4}=?$ $\dfrac{3}{0}=?$

Note: $\dfrac{0}{4}=0$ because $4\cdot 0=0$. So to find $\dfrac{3}{0}$, we need to find a number, ?, such that $0\cdot ?=3$. This is impossible, so we say that $\dfrac{3}{0}$ is "undefined" or "does not exist".

<u>"Special" Slopes – Slopes of Horizontal and Vertical Lines</u>



Ex 8: Find the slope of the line in the graph.

Step One: Find two points on the line. (See graph.)

Step Two: Start at one of the points. Count how many units up or down you would have to "step" to get to the other point. This is your "rise". Note: If you go down, the rise is *negative*; if you go up, the rise is *positive*.

Starting at the point on the right, we would have to step down 0 units, therefore the "rise" is 0.

Step Three: Now determine how many units right or left you would have to "step" to get to the other point. This is your "run". Note: If you go left, the run is *negative*; if you go right, the run is *positive*.

We would have to step right 4 units, therefore the "run" is -4.

Step Four: Write the slope as a fraction, $\dfrac{\text{rise}}{\text{run}}$. The slope is $\dfrac{0}{-4}=\boxed{0}$

Note: **All horizontal lines have a slope of 0**. If a car was driving from left to right, it would be going neither uphill nor downhill. Therefore, the slope is neither positive nor negative.

Ex 9: Find the slope of the line passing through the points $(5,6)$ and $(5,2)$.

We will use the slope formula $m=\frac{y_1-y_2}{x_1-x_2}$, where $(x_1,y_1)=(5,6)$ and $(x_2,y_2)=(5,2)$.

Substituting into the formula: $m=\frac{6-2}{5-5}=\frac{4}{0}$, so the slope is undefined.

⚠ Note: The points in the example above will graph a vertical line. **The slope of a vertical line is "undefined"**. If a car tried to drive on a vertical line it would crash!

Application of Slope: Rates of Change

Ex 10: In 1970, the price of a movie ticket at a particular theater was $1.50. In 1990 the price of a movie ticket at the same theater was $6.00. What is the rate of change of the cost of a movie ticket? Use correct units in your answer.

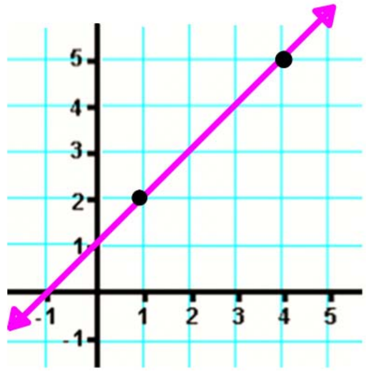
Using the slope formula, we will find the change in the cost of a ticket over the change in time.

$$\frac{\text{change in cost (\$)}}{\text{change in time (years)}}=\frac{6.00-1.50}{1990-1960}=\frac{4.50}{30}=0.15\text{ dollars per year}$$

QOD: Show algebraically using the slope formula why the slope of a horizontal line is 0 and the slope of a vertical line is undefined.

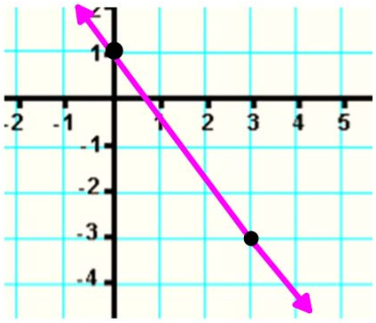
Sample Exam Questions

1. Find the slope of the line.



ANS: $\frac{rise}{run}=\frac{3}{3}=1$

2. Find the slope of the line.



ANS: $\frac{rise}{run} = \frac{-4}{3} = -\frac{4}{3}$

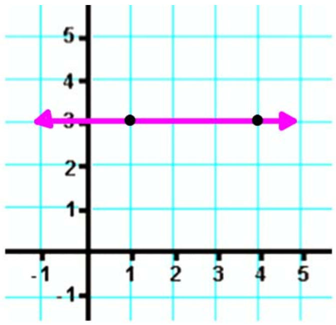
3. What is the slope of a line that goes through the points (−2, 5) and (6, 1)?
   A. −2
   B. $-\frac{1}{2}$
   C. $\frac{1}{2}$
   D. 2

ANS: B

4. What is the slope of a line that goes through the points (5, 0) and (7, 8)?
   A. −4
   B. $-\frac{1}{4}$
   C. $\frac{1}{4}$
   D. 4

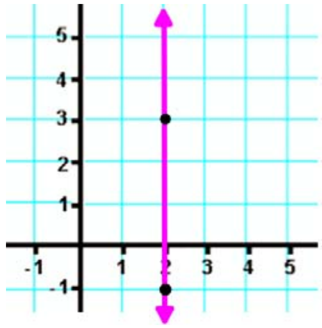
ANS: D

5. Find the slope of the line.



ANS: $\frac{rise}{run} = \frac{0}{3} = 0$

6. Find the slope of the line.



ANS: $\frac{rise}{run} = \frac{4}{0} = undefined$

7. What is the slope of a line that goes through the points (6, 3) and (6, −1)?

A. $\frac{1}{3}$
B. 3
C. 0
D. undefined

ANS: D

8. What is the slope of a line that goes through the points (1, 5) and (4, 5)?

A. $\frac{1}{3}$
B. 3
C. 0
D. undefined

ANS: C

9. Given the table of values below, determine whether or not the points all fall on a straight line. Explain your answer.

x	*y*
−2	1
1	3
4	5
7	7

ANS: Yes, it is linear. The slope is constant from point to point and equals $\frac{2}{3}$.

10. Find the value of y so that the line passing through the points (0,3) and (4,y) have a slope of −3.
    A. −15
    B. −9
    C. 9
    D. 15

ANS: D

11. Use the table of values to find the rate of change and then explain its meaning.

driving time (h)	distance traveled (m)
2	76
4	152
6	228

ANS: 38/1, which means the car is traveling at a speed of 38 mph.

12. Use the table of values to find the rate of change and then explain its meaning.

number of floor tiles	Area of tiled surface (in^2)
x	y
3	48
6	96
9	144

ANS: 16/1, which means that each floor tile is 16 in^2 .

13. When driving down a certain hill, you descend 15 feet for every 1000 feet you drive forward. What is the slope of the road?

ANS: $-\frac{3}{200}$

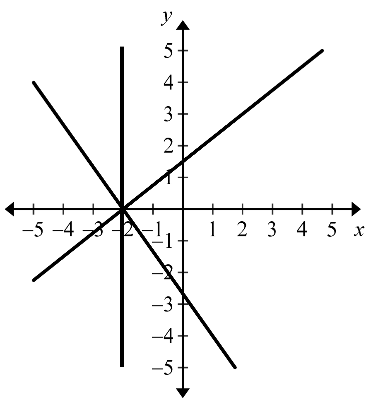
14. The point (−1, 8) is on a line that has a slope of −3. Is the point (4, −7) on the same line? Explain your reasoning.

ANS: Yes, the slope between the two points is also −3.

15. In 1996, a company had a profit of $153,000,000. In 2002, the profit was $186,000,000. If the profit increased the same amount each year, find the average rate of change of the company's profit in dollars per year.

ANS: $5,500,000 per year

16. Which statement is true about the characteristics of the linear functions in the graph below?



(A) They are the same function.

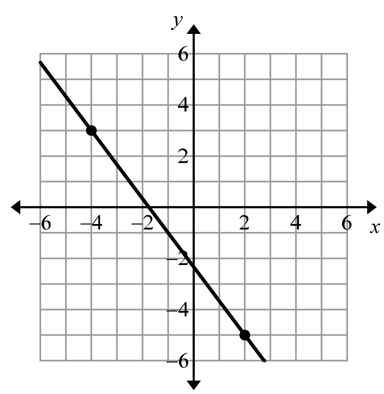
(B) They have the same slope.

(C) They have the same *x*-intercepts.

(D) They have the same *y*-intercepts.

ANS: C

17. Find the slope of the line in the graph.



(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

ANS: A

18. Find the slope of the line that contains the points $(7,-6)$ and $(4,-5)$.

(A) -3

(B) $-\frac{1}{3}$

(C) $\frac{1}{3}$

(D) $\frac{11}{3}$ ANS: B

19. Find the slope of the line that contains the points (8, 3) and (8, 8).

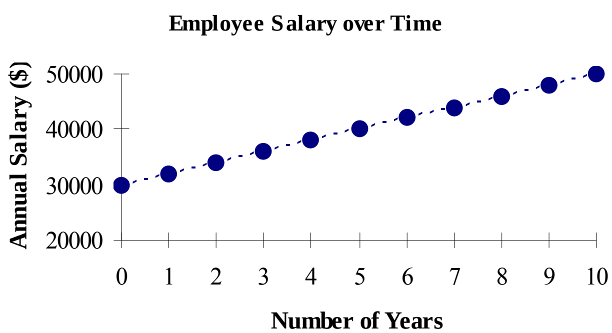
(A) $-\frac{5}{11}$

(B) 0

(C) $\frac{5}{16}$

(D) undefined ANS: D

20. A job pays a base salary of $30,000. Each year an employee will earn an additional $2,000. The graph of an employee's salary over 10 years is shown below.



What would happen to the graph if the base salary was changed from $30,000 to $40,000?

(A) The graph would translate down.

(B) The graph would translate up.

(C) The graph would rise less steeply from left to right.

(D) The graph would rise more steeply from left to right. ANS: B

For questions 21 and 22, use the table.

x	3	5	8	12	17
y	12	16	22	30	40

21. The ordered pairs (x, y) form a linear function.

(A) True ANS: A

(B) False

22. The value of y changes by increasingly larger amounts for each change of 1 in x.

(A) True ANS: B

(B) False

<u>Unit 5.4 To graph linear equations using slope intercept form.</u>

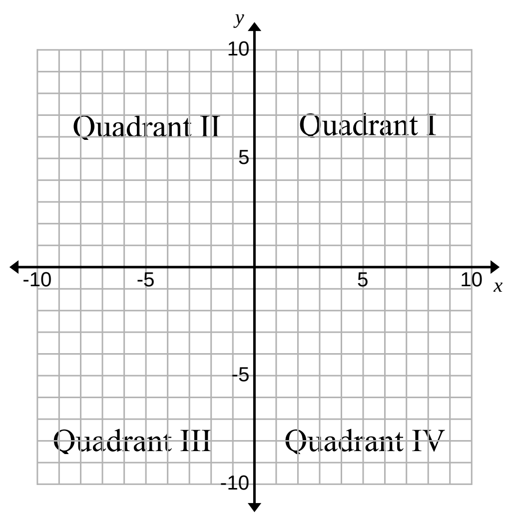
<u>Coordinate Plane</u> – a plane formed by two real number lines (axes) that intersect at a right angle

<u>Horizontal Axis</u> – x-axis <u>Vertical Axis</u> – y-axis

<u>Ordered Pair</u> – the coordinates of a point (x, y)

<u>Quadrants</u> – the four sections of the plane formed by the axes

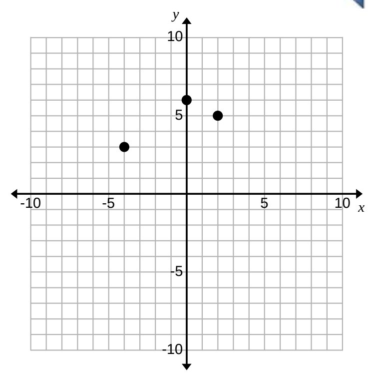
<u>The Cartesian (coordinate) Plane</u>:



<u>Origin</u> – the point $(0, 0)$

<u>Plotting Points on the Coordinate Plane</u>

Ex 1: Plot the points $A(2,5)$, $B(-4,3)$, and $C(0,6)$.



△Note: Point *A* is in Quadrant I, *B* is in Quadrant II, and *C* is on the *y*-axis.

<u>Solution of a Linear Equation in Two Variables</u> – an ordered pair, (x, y), that makes the equation true.

<u>Graph of a Linear Equations in Two Variables</u> – the set of all points, (x, y), that are solutions to the equation. The graph will be a line.

Ex 2: Is the point $(-4,3)$ a solution to the linear equation $-x+2y=10$?

Substitute the *x*- and *y*-coordinates of the point into the equation.

$$-(-4)+2(3)=10$$
$$4+6=10 \quad \text{true}$$

Because the values satisfy the equation, the ordered pair **is** a solution.

Graphing a Linear Equation

To graph a linear equation, we will choose values for the **independent variable**, *x*, and substitute these values into the equation to find the corresponding values for the **dependent variable**, *y*. It is helpful to organize the ordered pairs in a table (or "t-chart").

Ex 3: Graph the equation $3x+y=4$.

Step One: Solve the equation for *y*.

$$3x+y=4$$
$$-3x \quad -3x$$
$$y=-3x+4$$

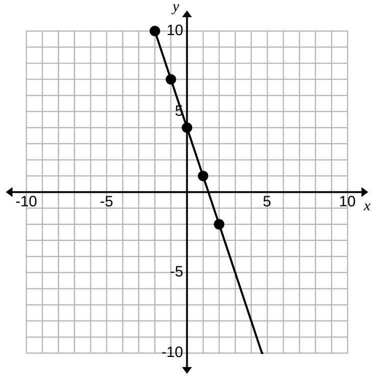
Step Two: Choose at least 5 values for *x*. Include 0, two positive integers, and two negative integers.

We will choose $-2,-1,0,1,2$.

Step Three: Make a table and substitute each *x*-value into the equation to find the corresponding *y*-value.

x	$y=-3x+4$
-2	$-3(-2)+4=10$
-1	$-3(-1)+4=7$
0	$-3(0)+4=4$
1	$-3(1)+4=1$
2	$-3(2)+4=-2$

Step Four: Plot each ordered pair from the table and connect to form a line.



Ex 4: Graph the linear function $2y=4-x$.

Step One: Solve the equation for *y*.

$$\frac{2y}{2}=\frac{4}{2}-\frac{x}{2}$$
$$y=2-\frac{1}{2}x$$

Step Two: Choose at least 5 values for *x*. Include 0, two positive integers, and two negative integers.

Because we are multiplying *x* by the fraction $\frac{1}{2}$, we will choose *x*-values that are multiples of 2.

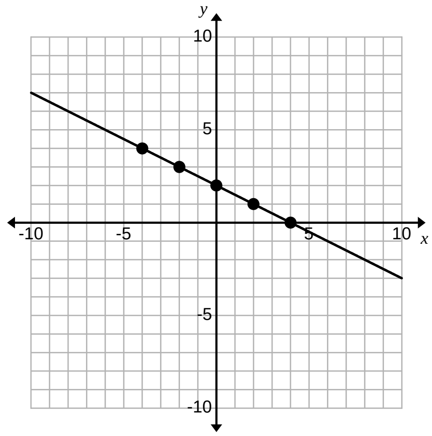
$-4,-2,0,2,4$

<u>Teacher Note</u>: Explain to students that it is easier to evaluate when we use *x*-values that are multiples of the denominator.

Step Three: Make a table and substitute each *x*-value into the equation to find the corresponding *y*-value.

x	$y = 2 - \frac{1}{2}x$
-4	$2 - \frac{1}{2}(-4) = 2 + 2 = 4$
-2	$2 - \frac{1}{2}(-2) = 2 + 1 = 3$
0	$2 - \frac{1}{2}(0) = 2 + 0 = 2$
2	$2 - \frac{1}{2}(2) = 2 - 1 = 1$
4	$2 - \frac{1}{2}(4) = 2 - 2 = 0$

Step Four: Plot each ordered pair from the table and connect to form a line.



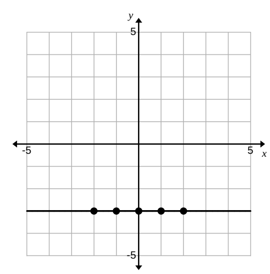
Ex 5: Graph the line $y = -3$.

x	-2	-1	0	1	2
y	-3	-3	-3	-3	-3

This equation is already solved for *y*, so we will simply make a table.

Note that the *y* value is always −3, regardless of the value of x.

Plotting the points and drawing the line, we see that the result is a **horizontal line.**

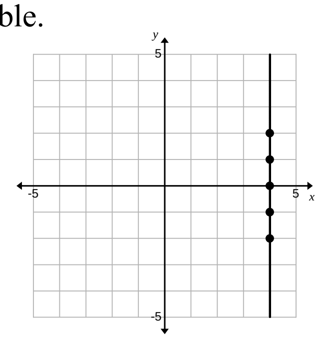


Ex 6: Graph the line $x = 4$.

x	4	4	4	4	4
y	-2	-1	0	1	2

This equation has no *y*, and the *x*-coordinate must equal 5. Because *y* is not restricted as part of the equation, we can choose any value for *y* in our table.

Plotting the points and drawing the line, we see that the result is a **vertical line**.



Note: This is the graph of a line, but it is not a function.

<u>Equation of a Vertical Line</u>: $x = a$, where a is a real number

<u>Equation of a Horizontal Line</u>: $y = b$, where b is a real number

<u>You Try:</u>

1. Use the linear equation $3x - \frac{1}{2}y = 1$. Is the ordered pair $(1, -4)$ a solution to the equation? Find five points that are solutions to the equation and graph the line.
2. Graph the lines $x = -2$ and $y = 5$. At what point do these two lines intersect?

<u>QOD:</u> Why isn't a vertical line a function?

Activity: Graph the following lines on the graphing calculator. Write down the slope and y-intercept of each line. Then, compare the steepness of the line to the parent function $y = x$.

To graph on the calculator, enter the function into the Y= screen. Use a Zoom Standard window.

1. $y = x$	Slope: $m = 1$	y-intercept = 0	
2. $y = 2x - 3$	Slope: $m = 2$	y-intercept = -3	steeper than $y = x$
3. $y = \frac{2}{3}x + 4$	Slope: $m = \frac{2}{3}$	y-intercept = 4	$y = x$ is steeper
4. $y = -\frac{1}{3}x + 5$	Slope: $m = -\frac{1}{3}$	y-intercept = 5	$y = x$ is steeper
5. $y = 4x - 1$	Slope: $m = 4$	y-intercept = -1	steeper than $y = x$
6. $y = -x$	Slope: $m = -1$	y-intercept = 0	steepness same as $y = x$

Write down your conclusions: The coefficient of x is the slope, and the constant term is the y-intercept.

<u>Slope-Intercept Form of a Linear Equation</u>: $y = mx + b$, $m =$ slope, $b = y$-intercept

Ex 7: Write the equation of the line $2x - 5y = 10$ in slope-intercept form. Identify the slope and y-intercept.

$$-5y = -2x + 10$$

To write in slope-intercept form, solve for y. $\boxed{y = \frac{2}{5}x - 2}$

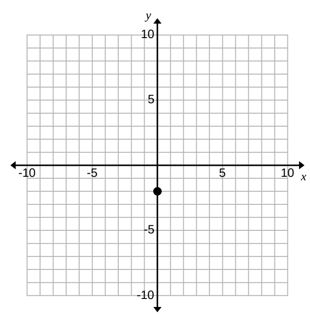
The slope is $\frac{2}{5}$, and the y-intercept is -2.

<u>Graphing a Line in Slope-Intercept Form</u>:

1. Plot the y-intercept.
2. Starting at the y-intercept, "step" to the next point on the line using the slope, $\frac{\text{rise}}{\text{run}}$.
3. "Begin at b, move m."

⚠ Note: If the slope is POSITIVE – you may rise "up" and run "right" $\left(\frac{+}{+}\right) = +$, or you may rise "down" and run "left" $\left(\frac{-}{-}\right) = +$. If the slope is NEGATIVE – you may rise "up" and run "left" $\left(\frac{+}{-}\right) = -$, or you may rise "down" and run "right" $\left(\frac{-}{+}\right) = -$.

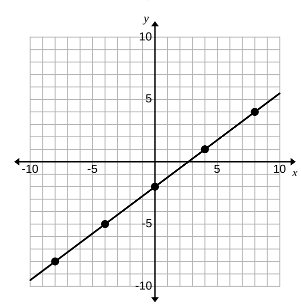
Ex 8: Graph the line $y = \frac{3}{4}x - 2$.



Step One: Identify the slope and y-intercept. $m = \frac{3}{4}$ y-intercept $= -2$

Step Two: Plot a point at the y-intercept.

Step Three: "Step" out the slope to plot another point. Do this a couple of times so that you can draw an accurate line.



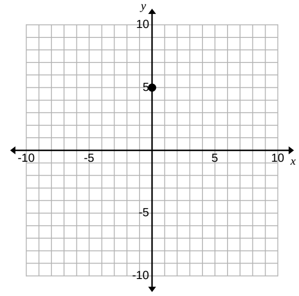
Note: Because the slope is positive, we can rise up 3 and run right 4, or rise down 3 and run left 4.

Draw the line connecting the points.

Ex 9: Graph the line $f(x) = -x + 5$.

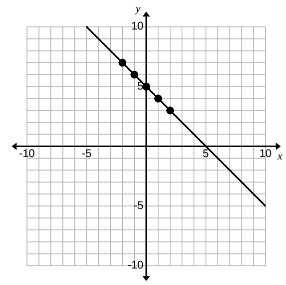
Step One: Identify the slope and y-intercept. $m = -1 = -\frac{1}{1}$ y-intercept = 5

Step Two: Plot a point at the y-intercept.



Step Three: "Step" out the slope to plot another point. Do this a couple of times so that you can draw an accurate line.

Note: Because the slope is negative, we can rise up 1 and run left 1, or rise down 1 and run right 1.



Draw the line connecting the points.

Note: Sometimes you must put the equation in slope-intercept form first.

Ex 10: Graph the equation of the line $y = \frac{4-x}{2}$ using slope-intercept form.

Step One: Write the equation in slope-intercept form.

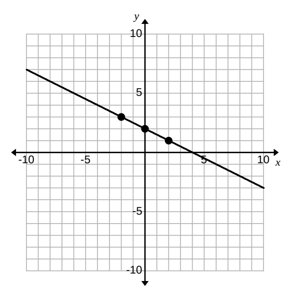
$$y = 2 - \frac{x}{2}$$

$$y = -\frac{1}{2}x + 2$$

Step Two: Identify the slope and y-intercept.

$m = -\frac{1}{2}$ y-intercept = 2

Step Three: Graph the line by plotting the y-intercept and then "stepping" out the slope.



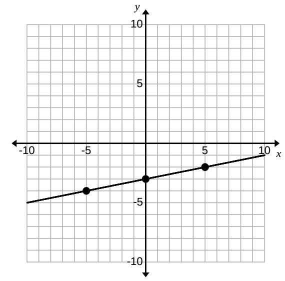
Ex 11: Graph the equation of the line $y = -3 + 0.2x$ using slope-intercept form.

Step One: Write the equation in slope-intercept form.

$$y = 0.2x - 3$$
$$y = \frac{1}{5}x - 3$$

Step Two: Identify the slope and *y*-intercept. $m = \frac{1}{5}$ *y*-intercept $= -3$

Step Three: Graph the line by plotting the *y*-intercept and then "stepping" out the slope.



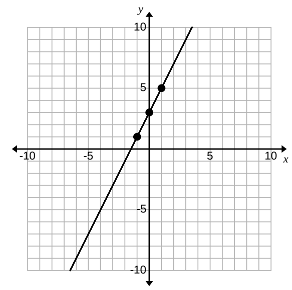
Ex 12: Graph the equation of the line $2x + 3y - 4 = 8x + 5$ using slope-intercept form.

Step One: Write the equation in slope-intercept form.

$$3y = 6x + 9$$
$$y = 2x + 3$$

Step Two: Identify the slope and *y*-intercept. $m = 2 = \frac{2}{1}$ *y*-intercept $= 3$

Step Three: Graph the line by plotting the *y*-intercept and then "stepping" out the slope.



Ex 13: Find the slope and *y*-intercept of the line $y = 5$. Describe the graph.

Slope = 0, *y*-intercept = 5

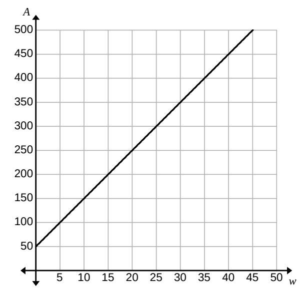
The graph will be a horizontal line that intersects the *y*-axis at 5.

Application Problems Using Slope-Intercept Form

Ex 14: Jack has \$50 in his savings account, and plans to save \$10 per week. Write an equation in slope-intercept form that represents the amount, *A*, Jack will have in his account after *w* weeks. Graph the equation in an appropriate window.

Amount = 10 × Number of Weeks + 50

$A = 10w + 50$



On Your Own: What does the slope and *y*-intercept represent on the graph in relation to the problem?

<u>You Try:</u> Graph the linear equation $y - 8 = -\frac{1}{2}(x + 6)$ using slope-intercept form.

<u>QOD:</u> Can you write the equation of a vertical line in slope-intercept form? Explain.

Sample Exam Questions

1. Use the graph below.

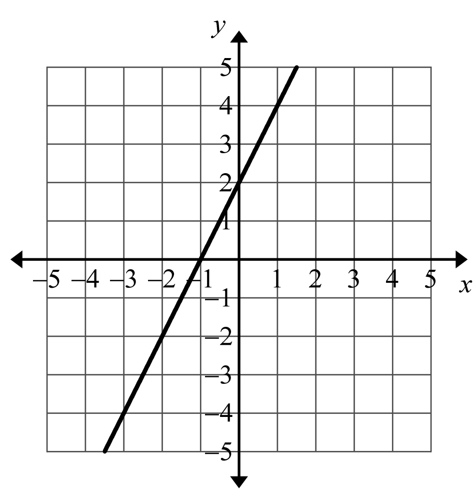
   What is the equation of the line in the graph?

   (A) $y = -2x - 1$

   (B) $y = -2x + 2$

   (C) $y = 2x - 1$

   (D) $y = 2x + 2$



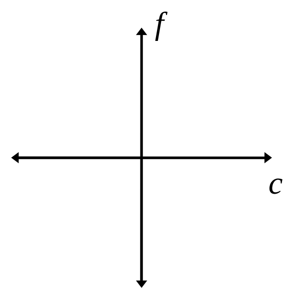
Ans: D

2. What is the equation of the horizontal line through the point (4, –7)?

(A) $x = 4$
(B) $x = -7$
(C) $y = 4$
(D) $y = -7$

Ans: D

3. When the function $f = k + ac$ is graphed on the axes shown, what quantity corresponds to the intercept on the vertical axis?



(A) f
(B) k
(C) $f - k$
(D) $\frac{f-k}{a}$

Ans: B

4. A line is defined by the equation $y = \frac{2}{5}x + 3$. Which ordered pair does NOT represent a point on the line?

(A) (–5, 0)
(B) (0, 3)
(C) $(1, \frac{17}{5})$
(D) (5, 5)

Ans: A

5. A certain child's weight was measured at 16.6 pounds. The child then gained weight at a rate of 0.65 pounds per month. On a graph of weight versus time, what would $0.65 \frac{\text{pounds}}{\text{month}}$ represent?

(A) The y-intercept of the graph
(B) The x-intercept of the graph
(C) The slope of the graph

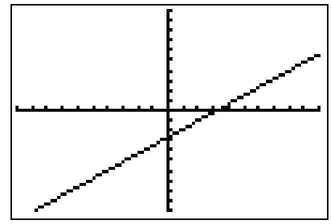
Ans: C

Graphing Calculator Activity: Solving linear equations on the graphing calculator.

Ex 15: Solve the equation $5 = 0.83x + 2.3$ graphically on the calculator.

Step One: Rewrite the equation in the form $ax + b = 0$.

$$5 = 0.83x + 2.3$$
$$0 = 0.83x - 2.7$$



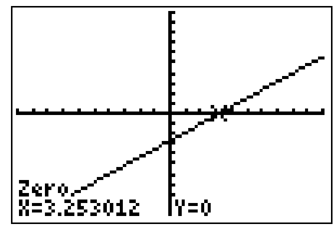
Step Two: Graph the line $y = 0.83x - 2.7$. Enter the line in the Y= screen..

Step Three: Find the zero using the CALC Menu. Keystrokes:

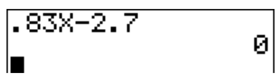
2nd TRACE 2 1 ENTER 7 ENTER 3 ENTER

Note: You may enter in values for the bounds and the "guess", or use the left and right arrows.

Solution: $\boxed{x \approx 3.253}$



Step Four: Check your answer on the home screen. Your calculator has the value of the zero stored as *x*.

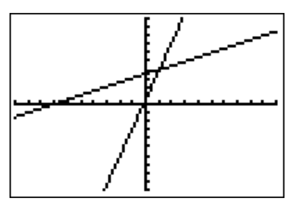


Alternate Method:

Ex 16: Solve the equation $\frac{1}{2}(x+7) = \frac{1}{3}(10x+2)$

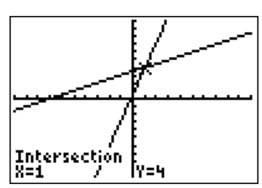
Because it is tedious to rewrite this equation in the form $ax + b = 0$ by hand, we will use an alternate method on the graphing calculator.

Step One: Graph each side of the equation as two separate lines.



$$y = \frac{1}{2}(x+7) \text{ and } y = \frac{1}{3}(10x+2)$$

Step Two: Find the point of intersection. This is the solution to the equation $\frac{1}{2}(x+7) = \frac{1}{3}(10x+2)$. Keystrokes: 2nd TRACE 5 ENTER ENTER ENTER



Note: The "guess" should be near the point of intersection. The *x*-coordinate is the solution.

$\boxed{x=1}$

Checking this by hand algebraically, we have

$$\frac{1}{2}(x+7)=\frac{1}{3}(10x+2) \qquad \frac{1}{2}(8)=\frac{1}{3}(12)$$
$$\frac{1}{2}(1+7)=\frac{1}{3}(10(1)+2) \qquad 4=4 \quad \text{True}$$

<u>Application Problem</u>

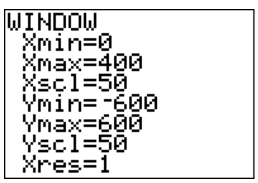
Ex 17: A small business makes and delivers box lunches. They calculate their average weekly cost *C* of delivering *b* lunches using the function $C=2.1b+75$. Last week their cost was \$600. How many lunches did they make last week? Solve algebraically and graphically.

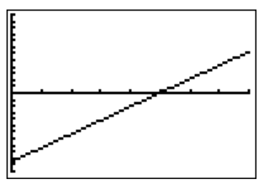
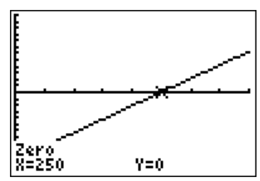
Because their cost was \$600, we will substitute this value in for *C*. $600=2.1b+75$

Solve for *b*.

$$525=2.1b$$
$$250=b$$

Solving graphically, we will rewrite the equation as $0=2.1b-525$, graph the line



$y=2.1x-525$, and find the zero.

Solution : The business made 250 lunch boxes last week.

<u>You Try:</u> Solve the equation $-9=6x-12$ graphically and check your solution algebraically.

<u>QOD:</u> What are three different terms to describe the graphical solution to a linear equation in the form $ax+b=0$?

<u>Unit 5.2 To use intercepts to graph linear functions in standard from</u>

<u>*x*-Intercept</u> – the *x*-coordinate of the point where the graph intersects the *x*-axis (Note: The *y*-coordinate of the *x*-intercept is 0.)

<u>*y*-Intercept</u> – the *y*-coordinate of the point where the graph intersects the *y*-axis (Note: The *x*-coordinate of the *y*-intercept is 0.)

Ex 1: Find the x- and y-intercept of the graph of the equation $2x-3y=9$.

Finding the x-intercept: Let $y=0$ and solve for x.

$$2x-3(0)=9$$
$$2x=9$$
$$x=\boxed{\frac{9}{2}}$$

Finding the y-intercept. Let $x=0$ and solve for y.

$$2(0)-3y=9$$
$$-3y=9$$
$$y=\boxed{-3}$$

⚠ Note: The line crosses the x-axis at the point $\left(\frac{9}{2},0\right)$ and the y-axis at the point $(0,-3)$.

Sketching the Graph of a Line Using Intercepts

Ex 2: Sketch the graph of the line $-5x+3y=15$.

To sketch a line, we only need two points. We will use the intercepts as the two points.

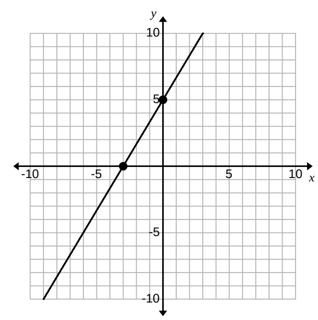
Step One: Find the x-intercept.

$$-5x+3(0)=15$$
$$x=-3$$

Step Two: Find the y-intercept.

$$-5(0)+3y=15$$
$$y=5$$

Step Three: Plot the intercepts on the axes and draw the line.

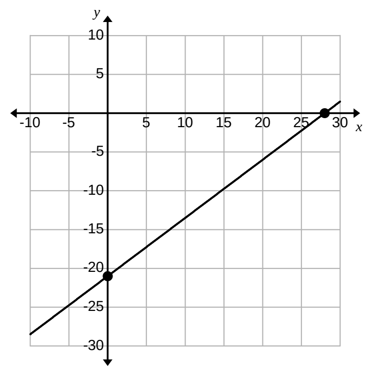


Ex 3: Sketch the graph of the line $f(x)=\frac{3}{4}x-21$.

To sketch a line, we only need two points. We will use the intercepts as the two points.

Step One: Find the x-intercept.

$$0=\frac{3}{4}x-21$$
$$21=\frac{3}{4}x$$
$$28=x$$



Step Two: Find the *y*-intercept.

$$y = \frac{3}{4}(0) - 21$$
$$y = -21$$

Step Three: Plot the intercepts on the axes and draw the line. Note: Use a "friendly" scale to view the graph.

<u>Application Problem: Using a Linear Model</u>

Ex 4: Adult tickets to a football game cost \$5, and student tickets cost \$3. A school collects \$3375 at Friday night's game. Write an equation and draw a line that represents the possible number of adult tickets, *a*, and student tickets, *s*, that were sold at the game.

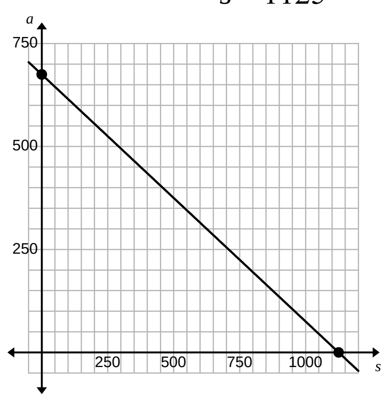
Write the equation: $5a + 3s = 3375$

Find the intercepts:

$5(0) + 3s = 3375$ $5a + 3(0) = 3375$

$s = 1125$ $a = 675$

Draw the graph:



Note: Every whole-number point on the graph of the line are ordered pairs that represent the possible number of adults and students who purchased a ticket for the football game.

<u>You Try:</u> Find the *x*- and *y*-intercepts of the graph of the equation $0.2x - 6y = 12$. Graph the line.

<u>QOD</u>: Describe the graph of a line that has no *x*-intercept. Write an equation of a line with no *x*-intercept.

Sample Exam Questions

1. What are the *x*- and *y*-intercepts of $9x - y = 3$?

   (A) *x*-intercept = 9; *y*-intercept = –3

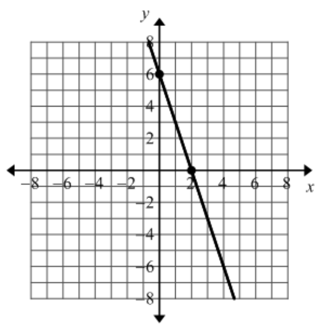
   (B) *x*-intercept = $\frac{1}{3}$; *y*-intercept = –3

   (C) *x*-intercept = –3; *y*-intercept = $\frac{1}{3}$

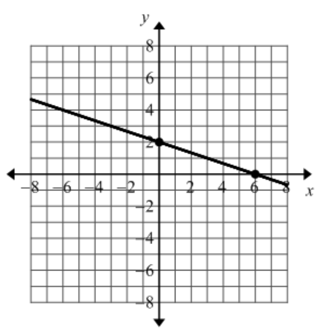
   (D) *x*-intercept = $\frac{1}{3}$; *y*-intercept = 9 Ans: B

2. Graph the equation: $6x + 2y = 12$.

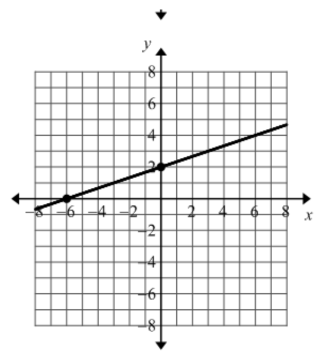
A.



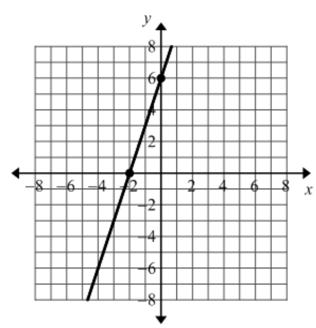
B.



C.



D.



Ans: A

3. What are the intercepts of the line with equation 2*x* – 3*y* = 30?

   (A) (–10, 0) and (0, 15)

   (B) (6, 0) and (0, –6)

   (C) (15, 0) and (0, –10)

   (D) (30, 0) and (0, –30) Ans: C

Unit 5.5 To graph linear inequalities in two variables.

Linear Inequality in Two Variables: an inequality that can be written in one of the following forms

$$ax+by<c,\ ax+by\le c,\ ax+by>c,\ ax+by\ge c$$

Solutions of a Linear Inequality: the ordered pairs, (x, y) that make the inequality true

Checking a Solution to a Linear Inequality

Ex 1: Is the ordered pair $(-2,3)$ a solution to the linear inequality $3x-2y>8$?

Substitute the ordered pair in for *x* and *y* in the inequality.

$$3x-2y>8$$
$$3(-2)-2(3)>8$$

Evaluate and determine if it makes the inequality true.

$$-6-6>8$$
$$-12>8 \quad \text{False}$$

The ordered pair does not make the inequality true, so it is NOT a solution.

Half-Plane: one of the two planes that a line separates the coordinate plane into

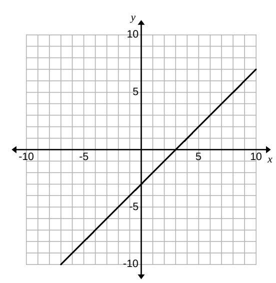
Graphing a Linear Inequality

1. Graph the line $ax+by=c$
2. Use a *dashed* line for $<$ or $>$, a *solid* line for $\le$ or $\ge$.
3. Choose a test point in one of the half-planes and substitute the ordered pair into the inequality for *x* and *y*. If the ordered pair makes the equation true, *shade* the half-plane that contains your test point. If it does not, shade the other half-plane.

Ex 2: Graph the linear inequality $x-y\ge 3$.

Step One: Graph the line $x-y=3$.

x-intercept = 3, *y*-intercept = −3

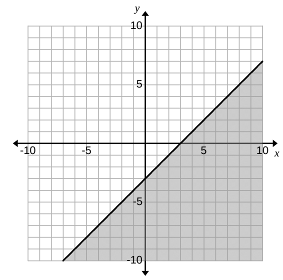


Note: We will use a solid line, because the inequality is $\ge$.

Step Two: Pick a test point in one of the half-planes and test it in the linear inequality.

We will choose $(0,0)$.

$$x - y \geq 3$$
$$0 - 0 \geq 3 \quad \text{False}$$



Step Three: Because $(0,0)$ is NOT a solution to the inequality, we will shade the other half-plane.

Note: All of the solutions of the linear inequality are all points on the line and in the shaded region.

Ex 3: Graph the linear inequality $y < \frac{1}{2}x$.

Step One: Graph the line $y = \frac{1}{2}x$.

Slope = $\frac{1}{2}$, *y*-intercept = 0

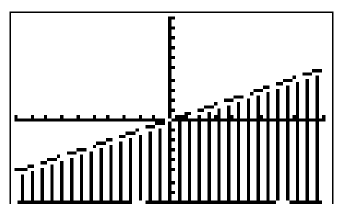
Note: We will use a dashed line, because the inequality is <.

Step Two: Pick a test point in one of the half-planes and test it in the linear inequality.

We will choose the point $(0,-2)$.

$$y < \frac{1}{2}x$$
$$-2 < \frac{1}{2}(0)$$
$$-2 < 0 \quad \text{True}$$

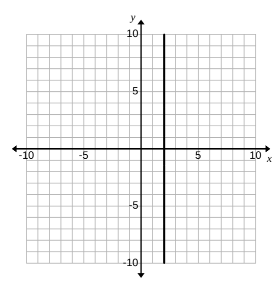
Step Three: Because $(0,-2)$ IS a solution to the inequality, we will shade the half-plane that the point lies in.



Ex 4: Sketch the graph of $x \geq 2$ in the coordinate plane.

Step One: Graph the line $x = 2$.

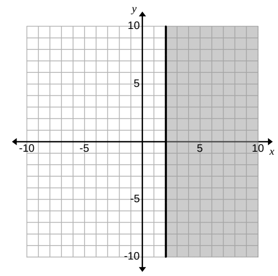
Note: We will use a solid line, because the inequality is $\geq$.



Step Two: Pick a test point in one of the half-planes and test it in the linear inequality.

We will choose the point $(0,0)$.

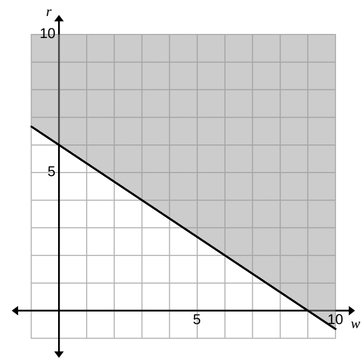
$$x \geq 2$$
$$0 \geq 2 \quad \text{False}$$



Step Three: Because $(0,0)$ IS NOT a solution to the inequality, we will shade the other half-plane.

<u>Application Problem</u>

Ex 5: A basketball team is 18 points behind with 5 minutes left in the game. Write an inequality that represents the number of 2-point and 3-point shots the team could score to earn at least 18 points. Graph the inequality and give two different numbers of 2-point and 3-point shots the team could score.



Assign labels: *w* = number of 2-point shots, *r* = number of 3-point shots

Write an inequality: $2w + 3r \geq 18$

Graph the inequality: *w*-intercept = 9, *r*-intercept = 6

Test point $(0,0)$:
$$2(0) + 3(0) \geq 18$$
$$0 \geq 18 \quad \text{False}$$

Two *possible* 2- and 3-point combinations:

Three 2-point shots and four 3-point shots (This corresponds to the ordered pair $(3,4)$, which lies on the line.)

Six 2-point shots and three 3-point shots (This corresponds to the ordered pair $(6,3)$, which lies in the shaded region.)

<u>You Try</u>: Graph the linear inequalities.

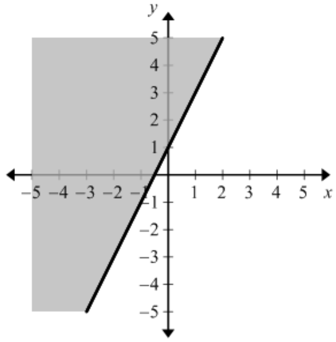
1. $3y > -9$
2. $2x + y \leq 6$

<u>QOD</u>: Explain how to determine if an ordered pair is a solution of a linear inequality.

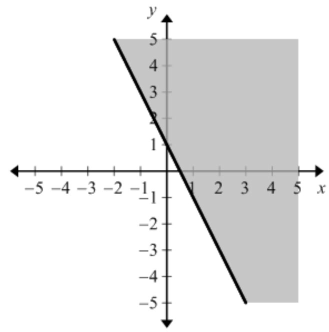
Sample Exam Questions

1. Graph the linear inequality $-y \geq 2x - 1$.

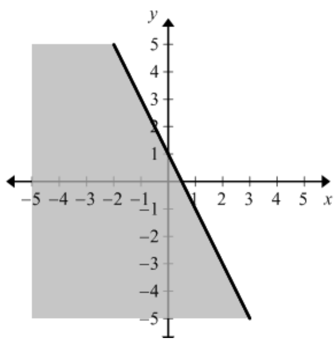
A.



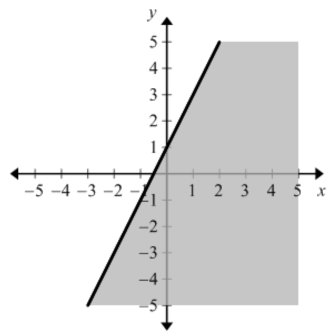
C.



B.



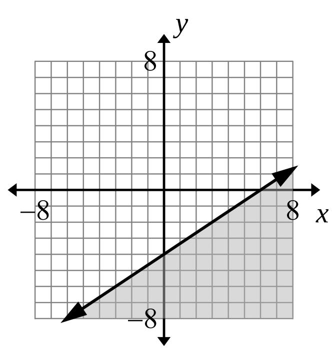
D.



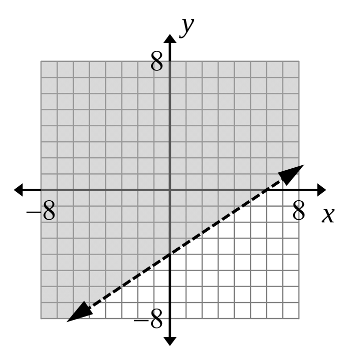
Ans: B

2. Which is the graph of $2x - 3y < 12$?

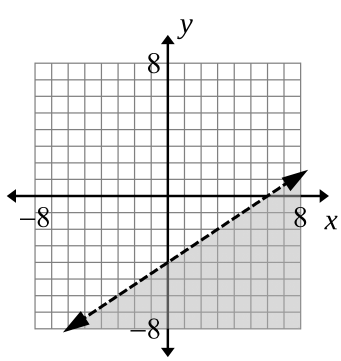
(A)



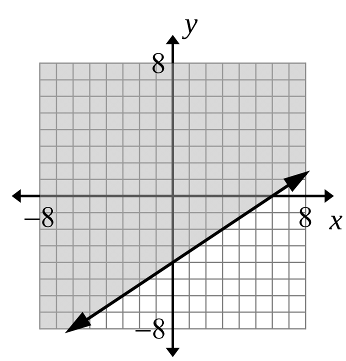
(B)



(C)

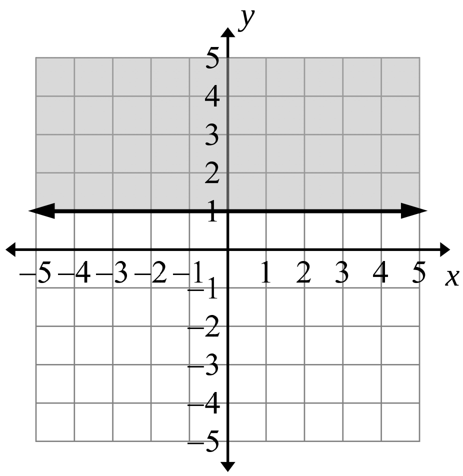


D)



Ans: B

3. Use the graph.



Which inequality is represented in the graph?

(A) $x \le 1$

(B) $x \ge 1$

(C) $y \le 1$

(D) $y \ge 1$

Ans: D

For questions 4-6, use the inequality $y < \frac{x}{2} + 1$.

4. (0, 1) is a solution of the inequality.

(A) True

(B) False

5. (1, 2) is a solution of the inequality.

(A) True

(B) False

6. (2, 0) is a solution of the inequality.

(A) True

(B) False

Ans: B, B, A