

Pre-Algebra Notes: Introduction to Functions

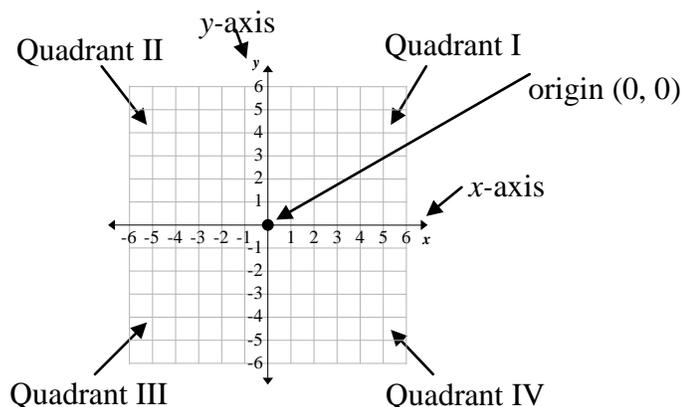


The Coordinate Plane

A coordinate plane is formed by the intersection of a horizontal number line called the x -axis and a vertical number line called the y -axis.

The x -axis and y -axis meet or intersect at a point called the *origin*.

The coordinate plane is divided into six parts: the x -axis, the y -axis, Quadrant I, Quadrant II, Quadrant III, and Quadrant IV. Refer to the diagram on the next page. Hint: one way to remember the order of the quadrants is to think of writing a “C” (for coordinate plane) around the origin. To create the “C” you start in quadrant I and move counterclockwise (and so does the numbering of the quadrants).



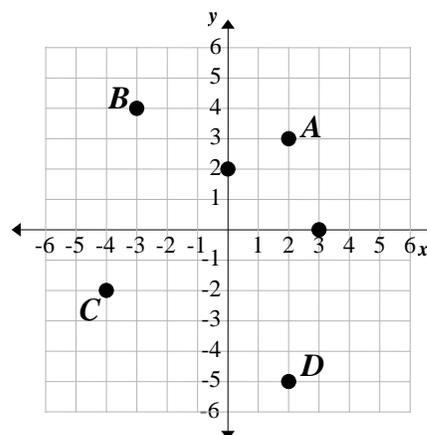
The coordinate plane consists of infinitely many points called *ordered pairs*. Each ordered pair is written in the form of (x, y) . The first coordinate of the ordered pair corresponds to a value on the x -axis and the second number of the ordered pair corresponds to a value on the y -axis. Our movements in the coordinate plane are similar to movements on the number line. As you move from left to right on the x -axis, the numbers are increasing in value. The numbers are increasing in value on the y -axis as you go up.

To find the coordinates of point A in Quadrant I, start from the origin and move 2 units to the right, and up 3 units. Point A in Quadrant I has coordinates $(2, 3)$.

To find the coordinates of point B in Quadrant II, start from the origin and move 3 units to the left, and up 4 units. Point B in Quadrant II has coordinates $(-3, 4)$.

To find the coordinates of the point C in Quadrant III, start from the origin and move 4 units to the left, and down 2 units. Point C in Quadrant III has coordinates $(-4, -2)$.

To find the coordinates of the point D in Quadrant IV, start from the origin and move 2 units to the right, and down 5 units. Point D in Quadrant IV has coordinates $(2, -5)$.



Here are a few phrases that teachers have used with students to help them remember how to determine the coordinates of a point. “Taxi before you take off” implies moving right or left before you move up or down. Same with “Run before you jump”.

Notice that there are two other points on the above graph, one point on the x -axis and the other on the y -axis. For the point on the x -axis, you move 3 units to the right and do not move up or down. This point has coordinates of $(3, 0)$. For the point on the y -axis, you do not move left or right, but you do move up 2 units on the y -axis. This point has coordinates of $(0, 2)$. Points on the x -axis will have coordinates of $(x, 0)$ and points on the y -axis will have coordinates of $(0, y)$. The *first* number in an ordered pair tells you *to move left or right* along the x -axis. The *second* number in the ordered pair tells you *to move up or down* along the y -axis.

Relations and Functions

NVACS 8.F.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. (Function notation is not required.)

Students that have read a menu have experienced working with ordered pairs. Menus are typically written with a food item on the left side of the menu, the cost of the item on the other side as shown:

| | |
|-----------------|--------|
| Hamburger | \$3.50 |
| Pizza..... | 2.00 |
| Sandwich..... | 4.00 |

Menus could have just as well been written horizontally:

Hamburger, \$3.50, Pizza, 2.00, Sandwich, 4.00

But that format (notation) is not as easy to read and could cause confusion. Someone might look at that and think you could buy a \$2.00 sandwich. To clarify that so no one gets confused, I might group the food item and its cost by putting parentheses around them:

(Hamburger, \$3.50), (Pizza, 2.00), (Sandwich, 4.00)

Those groupings would be called ordered pairs because there are two items. Ordered because food is listed first, cost is second.

By definition, a *relation* is any set of ordered pairs.

Another example of a set of ordered pairs could be described when buying cold drinks. If one cold drink cost \$0.50, two drinks would be \$1.00, and three drinks would be \$1.50. I could write those as ordered pairs:

(1, .50), (2, 1.00), (3, 1.50), and so on.

From this you would expect the cost to increase by \$0.50 for each additional drink. What do you think might happen if one student went to the store and bought 4 drinks for \$2.00 and his friend who was right behind him at the counter bought 4 drinks and only paid \$1.75?

My guess is the first guy would feel cheated, that it was not right, that this was not working, or this was not *functioning*. The first guy would expect anyone buying four drinks would pay \$2.00, just like he did.

Let's look at the ordered pairs that caused this problem.

(1, .50), (2, 1.00), (3, 1.50), (4, 2.00), (4, 1.75)

The last two ordered pairs highlight the malfunction, one person buying 4 drinks for \$2.00, the next person buying 4 drinks for a \$1.75. For this to be fair or functioning correctly, we would expect that anyone buying four drinks would be charged \$2.00. Or more generally, we would expect every person who bought the same number of drinks to be charged the same price. When that occurs, we'd think this is functioning correctly. So let's define a function.

A function is a special relation in which no two different ordered pairs have the same first element.

Since the last set of ordered pairs has the same first elements, those ordered pairs would not be classified as a function.

If I asked students how much 10 cold drinks would cost, many might realize the cost would be \$5.00. If I asked them how they got that answer, eventually, with prodding, someone might tell me they multiplied the number of cold drinks by \$0.50. That shortcut can be described by a rule:

$$\text{cost} = \$0.50 \times \text{number of cold drinks}$$

$$c = .50n$$

or the way you see it written in your math book

$$y = .50x \text{ or } y = \frac{1}{2}x$$

That rule generates more ordered pairs. So if I wanted to know the price of 20 cold drinks, I would substitute 20 for x . The result would be \$10.00. Written as an ordered pair, I would have (20,10).

Note: You can also use a couple examples to explain why you can have the situation (1, \$15), (2, \$15), (3, \$30), (4, \$30). Sometimes there can be the same “y” value for different “x” values. I tell them it is like “buy one, get one free”...one and two shirts still cost \$15, but the 3rd and 4th shirts will cost \$30 and this still follows the function rule. Also explain the situation (1, \$10), (2, \$10), (3, \$10), (4, \$10) – where all the “y” values are the same but the “x” value is different by using the “all you can eat example”. At Cici’s pizza, you can buy the “all you can eat special for \$10”. If you eat one piece of pizza, it is \$10. If you eat 3 pieces of pizza, it is still \$10. If you eat 100 pieces of pizza, it is still \$10. The function rule remains in place!

Let’s look at another rule.

Example: $y = 3x + 2$

If we plug 4 in, we will get 14 out, represented by the ordered pair (4, 14).

If we plug 0 in, we get 2 out, represented by (0, 2).

There is an infinite number of numbers I can plug in.

A *relation* is any set of ordered pairs. The *set of all first members of the ordered pairs* is called the domain of the relation. The set of *second numbers* is called the range of the relation.

Sometimes we put restrictions on the numbers we plug into a rule (the domain). Those restrictions may be placed on the relation so it fits real world situations.

For example, let’s use our cold drink problem. If each drink costs \$0.50, it would not make sense to find the cost of -2 drinks. You can’t buy negative two drinks, so we would put a restriction on the domain. The only numbers we could plug in are whole numbers: 0, 1, 2, 3...

The restriction on the domain also affects the range. If you can only use positive whole numbers for the domain, what values are possible for the range?

We defined a *function* as a special relation in which no two ordered pairs have the same first member.

What this means is that for every member of the domain, there is one and only one member of the range. That means if I give you a rule, like $y = 2x - 3$, when I plug in $x = 4$, I get 5 out. This is represented by the ordered pair (4, 5). Now if I plug in 4 again, I have to get 5 out. That makes sense—it’s expected—so just like the rule for buying cold drinks, this is working, this is functioning as expected, it’s a function.

Now, you are thinking, big deal, isn’t that what we would expect?

Looking at another rule might give us a clue, $x^2 + y^2 = 25$.

Solving that for y , we get $y^2 = 25 - x^2$
 $y = \pm\sqrt{25 - x^2}$

Now if we plug in a number like 3, we get two answers: (3, 4) and (3, -4). You can see there is not one and only one member in the range for each member in the domain. Therefore, this rule describes a relation that is *not* a function.

We can look at the graphs of relations that are nothing more than a bunch (set) of ordered pairs (points) and determine if it's a function.

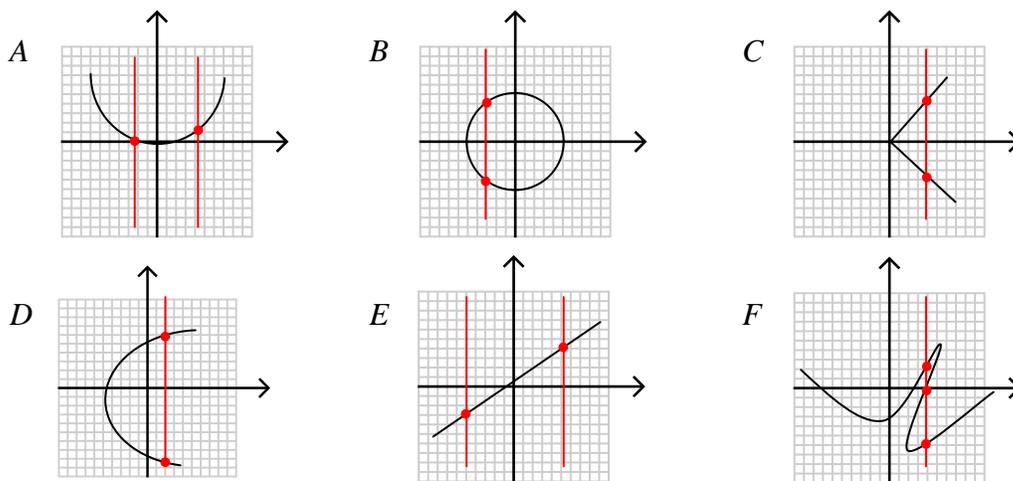
To determine if a graph describes a function, you use what we call the vertical line test. That is, you try to draw a vertical line through the graph so it intersects the graph in more than one point. If you can do that, then those two ordered pairs have the same first element, but a different second element. Therefore, the graph would *not* describe a function.

If there does not exist any vertical line which crosses the graph of the relation in more than one place, then the relation is a function. This is called the VERTICAL LINE TEST.

What we try to do is draw a vertical line so it intersects the graph in more than one place. If we can't, then we have a graph of a function.

Let's try a few problems.

Label the following as relations or functions.

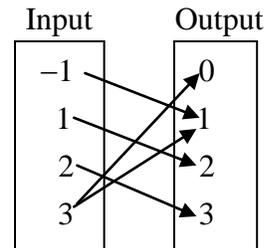


Only A and E are functions.

In addition to looking at a graph to represent a relation, you can use a *mapping diagram*.

Example: Represent the relation $(-1, 1)$, $(3, 0)$, $(3, 1)$, $(2, 3)$, $(1, 2)$ using a mapping diagram.

List the inputs and the outputs in order.
Draw arrows from the inputs to their outputs.



Is this relation a function? No, because the input 3 is paired with two outputs, 0 and 1. You could also verify this by graphing the ordered pairs and applying the vertical line test.

Graphing Linear Equations

In order to plot the graph of a linear equation, we solve the equation for y in terms of x . Then we assign values for x and find the value of y that corresponds to that x . Each x and y , called an ordered pair (x, y) , represents the coordinate of a point on the graph.

Example: Graph $3x + y = 2$.

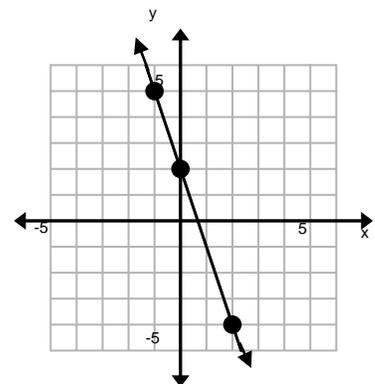
Solving for y , I subtract $3x$ from both sides.

$$y = 2 - 3x \text{ or } y = -3x + 2$$

When I assign values for x , I get these y values:

| x | y |
|-----|-----|
| 0 | 2 |
| 2 | -4 |
| -1 | 5 |

Rewriting as ordered pairs, I get $(0, 2)$, $(2, -4)$, $(-1, 5)$ which I plot on the graph. When I connect those three points, I get a straight line called a LINEAR equation.



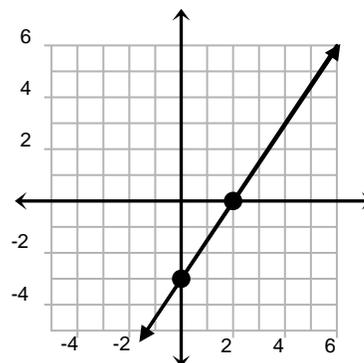
I could have chosen any values for x and found the corresponding y . However, it is easier to choose a convenient number, like zero, one, or two. Choosing a number like 100 would make my graph a lot larger; or I could have chosen a fraction, but that can be messy.

Example: Graph. $y = \frac{3}{2}x - 3$

When I assign values for x , I get these y values:

| x | y |
|-----|-----|
| 0 | -3 |
| 2 | 0 |
| -2 | -6 |

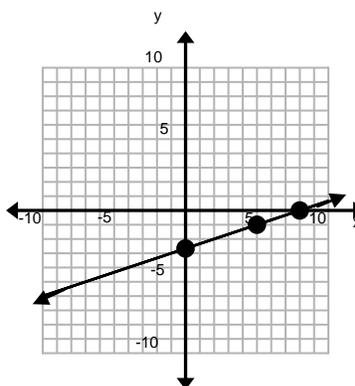
Rewriting as ordered pairs, I get $(0, -3)$, $(2, 0)$, $(-2, -6)$ which I plot on the graph. Again, when I connect those three points, I get a straight line called a LINEAR equation.



Example: Graph. $y = \frac{1}{3}x - \frac{8}{3}$

Assign values for x , and find the corresponding y 's. Choose numbers that will make the values for y easy to work with.

| x | y |
|-----|----------------|
| 0 | $-\frac{8}{3}$ |
| 8 | 0 |
| 5 | -1 |



The ordered pairs $(0, -\frac{8}{3})$, $(8, 0)$, and $(5, -1)$ represent the points on the graph.

If we did enough of these problems, we would see a quicker way of graphing linear equations. First, all linear equations are graphs of lines; therefore, all we need do is graph two points.

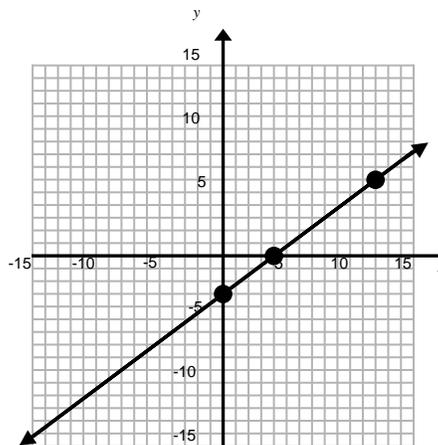
Second, we would see that the value of x when the graph crosses the y -axis is always zero. Look at the last two examples. What's the value of y when the graph crosses the x -axis? Look at the graphs we've already plotted. When the graph crosses the x -axis, the value of y is zero.

Let's look at another example. First we'll plot the graph as we did in the previous examples.

Example: Graph. $y = \frac{3}{4}x - 3$

Now assign values for x and find the corresponding y 's. Graph each ordered pair.

| x | y |
|-----|-----|
| 0 | -3 |
| 4 | 0 |
| 12 | 6 |



Horizontal and Vertical Lines

Don't forget to address graphing *horizontal* and *vertical* lines (students can have problems with these). The graph of the equation $y = b$ is the horizontal line through $(0, b)$. The graph of the equation $x = a$ is the vertical line through $(a, 0)$. Of course, remind students to simply set up a table of values if they are confused. ***If the line is horizontal, the slope is zero. If the line is vertical, the slope is undefined.***

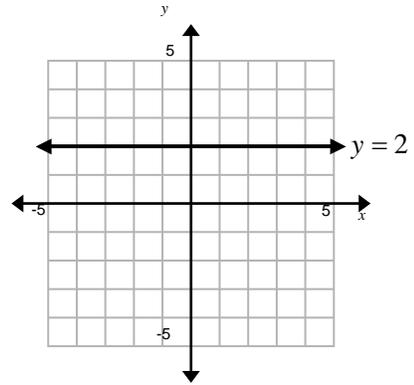
Example: Graph $y = 2$.

The graph of the equation $y = 2$ is the horizontal line through $(0, 2)$.

Or a quick table would give us points to plot, and then we could draw the line.

| x | y |
|-----|-----|
| -2 | 2 |
| 0 | 2 |
| 2 | 2 |

$$\begin{aligned} m &= \frac{2-2}{2-0} \\ &= \frac{0}{2} \\ &= 0 \end{aligned}$$



Example: Graph $x = -3$.

The graph of the equation $x = -3$ is the horizontal line through $(0, -3)$.

Or a quick table would give us points to plot and then draw the line.

| x | y |
|-----|-----|
| -3 | -2 |
| -3 | 0 |
| -3 | 2 |

$$\begin{aligned} m &= \frac{0-(-2)}{-3-(-3)} \\ &= \frac{2}{0} \\ &\text{undefined} \end{aligned}$$

