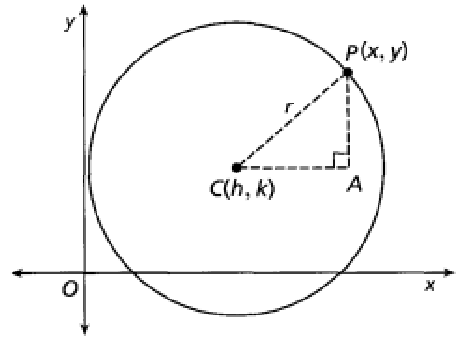
Geometry – Unit 10 Activity
Deriving the Equation of a Circle
G.GPE.A.1

Name: ______________________________!

Date: ___________ Pd: ____

By placing a circle in the coordinate plane with center *C*(*h*, *k*) and radius (*r*) we can derive the equation a circle. Chose a point on the circle, *P*(*x*, *y*) and create a right triangle with $\overline{CP}$ as the hypotenuse.



1) Find the coordinates of point *A*. (____, ____)

2) Write an expression for the lengths of each of the legs of Δ*CAP*.
CA = ________
PA = ________

3) Use the Pythagorean Theorem to write an expression that represents the length of $\overline{CP}$.
CP^2 = ______________________________
CP = ______________________________

4) Another name for $\overline{CP}$ is _____.

5) Rewrite your expression in Part 3, substituting your answer to Part 4 for *CP*. _________________________

6) Square both sides. _________________________ (Standard Form)

7) Suppose a circle has its center at the origin. What is the Standard Form equation of the circle in this case?
_________________________ (*this is considered a special case*)

General Form :

$x^2 + y^2 + ax + by + c = 0$, where *a, b, c* are constants, *c* cannot be equal to 0 while *a* and *b* can be.

8) Suppose a circle has its center at the origin. What is the General Form equation of the circle in this case?
_________________________ (*this is considered a special case*)

An equation in General Form must first be solved using the method of **Completing the Squares** to express it in Standard Form. This allows you to find the center and radius.

Characteristics of the Equation of a Circle

- there is always an ***x*** and a ***y*** to the second power
- the ***x*** and ***y*** to the second power must have the same sign when on the same side of the equation
- the ***x*** and ***y*** to the second power always have the same coefficient
- there could be an ***x*** and/or a ***y*** to the first power (means that the center is not at the origin) as well as constants

Let's examine the equation of a circle given in Standard Form:

9) $(x-2)^2 + (y+3)^2 = 6$
a) Find the center. (____, ____)
b) Find the radius. ______
c) Rewrite in General Form. ______________________________

Changing Standard Form to General Form:

Example: Rewrite in General Form: $(x+2)^2+(y-7)^2=9$.

Multiply the quadratic terms: $(x+2)(x+2)+(y-7)(y-7)=9$

$x^2+2x+2x+4+y^2-7y-7y+49=9$

Collect like terms: $x^2+4x+y^2-14y+53=9$

Reorder terms: $x^2+y^2+4x-14y+53=9$

Set equal to zero: $x^2+y^2+4x-14y+44=0$ (*subtract 9 from both sides*)

What if the equation is NOT given in Standard Form?

Example: Find the center and radius of a circle whose equation is $x^2-4x+y^2+2y=4$.

a) Complete the square to write the equation in the form $(x-h)^2+(y-k)^2=r^2$.

b) $x^2-4x+\_\_\_\_+y^2+2y+\_\_\_\_=4+\_\_\_\_\_\_\_\_$ (*Set up to complete the square.*)

c) $x^2-4x+\_\_\_\_+y^2+2y+\_\_\_\_=4+\_\_\_\_\_\_\_\_$ (*Add* $\left(\frac{-4}{2}\right)^2$ *and* $\left(\frac{2}{2}\right)^2$ *to both sides.*)

d) $x^2-4x+\_\_\_\_+y^2+2y+\_\_\_\_=4+\_\_\_\_\_\_\_\_$ (*Simplify.*)

e) $(x-\_\_\_\_)^2+(y+\_\_\_\_)^2=\_\_\_\_\_\_\_$ (*Factor.*)

f) Identify *h*, *k*, and *r* to determine the center and radius.

h = ________, *k* = ________, *r* = ________

So, the center is (_____, _____) and the radius is ______.

Translations in the Plane:

Suppose you translate any circle by the translation $(x,y)\rightarrow(x+3,y-2)$.

Describe what happens to each:

a) The graph of the circle. ____
b) *h* ____
c) *k* ____
d) *r* ____

Finding the radius:

A) Given the Standard Form equation: $(x+1)^2+(y-2)^2=9$.

Radius = $\sqrt{9}$ = 3 (*square root of the constant*)

B) Given General Form: $x^2+y^2+6x+8=0$.

Must be changed into Standard Form… (*complete the square*)

$x^2+6x+\_+y^2+0y+\_+8=0+\_+\_$

$x^2+6x+\underline{9}+y^2+0y+\underline{0}+8=0+\underline{9}+\underline{0}$

$(x+3)^2+(y+0)^2+8=9$

$(x+3)^2+(y+0)^2=1$

Radius = $\sqrt{1}$ = 1 (*square root of the constant*)

C) Given Center and Point on Circle:

Find the radius if the center is at (0, -5) and one point on the circle is (2, 3). (*Use the distance formula!*)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(2-0)^2 + (3-(-5))^2}$$

$$\sqrt{(2)^2 + (8)^2} = \sqrt{4+64} = \sqrt{68} = 2\sqrt{17}$$

Radius = $2\sqrt{17}$

D) Given endpoints of the diameter:

Write the equation of a circle whose diameter has endpoints (4, -1) and (-6, 7).

(*Use the distance formula, then divide by 2*)

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\sqrt{(4-(-6))^2 + ((-1)-7)^2}$$

$$\sqrt{(10)^2 + (-8)^2} = \sqrt{100+64} = \sqrt{164} = 2\sqrt{41}$$

Radius = $\sqrt{41}$ (*after you divide by 2!*)

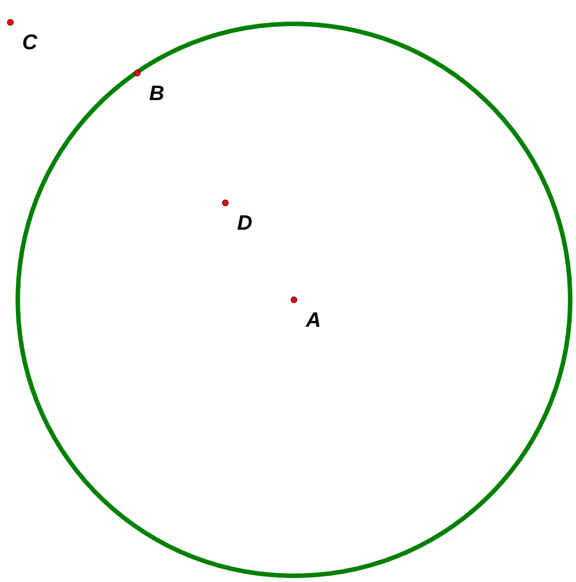
Determining whether a point is ON / IN / OUTSIDE the circle:

In order to determine the position of a point in relation to a given circle, you must compare the distance from the center to the point.

a) For points **ON** the circle: the distance _____ the radius

b) For points **IN** the circle: the distance _____ the radius

c) For points **OUTSIDE** the circle: the distance _____ the radius



In $\odot A$, *B* is ON / IN / OUTSIDE because *AB* ____ radius length. (chose <, >, or =)

In $\odot A$, *C* is ON / IN / OUTSIDE because *AC* ____ radius length. (chose <, >, or =)

In $\odot A$, *D* is ON / IN / OUTSIDE because *AD* ____ radius length. (chose <, >, or =)