Geometry – Unit 10 Activity **Name: ______________________________!**
The Angle Revolution
G.C.B.5 **Date: ___________ Pd: ____**

Revolution – full circle, turn, full turn or rotation.

1 revolution = 1 full circle = 1 turn = *1 rotation* = 1 circle = 360°

Degree – used in mathematics to measure angles, $\frac{1}{360}$ of the circumference of a circle.

1 full circle = 360°

It's an obvious fact that circles should have 360 degrees. Right?

Wrong. Most of us have *no idea* why there's 360 degrees in a circle. We memorize a magic number as the "size of a circle" and set ourselves up for confusion when studying advanced math or physics, with their so called "radians".

Here's a theory about how degrees came to pass:

- Humans noticed that constellations moved in a full circle every year
- Every day, they were off by a tiny bit ("a degree")
- Since a year has about 360 days, a circle had 360 degrees

But, but… why not *365 degrees* in a circle?

Cut 'em some slack: they had *sundials* and didn't know a year should have a convenient 365.242199 degrees like you do.

360 is close enough for government work. It fits nicely into the Babylonian base-60 number system, and divides well (by 2, 3, 4, 6, 10, 12, 15, 30, 45, 90… so you get the idea).

Radians – used in advanced mathematics to measure angles.

*Invented in the 1700's by mathematicians who wanted to define angles rationally, radians were determined from the circumference of a circle with a radius of 1. One **radian** is just under 57.3°.*

1 full circle = 2π radians

"Radians make math easier!" the experts say, without a simple reason why.

Imagine watching a friend run the track from a position inside the track, degrees measure the amount you need to turn your head to see him run. Radians measure angles by distance traveled.

But absolute distance isn't that useful, since going 10 miles is a different number of laps depending on the track. So we divide by radius to get a normalized angle:

$$Radian = \frac{distance \quad traveled}{radius}$$

You'll often see this as

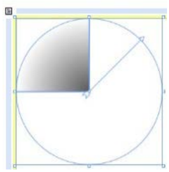
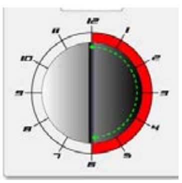
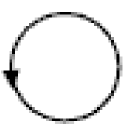
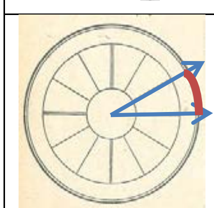
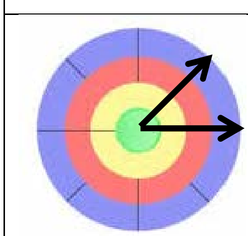
$$\theta = \frac{s}{r}$$

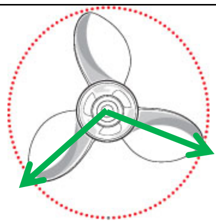
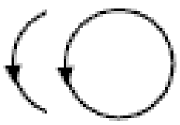
or angle in radians (theta) is arc length (s) divided by radius (r).

In plain English, 1 radian is what you get when the length of the segment of the circle's circumference being measured is equal to the length of the radius.

Radians are a count of distance in terms of "radius units"; think of "radian" as shorthand for that concept.

Fill in the radian measures to compare revolutions, degrees and radians:

DIAGRAM	WORDS	REVOLUTIONS	DEGREES	RADIANS
…?…	no turn	0	0°	0
	quarter turn	1/4	90°	______
	half turn	1/2	180°	______
	three-quarter turn	3/4	270°	______
	full turn	1	360°	______
	twelfth turn	1/12	30°	______
	eighth turn	1/8	45°	______
	sixth turn	1/6	60°	______

	fifth turn	1/5	72°	______
	third turn	1/3	120°	______
	two turns	2	720°	______
	three turns	3	1080°	______

Conversions:

In one full circle, there are 360 degrees or 2π radians. Therefore we know that 2π radians = 360 degrees. Since the angles within the circle divide the 2π radians proportionately, we can use simple proportions to convert between degrees and radians.

$$\frac{\textit{degree measure}}{360} = \frac{\textit{radian measure}}{2\pi}$$

Example: Convert 270° to radians.

$\frac{270}{360} = \frac{x}{2\pi}$ — fill in known and unknown values

$540\pi = 360x$ — find cross-product

$\frac{540}{360}\pi = x$ — solve for *x*

$\frac{3}{2}\pi = x$ — simplify

SO… 270° = $\frac{3}{2}\pi$ radians

Example: Convert from $\frac{2}{3}\pi$ radians to degrees.

$\frac{x}{360} = \frac{\frac{2}{3}\pi}{2\pi}$ fill in known and unknown values

$2\pi x = 240\pi$ find cross-product

$x = \frac{240\pi}{2\pi}$ solve for x

$x = 120$ simplify SO... $\frac{2}{3}\pi$ radians = 120°

We can also use the conversion equations:

$$\textbf{\textit{degree measure}} = \textbf{\textit{radian measure}}\left(\frac{180}{\pi}\right) \quad \text{OR} \quad \textbf{\textit{radian measure}} = \textbf{\textit{degree measure}}\left(\frac{\pi}{180}\right)$$

<u>Compile your results here, you may need to calculate some conversions:</u>

{degrees, radians}