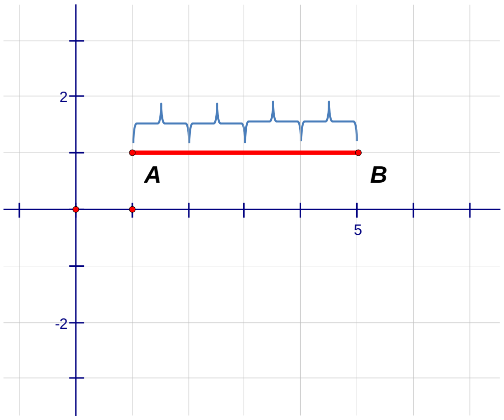
Deriving the Distance Formula

Any two points on the coordinate plane can be connected with a line segment.

If that segment is vertical or horizontal the length can simply be counted.

Example:



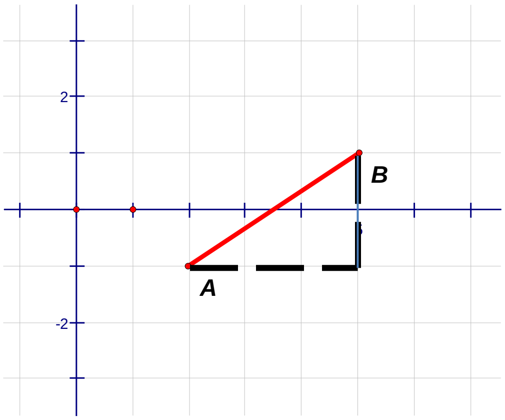
We say the length of $\overline{AB}$ is 4 units.

OR

$AB = 4$

NOTE: some books or worksheets use the notation $m\overline{AB} = 4$, m stands for the measure of the line.

If the segment connecting the two points is slanted then another approach MUST be used.



Start by creating a right triangle with your segment as the hypotenuse. It can be either above or below the segment.

The length of each of the legs can now be determined by counting like above.

The Pythagorean Theorem can now be used to determine the length of $\overline{AB}$.

$$c^2 = a^2 + b^2$$
$$AB^2 = 2^2 + 3^2$$
$$AB = \sqrt{4+9}$$
$$AB = \sqrt{13}$$

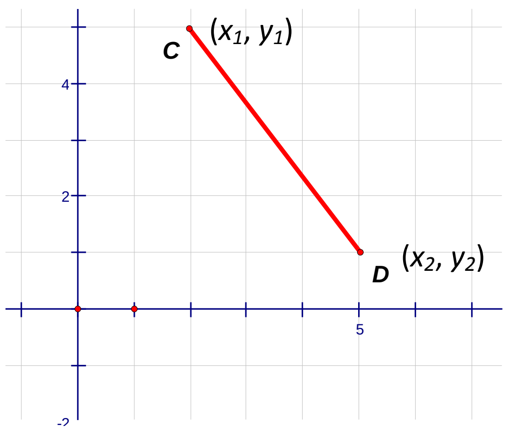
The coordinates of all ordered pairs are arranged (*x*, *y*). Notice the coordinates of *A* and *B*.

A(2, -1) and *B*(5, 1).

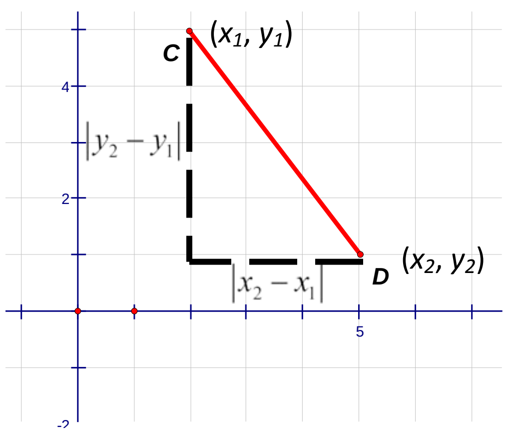
It is not a coincidence that the *x*-values change by 3 and the *y*-values change by 2.

This allows us to look for the general case…

GENERAL CASE:



Label the endpoints of the segment as (x_1, y_1) and (x_2, y_2).



Draw a right triangle with this segment as the hypotenuse.

Represent the lengths of the legs of the right triangle as the absolute value of the difference of the *x* and *y* coordinate values.

Apply the Pythagorean Theorem:

$$c^2 = a^2 + b^2$$

$$CD^2 = |x_2 - x_1|^2 + |y_2 - y_1|^2$$

$$CD = \sqrt{|x_2 - x_1|^2 + |y_2 - y_1|^2}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

OR we can say…

$$\text{distance} = \sqrt{(\text{change in x})^2 + (\text{change in y})^2}$$