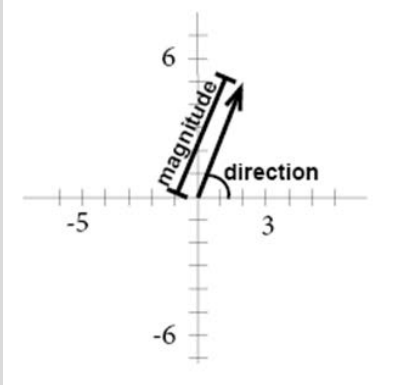
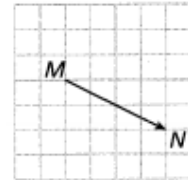


Some mathematical quantities, such as force and velocity, involve both **magnitude** and **direction** and cannot be completely characterized by a single real number. To represent such a quantity, we use a directed line segment (arrow) called a **VECTOR**.



1) To name the vector, identify the initial point and the terminal point.

- a) The initial point is \_\_\_\_\_ .
- b) The terminal point is \_\_\_\_\_ .
- c) The name of the vector is \_\_\_\_\_ .



2) To write the vector in component form, identify the horizontal change and vertical change from the initial to the terminal point.

- a) The horizontal change is \_\_\_\_\_ .
- b) The vertical change is \_\_\_\_\_ .
- c) The component form for the vector is \_\_\_\_\_ .

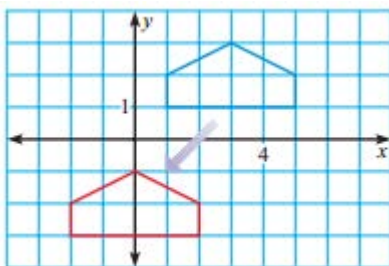
3) Is  $\overline{XY}$  the same as  $\overline{YX}$  ?

4) How is  $\overline{CD}$  different than  $\overline{DC}$  ?

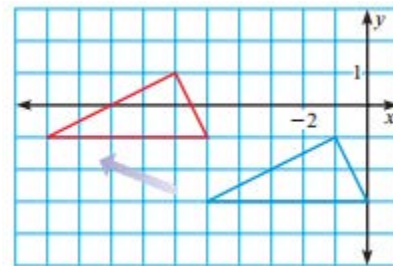
5)  $P'(7, 2)$  is the image of  $P(2, 11)$  after a translation. What vector describes this translation?

6) Describe the translation using (i) coordinate notation and (ii) a vector component form.

a)



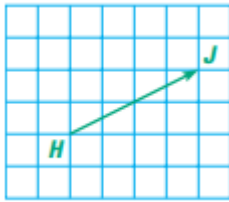
b)



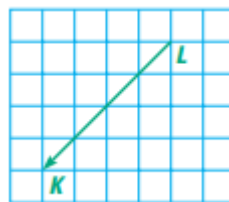


7) Name the vector and write its component form.

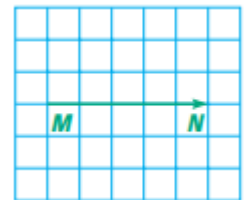
a)



b)



c)



8) Find the component form of the vector with initial point  $P(1, 2)$  and terminal point  $Q(4, 4)$ .

9) Find the component form of the vector with initial point  $P(-1, -3)$  and terminal point  $Q(-6, 5)$ .

10) Find the component form of the vector  $u$  with initial point  $(-1, 3)$  and terminal point  $(-6, 1)$ .  
Find the magnitude of the vector  $u$ .

To **add or subtract vectors**, we just add or subtract their corresponding horizontal and vertical components.

To **multiply a vector by a scalar**, we multiply both the horizontal and vertical components by this scalar.

**Example:** Given  $u = \langle 1, -3 \rangle$  and  $v = \langle 2, 4 \rangle$ .

$$u + v = \langle 1, -3 \rangle + \langle 2, 4 \rangle = \langle 3, 1 \rangle$$

$$u - v = \langle 1, -3 \rangle - \langle 2, 4 \rangle = \langle -1, -7 \rangle$$

$$3u - 2v = 3\langle 1, -3 \rangle - 2\langle 2, 4 \rangle = \langle 3, -9 \rangle - \langle 4, 8 \rangle = \langle -1, -17 \rangle$$

11) Given  $u = \langle -3, 3 \rangle$  and  $v = \langle -5, -1 \rangle$ . **Find:** (Write answers in component form as  $\langle x, y \rangle$ )

a)  $u + v$

b)  $u - v$

c)  $-4u - v$

12) Let  $\vec{v} = \langle -2, 5 \rangle$  and  $\vec{w} = \langle 3, 4 \rangle$ . **Find:**

a)  $2\vec{v}$

b)  $\vec{w} - \vec{v}$

c)  $\vec{v} + 2\vec{w}$

13) Let  $u = \langle -2, 5 \rangle$  and  $v = \langle 2, -8 \rangle$ . **Calculate:**

a)  $-5v$

b)  $u + v$

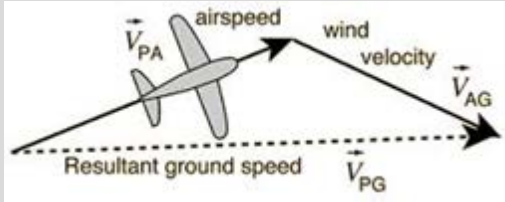
c)  $-u + 3v$



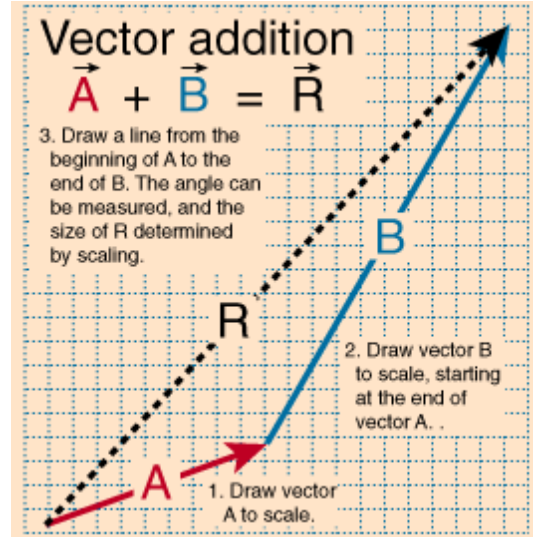
**Another way...**

By graphing the vectors end to end, you can draw the resultant vector which is equivalent to the sum of the given vectors. See diagram to the right...

When is this used?



Here's an example involving speed, notice the airplane has speed but the wind pushes against it at an additional speed. The ground speed can be determined by finding the length of the resultant vector.



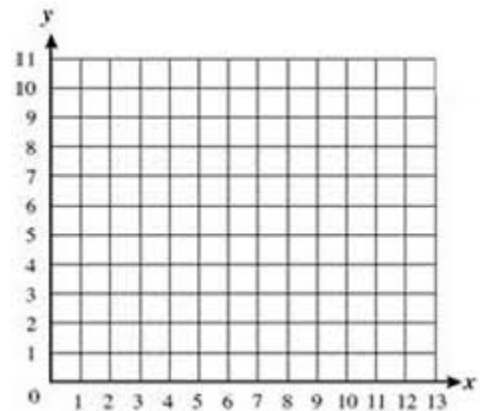
**14) Is addition of vectors commutative? Is  $u + v$  equivalent to  $v + u$ ?  $u = \langle 1, 4 \rangle$ ,  $v = \langle 2, 2 \rangle$ .**

Use the space below to add the two vectors algebraically and geometrically.

$u + v =$  \_\_\_\_\_

$v + u =$  \_\_\_\_\_

Find the coordinates of your ending position on the graph. \_\_\_\_\_

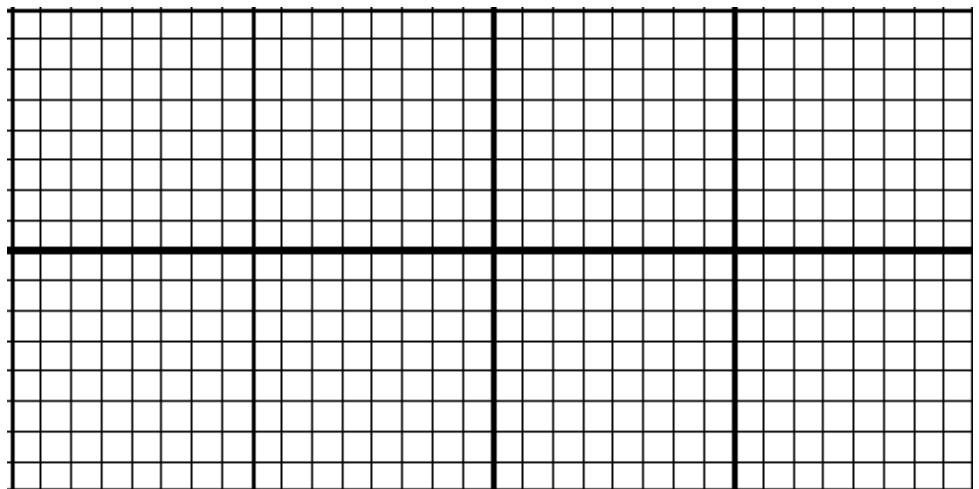
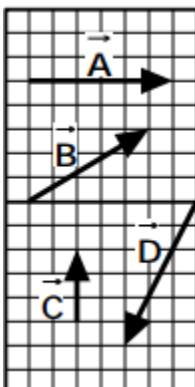


**15) Using the vectors given below, geometrically add:**

- a)  $A + B$
- b)  $C + D$

c)  $\frac{1}{2}A + 2C$

d)  $B + A + D$





### Magnitude of a Vector:

This is the "length" of the vector. It can also represent speed, weight, etc.

The magnitude  $\|\mathbf{v}\|$  of a vector  $\mathbf{v} = \langle x, y \rangle$  is  $\|\mathbf{v}\| = \sqrt{x^2 + y^2}$ .

#### 16) Calculate the EXACT magnitude of:

- a)  $\bar{\mathbf{v}} = \langle 3, 2 \rangle$
- b)  $\bar{\mathbf{v}} = \langle 0, -7 \rangle$
- c)  $\bar{\mathbf{v}} = \langle -4, -3 \rangle$
- d)  $\bar{\mathbf{v}} = \langle 8, 0 \rangle$

#### 17) Given that the vector $\mathbf{v}$ has an initial point at (9, -3) and a terminal point at (4, 6),

- a) What is the component form of  $\mathbf{v}$ ?
- b) What is the magnitude of vector  $\mathbf{v}$ ? Round your answer to one decimal point.

### Unit Vector:

A vector with length (magnitude)  $1$  is called a unit vector. If we have a vector  $\mathbf{v}$  and we want a unit vector pointing in the same direction as  $\mathbf{v}$ , all we do is calculate  $\frac{\mathbf{v}}{\|\mathbf{v}\|} = \frac{1}{\|\mathbf{v}\|} \mathbf{v}$ .

#### 18) Find a unit vector in the direction of:

- a)  $\bar{\mathbf{v}} = \langle 3, 2 \rangle$  (Note: It is common practice to rationalize denominators!)
- b)  $\bar{\mathbf{v}} = \langle 0, -7 \rangle$
- c)  $\mathbf{v} = \langle -2, 3 \rangle$