Pre-Algebra 8 Notes – Unit 02A: Linear Equations in One Variable—Properties and One-Step Equations

Properties of Real Numbers

Commutative sounds like *commute* which means to go back and forth. Addition and multiplication are both commutative; they work the same either forward or backward.

You *associate* with friends and family in groups. The Associative Properties are ways you can group addends and factors.

A teacher can *distribute* worksheets to students, separating the stack of worksheets into one worksheet per student ("passing out papers"). Often times a teacher needs to do the opposite and "pass in papers" , collecting papers from individual students and putting them in one stack. The Distributive Property will let you "pass out" and "pass in" parts.

When you add 0 to any number or multiply any number by 1, the result is *identical* to the original number. The numbers 0 and 1 are called *identities*.

The properties of addition and multiplication are summarized in the chart below.

Property	Operation	Algebra	Numbers	Key Words
Commutative (**Change o**rder)	$+$	$a + b = b + a$	$3 + 4 = 4 + 3$	**Change O**rder
Commutative	$\times$	$a \cdot b = b \cdot a$	$5 \cdot 2 = 2 \cdot 5$	**Change O**rder
Associative	$+$	$(a + b) + c = a + (b + c)$	$(9 + 4) + 6 = 9 + (4 + 6)$	Change Grouping
Associative	$\times$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$	$(9 \cdot 5) \cdot 2 = 9 \cdot (5 \cdot 2)$	Change Grouping
Identity	$+$	$a + 0 = a$	$7 + 0 = 7$	Add 0
Identity	$\times$	$a \cdot 1 = a$	$6 \cdot 1 = 6$	Multiply by 1
Distributive	multiplication over addition	$a(b + c) = a \cdot b + a \cdot c$	$5(23) = 5(20) + 5(3)$ $100 + 15$ 115	Distribute Over ( )

The properties allow you to manipulate expressions and compute mentally.

Example: Simplify $4 \cdot 13 \cdot 25$.

Using only the Order of Operations to simplify the expression above, you would multiply 4 by 13, then multiply that result by 25. Most students would need pencil & paper. However, using the properties we can solve this problem using mental math.

$(4 \cdot 13)25$ Order of Operations, multiply in order from left to right

$= 4(13 \cdot 25)$ Associative property of multiplication changes grouping

$= 4(25 \cdot 13)$ Commutative property of multiplication changes order

$= (4 \cdot 25)13$ Associative property of multiplication changes grouping

$= 100 \cdot 13$ Simplify

$= 1300$ Simplify

Example: Simplify $25 \cdot 12$.

That could be simplified by just multiplying 25 by 12 using the standard algorithm. Or, we could use the distributive property to break the number apart to perform the multiplications mentally.

$$
\begin{aligned}
25 \cdot 12 &= 25 \cdot (10 + 2) \\
&= 25(10) + 25(2) \\
&= 250 + 50 \\
&= 300
\end{aligned}
$$

Properties allow us to simplify expressions.

Example: Simplify the expression $x + 4 + 2$.

$x + 4 + 2$

$= x + (4 + 2)$ Use associative property of addition.

$= x + 6$ Simplify.

Example: Simplify the expression $4(8y)$.

$$4(8y)$$
$$= (4 \cdot 8)y \quad \text{Use associative property of addition.}$$
$$= 32y \qquad \text{Simplify.}$$

Example: Write an equivalent expression $3(x + 7)$.

$$3(x + 7) = 3x + 3(7) \quad \text{Use the distributive property.}$$
$$= 3x + 21 \quad \text{Simplify.}$$

Often standardized tests use geometry to assess knowledge of these properties.

Example: Find the perimeter of a square whose sides are represented by $y - 5$ and simplify the expression. Explain your thinking.
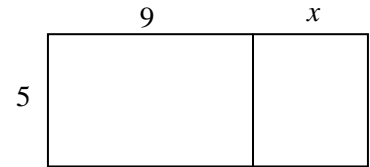
Work	Explanation
$P = 4(y - 5)$ $P = 4y - 20$	To find the perimeter, you add the measures of all 4 sides. Since it is a square, I know all 4 sides are equal, so I can multiply the length of one side by 4. I will use the distributive property to simplify.

Example: Find the area of the triangle and simplify the expression. Show your work.
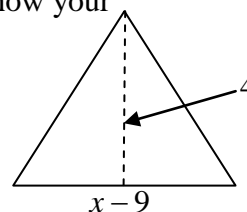
Work	Explanation
$A = \dfrac{1}{2}bh$ $A = \dfrac{1}{2}(x - 9)(4)$ $A = \dfrac{1}{2}(4)(x - 9)$ $A = 2(x - 9)$ $A = 2x - 18$	To find the area, I would use the triangle formula and substitute the values for b and h. To simplify the expression, I would use the commutative property to rewrite the order, and then simplify one-half times four. Then I would use the distributive property.

Example: Find the area of the rectangle and simplify the expression. Show your work.

| 9 | x |
| 5 | |

Work	Explanation
$A = bh$ $A = (9 + x)(5)$ $A = (9 \cdot 5) + (x \cdot 5)$ $A = 45 + 5x$ or $A = 5x + 45$	To find the area, I would use the area formula for a rectangle and substitute the values for b and h. I would use the distributive property, and then simplify the expression.

Simplifying Variable Expressions

A *term* is a number, variable, product or quotient in an expression.

Example: In the variable expression $2x + 3y - 5$, the $2x$, $3y$, and -5 are terms of that expression.

A *coefficient* is a numerical factor in a term.

Example: In the variable expression $2x + 3y - 5$, the 2 is the coefficient of the x in the first term; the 3 is the coefficient of the y in the second term.

A *constant* term has a number but no variable. It is a quantity that always stays the same.

Example: In the expression $2x + 3y - 5$, the only number without a variable attached is -5, so -5 is the constant.

A *variable* term has a number and a variable.

Example: In the expression $2x + 3y - 5$, the $2x$ and $3y$ are variable terms.

Like terms are terms that have identical variable parts (i.e., same variable raised to the same power)

Example: Identify the like terms of the expression $3x + 4 + 5x$.

$3x$ and $5x$ have identical variable parts, so they are like terms.

Just to review, let's identify the terms, coefficients, and constant terms of the unsimplified expression $x + 7 - 4y - 3$. There are four terms: $x, \; 7, -4y, -3$. The coefficient of x is 1 and the coefficient of y is -4. The constant terms are 7 and -3.

We can write an expression such as $2y + 6y$ as a single term. The distributive property allows us to do this: $2y + 6y = (2 + 6)y = 8y$.

To simplify expressions or combine like terms, group the terms with the same variable raised to the same power. Then add or subtract the coefficients as indicated.

Example: $\underline{8x^2} + \underline{\underline{9x}} + 7 + \underline{2x^2} - \underline{\underline{4x}} + 3$ Mark like terms.**

$(8x^2 + 2x^2) + (9x - 4x) + (7 + 3)$ Group like terms (applying the commutative and associative properties).

$(8 + 2)x^2 + (9 - 4)x + (7 + 3)$ Use the distributive property.

$10x^2 + 5x + 10$ Simplify.

**Another way to mark like terms (suggested by teachers) is to use boxes as shown below. (It helped students to visualize that the sign in front of the number "belongs" to that number.) $\boxed{8x^2}\boxed{+9x} + 7 \boxed{+2x^2}\boxed{-4x} + 3$

An important link can be made to previous learning for students. Review place value and expanded notation.

Example:
$$672 = 6(100) + 7(10) + 2(1)$$
$$= 6(10^2) + 7(10) + 2(1)$$

This parallels polynomials $= 6x^2 + 7x + 2$

Now show that you combine like terms in algebra the same way you combine terms in arithmetic.

To add horizontally, from left to right, group the hundreds, the tens and the ones. Example:

$241 + 352$
$= 2(100) + 4(10) + 1(1) + 3(100) + 5(10) + 2(1)$
$= (2 + 3)100 + (4 + 5)10 + (1 + 2)(1)$
$= 5(100) + 9(10) + 3(1)$
$= 593$

To add polynomials, you add from left to right, grouping terms. Using the same numerals, an example:

$(2x^2 + 4x + 1) + (3x^2 + 5x + 2)$
$= 2x^2 + 4x + 1 + 3x^2 + 5x + 2$
$= (2 + 3)x^2 + (4 + 5)x + (1 + 2)1$
$= 5x^2 + 9x + 3$

Example: Combine like terms for the expression $4x + 3 + 5x$.
$$4x + 3 + 5x = (4x + 5x) + 3$$
$$= 9x + 3$$

Example: Simplify $5(x + 4) - 2x + 5$.

Use the distributive property to get rid of the parentheses.

$$= 5x + 20 - 2x + 5$$

Now combine like terms

$$= 5x - 2x + 20 + 5$$

$$= 3x + 25$$

Equations and Their Solutions

An *equation* is a mathematical statement that shows two expressions are equivalent. Another way to define it: an *equation* is a mathematical sentence formed by placing an *equal sign* between two expressions.

A *solution* is a number that produces a true statement when it is substituted for the variable in an equation. (I.e., a solution makes your equation a true statement.)

So to determine if a value is a solution of an equation simply substitute the given value in place of the variable and simplify to see if this value makes a true statement. If it is true, it is a solution of the equation.

Example: Given $15 = x - 7$, determine whether or not $x = 8$ is a solution.

$15 = 8 - 7$ Substitute the 8 (value of *x*)

$15 = 1$ Simplify and determine if statement is true or false.

$15 \neq 1$ This is false, so 8 is NOT a solution to this equation.

Note: students may have trouble identifying the difference between an equation and an expression—emphasize the "=" in an equation.

It is students also important that can write verbal sentences as equations. The following chart provides some verbal expressions that are commonly found in algebra. Please note the * indicates particularly difficult ones for students to understand.

Operation	Verbal Expression	Algebraic Expression
Addition +	a number plus 7	$n + 7$
Addition +	8 added to a number	$n + 8$
Addition +	a number increased by 4	$n + 4$
Addition +	5 more than a number	$n + 5$
Addition +	the sum of a number and 6	$n + 6$
Addition +	Tom's age 3 years from now	$n + 3$
Addition +	two consecutive integers	$n, n+1$
Addition +	two consecutive odd integers	Let $x = $ 1st odd, $x + 2 = $ 2nd odd
Addition +	2 consecutive even integers	Let $x = $ 1st even, $x + 2 = $ 2nd even
Subtraction –	a number minus 7	$x - 7$
Subtraction –	8 subtracted from a number*	$x - 8$
Subtraction –	a number decreased by 4	$x - 4$
Subtraction –	4 decreased by a number	$4 - x$
Subtraction –	5 less than a number*	$x - 5$
Subtraction –	the difference of a number and 6	$x - 6$
Subtraction –	Tom's age 3 years ago	$x - 3$
Subtraction –	separate 15 into two parts*	$x,\ 15 - x$
Multiplication • ( )	12 multiplied by a number	$12n$
Multiplication • ( )	9 times a number	$9n$
Multiplication • ( )	the product of a number and 5	$5n$
Multiplication • ( )	Distance traveled in x hours at 50 mph	$50x$
Multiplication • ( )	twice a number	$2n$
Multiplication • ( )	half of a number	$\frac{1}{2}n$
Multiplication • ( )	number of cents in x quarters	$25x$
Division ÷	a number divided by 12	$\frac{x}{12}$
Division ÷	the quotient of a number and 5	$\frac{x}{5}$
Division ÷	8 divided into a number	$\frac{x}{8}$

Examples: Write the verbal sentence as an equation. Solve using mental math.
(a) The sum of a number and 5 is 12. $n + 5 = 12,\ n = 7$
(b) The product of a number and 4 is 20. $4x = 20,\ n = 5$
(c) 12 equals what number plus 2? $12 = a + 2,\ a = 10$
(d) 7 is the quotient of what number divided by 5? $7 = \dfrac{y}{5},\ y = 35$

Two other commonly used words are to "*square*" or "*cube*" a number. Make sure students understand these to represent x^2, x^3 respectively.

Solving Equations

To *solve an equation* is to find the value(s) of *x* which make the equation a true statement.

Strategy for Solving Equations: To solve linear equations, put the variable terms on one side of the equal sign, and put the constant (number) terms on the other side. To do this, use opppsite or inverse operations. *Inverse operations* are two operations that undo each other, such as addition and subtraction. There are several properties that will help us obtain equivalent equations by performing inverse operations.

Properties of Equality

Addition Property of Equality $\qquad$ if $a = b$, then $a + c = b + c$

Subtraction Property of Equality $\qquad$ if $a = b$, then $a - c = b - c$

Multiplication Property of Equality $\qquad$ if $a = b$, then $ac = bc$

Division Property of Equality $\qquad$ if $a = b$, $c \neq 0$, then $\dfrac{a}{c} = \dfrac{b}{c}$

We used the Order of Operations to evaluate expressions such as $5 + 3x$ when $x = 4$.

The expression $5 + 3x$ was rewritten substituting 4 for *x*.

$$
\begin{aligned}
&5 + 3x \\
&= 5 + 3(4) \\
&= 5 + 12 \\
&17
\end{aligned}
$$

So we now know that $5 + 3x = 17$ when $x = 4$.

The question becomes, can we find the value of *x*, the solution, if we know that $5 + 3x = 17$?

To undo a variable expression and isolate the variable, we must use the Order of Operations in reverse using the opposite (inverse) operations. Let's look at a gift wrapping analogy to better understand this strategy. When a present is wrapped, it is placed in a box, the cover is put on, the box is wrapped in paper, and finally a ribbon is added to complete the project. To get the present out of the box, everything would be done in reverse order, performing the OPPOSITE (INVERSE) OPERATION. First we take off the ribbon, then take off the paper, next take the cover off, and finally take the present out of the box.

To isolate the variable means to have all the variables on one side of an equation and the numbers on the other side. In this chapter we will practice one-step equations.

Example: Solve $x + 5 = 9$.

To solve this equation, undo the variable expression on the left side by using the order of operations in reverse using the opposite (inverse) operation.

We have addition and to get rid of that we subtract 5 from both sides. This is applying the Subtraction Property of Equality. Check your answer!

$$x + 5 = 9$$
$$x + 5 - 5 = 9 - 5$$
$$x = 4$$
$$4 + 5 = 9✔$$

It is also common practice to simply show subtracting five from both sides as shown below.

$$x + 5 = 9$$
$$-5 = -5$$
$$x = 4$$
$$4 + 5 = 9✔$$

We can also introduce what the graph of the solution would look like at this point, but this will also be covered in unit 3.
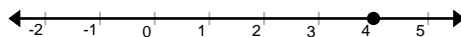
Example: Find the solution of $y - 3 = 12$.

There is a subtraction of 3; to get rid of that, add 3 to both sides (Addition Property of Equality).

$$y - 3 = 12 \qquad\qquad y - 3 = 12$$
$$y - 3 + 3 = 12 + 3 \qquad\qquad +3 = +3$$
$$\text{or}$$
$$y = 15 \qquad\qquad y = 15$$
$$15 - 3 = 12✔ \qquad\qquad 15 - 3 = 12$$

Example: Solve the equation $-4x = 32$.

There are no additions or subtractions. To isolate the variable, undo the multiplication by dividing both sides of the equation by –4.

$$-4x = 32$$
$$\frac{-4x}{-4} = \frac{32}{-4}$$
$$x = -8$$
$$-4(-8) = 32 ✔$$

Example: Solve for y. $\frac{y}{3} = 10$

There are no additions or subtractions, so we undo the division by multiplying both sides by 3.

$$\frac{y}{3} = 10$$
$$3\left(\frac{y}{3}\right) = 3(10)$$
$$y = 30$$
$$\frac{30}{3} = 10 ✔$$

The rules of math do not change when using different number sets. So we solve equations the same whether there are fractions, decimals, or integers in the equation. This is an excellent time to review operations with decimals by solving one-step equations with decimals.

Example:

$$x + 4.25 = -2.5$$
$$x + 4.25 - 4.25 = -2.5 - 4.25$$
$$x = -6.75$$
$$-6.75 + 4.25 = -2.5 ✔$$

Or

$$x + 4.25 = -2.5$$
$$-4.25 = -4.25$$
$$x = -6.75$$
$$-6.75 + 4.25 = -2.5 ✔$$