# Pre-Algebra 8 Notes – Unit 02B: Linear Equations in One Variable—Multi-Step Equations

## Solving Two-Step Equations

The general strategy for solving a multi-step equation in one variable is to rewrite the equation in $ax+b=c$ format (where *a*, *b,* and *c* are real numbers), then solve the equation by isolating the variable using the Order of Operations in reverse and using the opposite operation. (Remember the analogy to unwrapping a gift…)

**<u>Order of Operations</u>**

1. Parentheses (Grouping)
2. Exponents
3. Multiply/Divide, left to right
4. Add/Subtract, left to right

Evaluating an arithmetic expression using the Order of Operations will suggest how we might go about solving equations in the $ax+b=c$ format.

To evaluate an arithmetic expression such as $4+2\cdot 5$, we'd use the Order of Operations.

| | |
|---|---|
| $4+\underline{2\cdot 5}=$ | First we multiply, $2\cdot 5$ |
| $4+10=$ | Second we add, $4+10$ |
| $14$ | |

Now, rewriting that expression, we have $2\cdot 5+4=14$, a form that leads to equations written in the form $ax + b = c$. If I replace 5 with *n*, I have

$2\cdot n+4=14$ *or*
$2n+4=14$,
an equation in the $ax+b=c$ format.

To solve that equation, I am going to "undo" the expression "$2n + 4$***".*** ***I will isolate the variable by using the Order of Operations in reverse and using the opposite operation.***

That translates to getting rid of any addition or subtraction first, then getting rid of any multiplication or division next. Undoing the expression and isolating the variable results in finding the value of *n*.

This is what it looks like:

$$\begin{aligned} 2n+4&=14 \\ 2n+4-4&=14-4 \\ 2n&=10 \\ \frac{2n}{2}&=\frac{10}{2} \\ n&=5 \end{aligned}$$

subtract 4 from each side to "undo" the addition

divide by 2 to "undo" the mulitplication

$$\begin{aligned} 2n+4&=\ 14 \\ -4&=-4 \\ 2n&=10 \\ \frac{2n}{2}&=\frac{10}{2} \\ n&=5 \end{aligned}$$

Check your solution by substituting the answer back into the original equation.

$2n+4=14$ original equation

$2(5)+4=14$ substitute '5' for '$n$'

$10+4=14$

$14=14$ true statement, so my solution is correct

*Example*: Solve for *x*, $3x-4=17$.

Using the general strategy, we always want to "undo" whatever has been done in reverse order. We will undo the subtracting first by adding, and then undo the multiplication by dividing.

$$\begin{aligned} 3x-4&=17 \\ 3x-4+4&=17+4 \\ 3x&=21 \\ \frac{3x}{3}&=\frac{21}{3} \\ x&=7 \end{aligned}$$

*or*

$$\begin{aligned} 3x-4&=17 \\ +4&=+4 \\ 3x&=21 \\ \frac{3x}{3}&=\frac{21}{3} \\ x&=7 \end{aligned}$$

Check:

$$\begin{aligned} 3x-4&=17 \\ 3(7)-4&=17 \\ 21-4&=17 \\ 17&=17 \checkmark \end{aligned}$$

*Example:* Solve for *x*, $\frac{x}{4}+5=12$

$$\frac{x}{4}+5=12 \qquad \textit{or} \qquad \frac{x}{4}+5=12$$

$$\frac{x}{4}+5-5=12-5 \qquad\qquad -5=-5$$

$$\frac{x}{4}=7 \qquad\qquad \frac{x}{4}=7$$

$$(4)\left(\frac{x}{4}\right)=(4)(7) \qquad\qquad (4)\left(\frac{x}{4}\right)=(4)(7)$$

$$x=28 \qquad\qquad x=28$$

Check:

$$\frac{x}{4}+5=12$$

$$\frac{(28)}{4}+5=12$$

$$7+5=12$$

$$12=12\checkmark$$

*****NOTE:** Knowing how to solve equations in the $ax+b=c$ format is extremely important for success in algebra. All other equations will be solved by converting equations to $ax+b=c$. To solve systems of equations, we rewrite the equations into one equation in the $ax+b=c$ form and solve. (In the student's algebraic future, to solve quadratic equations we will rewrite the equation into factors using $ax+b=c$, then solve the resulting equation letting $c=0$. It is important that students are comfortable solving equations in the $ax+b=c$ format.)

## Solving Equations that Are NOT in the $ax+b=c$ Format

The general strategy for solving equations NOT in the $ax+b=c$ format is to rewrite the equation in $ax+b=c$ format using the Properties of Real Numbers.

An important problem solving/learning strategy is to take a problem that you don't know how to do and transform that into a problem that you know how to solve. Right now, we know how to solve problems such as $ax+b=c$. I cannot make that problem more difficult, but what I can do is make it look different.

For instance, if I asked you to solve $5(2x+3)-4=21$, that problem looks different from the $2x+4=14$ that we just solved by using the Order of Operations in reverse. The question you have to ask is "what is physically different in these two problems?"

The answer is the new problem has parentheses. That means to make these problems look alike, we need to get rid of those parentheses. We'll do that by using the distributive property.

$$5(2x+3)-4=21$$

$$10x+15-4=21 \qquad \text{we applied the distributive property}$$

$$10x+11=21 \qquad \text{we combined like terms}$$

Now I have converted $5(2x+3)-4=21$ into $ax+b=c$ format as $10x+11=21$. We recognize this format and know how to solve it.

$$\begin{aligned} 10x+11&=21 \\ 10x+11-11&=21-11 \\ 10x&=10 \\ \frac{10x}{10}&=\frac{10}{10} \\ x&=1 \end{aligned} \qquad \textit{or} \qquad \begin{aligned} 10x+11&=\ \ 21 \\ -11&=-11 \\ 10x&=10 \\ \frac{10x}{10}&=\frac{10}{10} \\ x&=1 \end{aligned}$$

Check:

$$\begin{aligned} &5(2x+3)-4=21 \\ &5\left[2(1)+3\right]-4=21 \\ &5\left[2+3\right]-4=21 \\ &5\left[5\right]-4=21 \\ &25-4=21 \\ &21=21\checkmark \end{aligned}$$

*Example:* Solve for *x*, $4(3x-2)+5=45$.

The parentheses make this problem look different than the $ax + b = c$ problems.
Get rid of the parentheses by using the distributive property, and then combine terms.

$$\begin{aligned} 4(3x-2)+5&=45 \\ 12x-8+5&=45 \\ 12x-3&=45 \\ 12x-3+3&=45+3 \\ 12x&=48 \\ \frac{12x}{12}&=\frac{48}{12} \\ x&=4 \end{aligned}$$

We will leave the check to you!

***Caution!*** Note that when we solved these equations we got rid of the parentheses first. Students will often want to get rid of the addition/subtraction first because they are using the Order of Operations in reverse. The reason we get rid of the parentheses first is because the strategy is to rewrite the equation in $ax+b=c$ format <u>before</u> using the Order of Operations in reverse.

**Solving Linear Equations with Variables on Both Sides**

Using the same strategy, we rewrite the equation so it is in $ax+b=c$ format.

*Example:* Solve for *x*, $5x+2=3x+14$.

We ask ourselves, "what is physically different in this problem?" This equation is different because there are variables on both sides of the equation. The strategy is to rewrite the equation in $ax+b=c$ format which calls for variables on only one side of the equation.

What's different? There is a $3x$ on the other side of the equation. How do we get rid of the addition of $3x$?

$$\begin{array}{rl}
5x+2=3x+14 & \\
5x+2-\mathbf{3x}=3x+14-\mathbf{3x} & \text{Subtraction Property of Equality}\\
5x-\mathbf{3x}+2=3x-\mathbf{3x}+14 & \text{Commutative Property of Addition}\\
2x+2=14 & \text{Combining like terms}
\end{array}$$

Now the equation is in $ax+b=c$ format.

$$\begin{array}{rl}
2x+2=14 & \\
2x+2-2=14-2 & \text{Subtraction Property of Equality}\\
2x=12 & \text{Arithmetic fact}\\
\dfrac{2x}{2}=\dfrac{12}{2} & \text{Division Property of Equality}\\
x=6 & \text{Arithmetic fact}
\end{array}$$

Let's try a few more examples:

$$\begin{aligned}
5x-17 &= 3x-9\\
-3x \quad &= -3x\\
2x-17 &= -9\\
+17 &= +17\\
2x &= 8\\
\frac{2x}{2} &= \frac{8}{2}\\
x &= 4
\end{aligned}$$

$$\begin{aligned}
-2x-17 &= 6-x\\
+x \quad &= \quad +x\\
-1x-17 &= 6\\
+17 &= +17\\
-1x &= 23\\
\frac{-1x}{-1} &= \frac{23}{-1}\\
x &= -23
\end{aligned}$$

$$\begin{aligned}
3m+8-5m &= 9+4m+29\\
-2m+8 &= 4m+38\\
-2m-4m+8 &= 4m-4m+38\\
-6m+8 &= 38\\
-8 &= -8\\
-6m &= 30\\
\frac{-6m}{-6} &= \frac{30}{-6}\\
m &= -5
\end{aligned}$$

***Now, I can make equations longer, but I cannot make them more difficult!***

***CCSS 8.EE.7b: Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.***

*Example:* Solve for *x*, $5(x+2)-3=3(x-1)-2x$.

In this problem, there are parentheses and variables on both sides of the equation. It is clearly a longer problem. The strategy remains the same: rewrite the equation in $ax+b=c$ format, and then isolate the variable by using the Order of Operations in reverse using the opposite operation.

Physically, this equation looks different because there are parentheses, so let's get rid of them by using the distributive property.

| | |
|---|---|
| $5(x+2)-3=3(x-1)-2x$ | |
| $5x+10-3=3x-3-2x$ | Distributive Property |
| $5x+7=x-3$ | Combine like terms |
| $5x+7-x=x-3-x$ | Subtraction Property of Equality |
| $4x+7=-3$ | Combine like terms |
| $4x+7-7=-3-7$ | Addition Property of Equality |
| $4x=-10$ | Simplify |
| $\frac{4x}{4}=\frac{-10}{4}$ | Division Property of Equality |
| $x=\frac{-5}{2}\,or-2.5$ | Simplify |

***Caution!*** When solving equations, we often write arithmetic expressions such as $-3+(-7)$ as $-3-7$ as we did in the above problem. Students need to be reminded that when there is not an "extra" minus sign, the computation is understood to be an addition problem.

*Examples:*

$$(+5)+(+6)=5+6=11$$
$$(-5)+(-6)=-5-6=-11$$
$$(-8)+(+3)=-8+3=-5$$

Emphasizing this will help students to solve equations correctly. If not taught, students often solve the equation correctly only to make an arithmetic mistake—then they think the problem they are encountering is algebraic in nature (instead of arithmetic).

Also make sure students understand simplifying expressions with several negatives.

*Examples:*

$$5-(-4)=5+4=9$$
$$-3-(-8)=-3+8=5$$
$$-2-5=-2+(-5)=-7$$

## Solving Equations with "No Solutions" or "Infinitely Many" as a Solution

***CCSS 8.EE.7a-1: Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. CCSS 8.EE.7a-2: Show whether a linear equation in one variable has one solution, infinitely many solutions or no solutions by successively transforming the given equation into simpler forms, until an equivalent equations of the form*** $x=a,\ a=a,$ or $a=b$ ***results (where*** $a$ ***and*** $b$ ***are different numbers).***

When you solve an equation, you *may* find something unusual happens: when you subtract the variable from both sides (in an effort to get the variable term on one side of the equation), no variable term remains! Your solution is either "no solution" or "infinitely many". Look at the following examples.

*Example:*

Solve $3(3x-1)=9x$.

$$3(3x-1)=9x$$
$$9x-3=9x$$

Notice we could stop now if we recognized that it is impossible for a number 9*x* to be equal to 3 less than itself. If we continue…..

$$\begin{aligned} 9x-3 &= \ \ 9x \\ -9x \quad &= -9x \\ -3 &= 0 \end{aligned}$$

….we still have a statement that is <u>not</u> true.
So the equation is said to have NO SOLUTION.

*Example:*

Solve $8x-2=2(4x-1)$.

$$8x-2=2(4x-1)$$
$$8x-2=8x-2$$

Notice that this statement would be true no matter what value we substitute for *x*. We could go further….

$$\begin{aligned} 8x-2 &= 8x-2 \\ -8x &= -8x \\ -2 &= -2 \end{aligned}$$

…. We still get a true statement. So the equation is said to have a solution of INFINITELY MANY.