Lesson 4: Comparing Methods—Long Division, Again?

Student Outcomes

- Students connect long division of polynomials with the long division algorithm of arithmetic and use this algorithm to rewrite rational expressions that divide without a remainder.

Lesson Notes

This lesson reinforces the analogous relationship between arithmetic of numbers and the arithmetic of polynomials (**A-APR.6**, **A-APR.7**). These standards address working with rational expressions and focus on using a long division algorithm to rewrite simple rational expressions. In addition, it provides another method for students to fluently calculate the quotient of two polynomials after the Opening Exercises.

Classwork

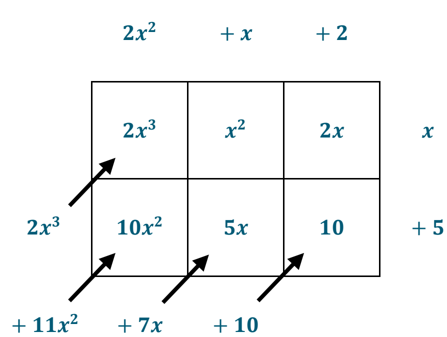
Opening

Have students work individually on the Opening Exercises to confirm their understanding of the previous lesson's outcomes. Circulate around the room to observe their progress, or have students check their work with a partner after a few minutes. Today's lesson will transition to another method for dividing polynomials.

Opening Exercises (5 minutes)

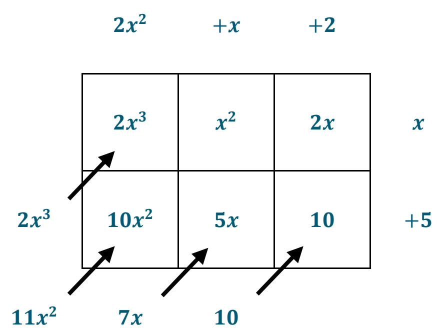
Opening Exercises

1. **Use the reverse tabular method to determine the quotient $\frac{2x^3+11x^2+7x+10}{x+5}$.**



2. **Use your work from Exercise 1 to write the polynomial $2x^3 + 11x^2 + 7x + 10$ in factored form, and then multiply the factors to check your work above.**

$(x + 5)(2x^2 + x + 2)$



The product is $2x^3 + 11x^2 + 7x + 10$.

Division and multiplication of polynomials is very similar to those operations with real numbers. In these problems, if you let $x = 10$, the result would match an arithmetic problem. Yesterday, we divided two polynomials using the reverse tabular method. Today, we want to see how polynomial division is related to the long division algorithm learned in elementary school.

Discussion (5 minutes)

We have seen how division of polynomials relates to multiplication and that both of these operations are similar to the arithmetic operations you learned in elementary school.

- Can we relate division of polynomials to the long division algorithm?
  - *We would need to use the fact that the terms of a polynomial expression represent place value when* $x = 10$.

Prompt students to consider the long division algorithm they learned in elementary school, and ask them to apply it to evaluate $1573 \div 13$. Have a student model the algorithm on the board as well. The solution to this problem is included in the example below.

Example 1 (5 minutes): The Long Division Algorithm for Polynomial Division

MP.7

When you solve the problem in Example 1, be sure to record the polynomial division problem next to the arithmetic problem already on the board. Guide students through this example to demonstrate the parallels between the long division algorithm for numbers and this method. You can emphasize that the long division algorithm they learned in elementary school is a special case of polynomial long division. They should record the steps on their handouts or in their notebooks. Have students check their work by solving this problem using the reverse tabular method. Use the questions below as you work the example.

See the sample problem written out after Example 1, and use the questions that follow as discussion points while you model this algorithm.

- What expression multiplied by x will result in x^3?
  - x^2
- When you do long division, you multiply the first digit of the quotient by the divisor and then subtract the result. It works the same with polynomial division. How do we represent multiplication and subtraction of polynomials?
  - *You apply the distributive property to multiply, and to subtract you add the opposite.*
- Then, we repeat the process to determine the next term in the quotient. What do we need to bring down to complete the process?
  - *You should bring down the next term.*

> *Scaffolding:*
>
> - For further scaffolding consider starting with a simpler problem such as $126 \div 18$. Have students compare this problem to the polynomial division problem $(x^2 + 2x + 6) \div (x + 8)$ by explaining the structural similarities. Have students consider this when you place them side by side on the board. This will show students that if we let $x = 10$, the polynomial division problem is analogous to the integer division problem.
> - For advanced learners, you could challenge them to create two examples, a numerical one and a polynomial one, that illustrate the structural similarities. Note, however, that not every problem will work nicely. For example, $800 \div 32 = 25$, but $8x^2 \div (3x + 2) \neq 2x + 5$ because there are many polynomials in x that evaluate to 25 when $x = 10$.

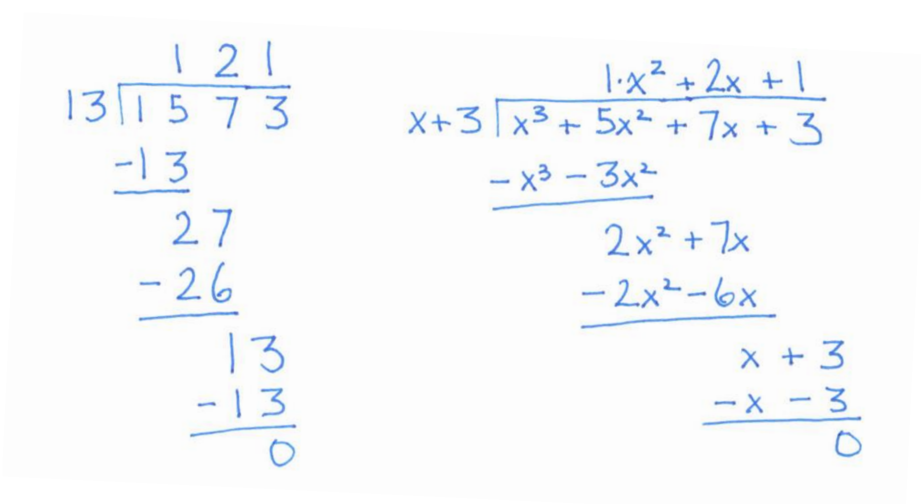
Example 1

If $x = 10$, then the division $1573 \div 13$ can be represented using polynomial division.

$$x+3\overline{)x^3+5x^2+7x+3}$$

The quotient is $x^2 + 2x + 1$.

The completed board work for this example should look something like this:



Example 2 (5 minutes): The Long Division Algorithm for Polynomial Division

We can divide any two numbers as long as the divisor is not equal to 0. Similarly, we can divide any two polynomials as long as the divisor is not equal to 0. Note: The number 0 is also a polynomial. Because we are now dealing with a general case of polynomials and not simply numbers, we can solve problems where the coefficients of the terms are any real numbers. It would difficult, but not impossible, if the coefficients of the terms of our polynomials were irrational. In the next example, model again how this process works. Be sure to point out that students must use a 0 coefficient place holder for the missing x term.

Example 2

Use the long division algorithm for polynomials to evaluate

$$\frac{2x^3 - 4x^2 + 2}{2x - 2}.$$

The quotient is $x^2 - x - 1$.

Before beginning the next exercises, take the time to reinforce the idea that polynomial division is analogous to whole number division by posing a reflection question. Students can discuss this with a partner or respond in writing.

- Why are we able to do long division with polynomials?
  - *Polynomials form a system analogous to the integers. The same operations that hold for integers hold for polynomials.*

Exercises 1–8 (15 minutes)

These problems start simple and become more complicated. Monitor student progress as they work. Have students work these problems independently or in pairs and use this as an opportunity to informally assess their understanding. After students have completed the exercises, post the solutions on the board but not the work. Have students with errors team up with a partner and trade papers. Ask students to find the mistakes in their partner's work. You may choose an incorrect solution to display on the board and then lead a class discussion to point out where students are likely to make errors and how to prevent them. Students will typically make careless errors in multiplying or subtracting terms. Other errors can occur if they forget to include the zero coefficient place holder terms when needed. If students appear to be running short on time, have them check every other result using the reverse tabular method. Alternately, students could check their work using multiplication.

Exercises 1–8

Use the long division algorithm to determine the quotient. For each problem, check your work by using the reverse tabular method.

1. $\frac{x^2+6x+9}{x+3}$

   $x + 3$

2. $(7x^3 - 8x^2 - 13x + 2) \div (7x - 1)$

   $x^2 - x - 2$

3. $(x^3 - 27) \div (x - 3)$

   $x^2 + 3x + 9$

4. $\frac{2x^4+14x^3+x^2-21x-6}{2x^2-3}$

   $x^2 + 7x + 2$

5. $\frac{5x^4-6x^2+1}{x^2-1}$

$5x^2-1$

6. $\frac{x^6+4x^4-4x-1}{x^3-1}$

x^3+4x+1

7. $(2x^7+x^5-4x^3+14x^2-2x+7)\div(2x^2+1)$

x^5-2x+7

8. $\frac{x^6-64}{x+2}$

$x^5-2x^4+4x^3-8x^2+16x-32$

Closing (5 minutes)

Ask students to summarize the important parts of this lesson either in writing, to a partner, or as a class. Use this opportunity to informally assess their understanding prior to starting the Exit Ticket. Important elements are included in the Lesson Summary box below. The questions that follow are recommended to guide the discussions with sample student responses included in italics. Depending on the structure of your closure activity, the sample responses would be similar to student-written, partner, or whole-class summaries.

- Which method do you prefer, long division or the reverse tabular method?
  - *Student responses will vary. The reverse tabular method may appeal to visual learners. The long division algorithm works well as long as you avoid careless mistakes.*
- Is one method "easier" than another?
  - *This will depend on student preferences, but some will like the connection to prior methods for dividing and multiplying. Perhaps when many terms are missing (as in Exercise 8), the reverse tabular method can go more quickly than long division.*
- What advice would you give to a friend that is just learning how to do these problems quickly and accurately?
  - *Be careful when multiplying terms and working with negative terms.*

Lesson Summary

The long division algorithm to divide polynomials is analogous to the long division algorithm for integers. The long division algorithm to divide polynomials produces the same results as the reverse tabular method.

Exit Ticket (5 minutes)

Name ______________________________ Date ______________

Lesson 4: Comparing Methods—Long Division, Again?

Exit Ticket

Write a note to a friend explaining how to use long division to find the quotient.

$$\frac{2x^2 - 3x - 5}{x + 1}$$

Exit Ticket Sample Solutions

Write a note to a friend explaining how to use long division to find the quotient.

$$\frac{2x^2 - 3x - 5}{x + 1}$$

Set up the divisor outside the division sign and the dividend underneath it. Then ask yourself what number multiplied by x is $2x^2$. Then multiply that number by $x + 1$, and record the results underneath $2x^2 - 3x$. Subtract these terms and bring down the -5. Then repeat the process.

Problem Set Sample Solutions

Use the long division algorithm to determine the quotient.

1. $\frac{2x^3 - 13x^2 - x + 3}{2x + 1}$

   $x^2 - 7x + 3$

2. $\frac{3x^3 + 4x^2 + 7x + 22}{x + 2}$

   $3x^2 - 2x + 11$

3. $\frac{x^4 + 6x^3 - 7x^2 - 24x + 12}{x^2 - 4}$

   $x^2 + 6x - 3$

4. $(12x^4 + 2x^3 + x - 3) \div (2x^2 + 1)$

   $6x^2 + x - 3$

5. $(2x^3 + 2x^2 + 2x) \div (x^2 + x + 1)$

   $2x$

6. **Use long division to find the polynomial, p, that satisfies the equation below.**

   $$2x^4 - 3x^2 - 2 = (2x^2 + 1)(p(x))$$

   $x^2 - 2$

7. **Given $q(x) = 3x^3 - 4x^2 + 5x + k$.**

   a. **Determine the value of k so that $3x - 7$ is a factor of the polynomial q.**

      $k = -28$

   b. **What is the quotient when you divide the polynomial q by $3x - 7$?**

      $x^2 + x + 4$

MP.2 & MP.8

8. In parts (a)–(b) and (d)–(e), use long division to evaluate each quotient. Then, answer the remaining questions.

a. $\frac{x^2-9}{x+3}$

$x-3$

b. $\frac{x^4-81}{x+3}$

$x^3-3x^2+9x-27$

c. Is $x+3$ a factor of x^3-27? Explain your answer using the long division algorithm.

No. The remainder is not 0 when you perform long division.

d. $\frac{x^3+27}{x+3}$

x^2-3x+9

e. $\frac{x^5+243}{x+3}$

$x^4-3x^3+9x^2-27x+81$

f. Is $x+3$ a factor of x^2+9? Explain your answer using the long division algorithm.

No. The remainder is not 0 when you perform long division.

g. For which positive integers n is $x+3$ a factor of x^n+3^n? Explain your reasoning.

Only if n is an odd number. By extending the patterns in parts (a)–(c) and (e), we can generalize that $x+3$ divides evenly into x^n+3^n for odd powers of n only.

h. If n is a positive integer, is $x+3$ a factor of x^n-3^n? Explain your reasoning.

Only for even numbers n. By extending the patterns in parts (a)–(c), we can generalize that $x+3$ will always divide evenly into the dividend.