Objectives for Inverse Function Activity

- Find the input of a function given an output
- Find the inverse function
- Determine the domain and range of function and its inverse
- Determine whether or not an inverse function exists
- Use and interpret inverse function notation

Inverse Functions

Inverse Function Notation

1. Explain the difference in meaning of the notation $f(2)=5$ versus the notation $f^{-1}(5)=2$.

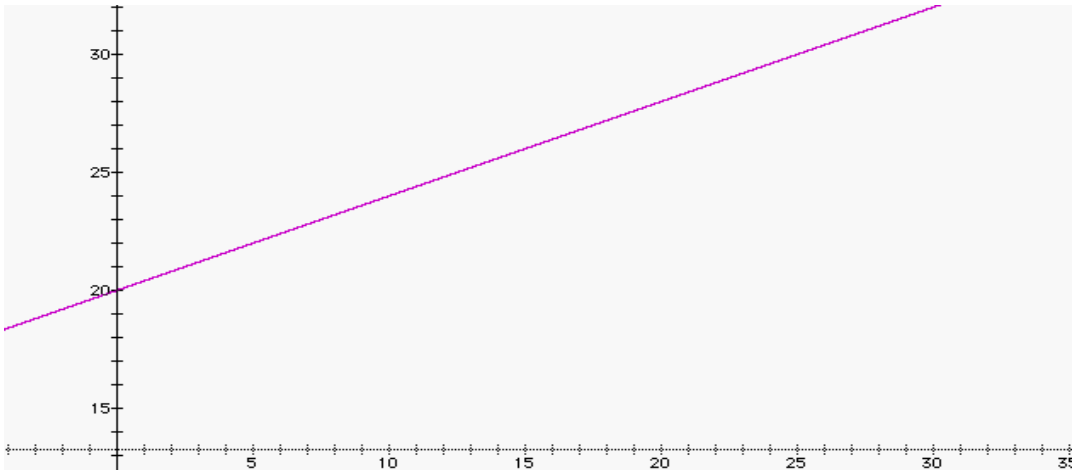
2. Suppose the point (10, -5) lies on the graph of a function f. What point lies on the graph of f^{-1}?

3. The number of people (in thousands) in a city is given by the function $f(t) = 20 + 0.4t$, where t is the number of years since 1970.

a. In the context of this problem, explain what $f(25)$ and $f^{-1}(25)$ mean (no calculations required). What is the unit of measure (number of people or number of years) for $f(25)$ and $f^{-1}(25)$?

b. Now calculate $f^{-1}(25)$.

4. The graph of f from problem 3 is shown below. Estimate $f^{-1}(25)$ by reading the graph below.



5. Suppose we have the function $w = j(x)$ where *w* represents the average daily quantity of water (in gallons) required by an oak tree of height *x* feet.

a. What does the expression $j(25)$ represent? What are its units of measure?

b. What does the expression $j^{-1}(25)$ represent? What are its units of measure?

c. What does the following equation tell you about *v*: $j(v) = 50$

d. Re-write the statement $j(v) = 50$ in terms of j^{-1}.

e. On a certain acreage, oak trees on average measure *z* feet high and an oak tree of average height requires *p* gallons of water. Represent this statement first in terms of j and then in terms of j^{-1}.

6. The total cost, *C*, in dollars for a clothing factory to make '*j*' jackets is given by the function *C* = *f*(*j*). Interpret the meaning of the following notation within the context of the story just given.

a. $f(30)=678$

b. $f^{-1}(30) = 678$

Calculating Inverses Numerically

1. Using the chart, find

x	$h(x)$
-2	3
-1	-2
0	5
1	0
2	-1
7	8

a. $h(0)$

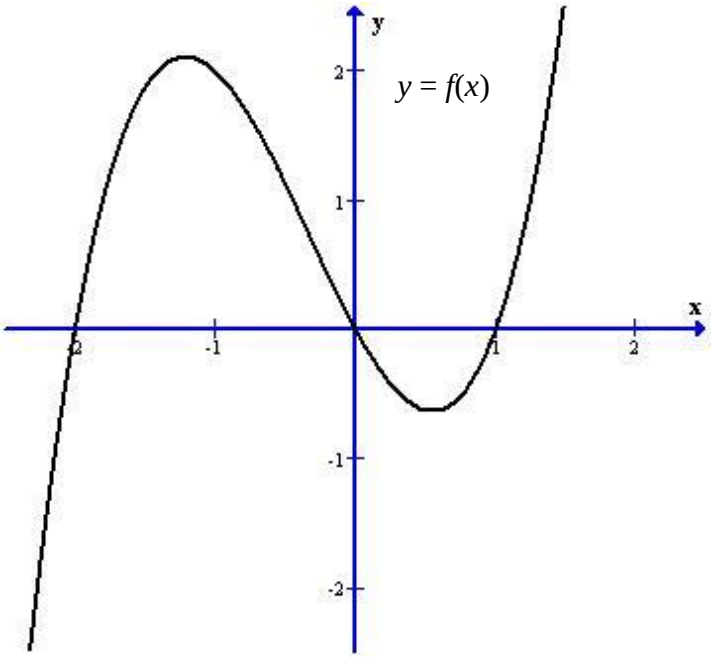
b. $h^{-1}(-1)$

c. $h(-2)$

d. $h^{-1}(-2)$

e. $(h(2))^{-1}$

2. Using the graph estimate:



a. $f(0)$

b. $f^{-1}(0)$

c. $f(-1)$

d. $f^{-1}(-1)$

Calculating Inverse Functions

For the following functions find:

a. The inverse function
b. Write the inverse function using inverse function notation
c. State the domain and range of the original function
d. State the domain and range of the inverse function

1. $f(x) = 3x - 2$

2. $g(x) = \frac{1}{x} - 2$

3. $h(x) = \sqrt{1+x}$

4. The formula $F = f(C) = 1.8C + 32$ converts temperatures in degrees Celsius, *C*, to degrees Fahrenheit, *F*.

a. What is the *input* to the function *f*? What is the *output*?

b. Find a formula for the inverse function giving Celsius as a function of Fahrenheit.

c.Use inverse function notation to write your formula.

$f^{-1}(\_\_) = \_\_\_\_$

d.What is the *input* to the function f^{-1}? the *output*?

e. Interpret the meaning of the notation: *f*(50) = 122

f. Interpret the meaning of the notation: $f^{-1}(200) = 93.3$

5. The formula $V = f(r) = \frac{4}{3}\pi r^3$ gives the volume of a sphere of radius *r*.

a. What is the *input* to the function *f*? What is the *output*?

b. Find a formula for the inverse function giving radius as a function of volume.

c. Use inverse function notation to write your formula found in #2 above.

$f^{-1}(\_\_) =$ _____

d. What is the *input* to the function f^{-1}? the *output*?

e. Suppose you already know the radius of the sphere. Which function gives you the volume?

f. Now suppose you already know the volume. Which function gives you the radius?

g. Explain the meaning of $f^{-1}(V) = 5$.

Verifying Inverse Functions

1. Suppose $f\ x\ = 2x-4$ and $g\ x\ = \frac{x+4}{2}$. Are *f* and *g* inverse functions?

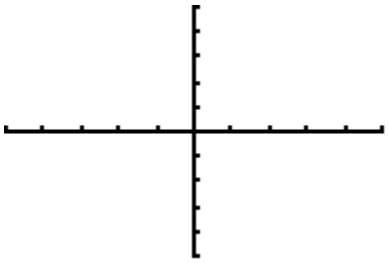
a) Use algebraic methods to verify. That is, find *f*(*g*(*x*)) and then find *g*(*f*(*x*)).

First find *f*(*g*(*x*)):

Now find *g*(*f*(*x*)):

What do you conclude?

b) Demonstrate the inverse relationship by means of a graph:



c) Explain verbally:

Describe in words what ***f*** "does to its input." i. ii.	Describe in words what ***g*** "does to its input." i. ii.

d) Fill in the cells for the output and then explain the inverse relationship:

$f\ x\ = 2x-4$							
input	2	3	4	5	6	7	8
output							
$g\ x\ = \frac{x+4}{2}$							
input	0	2	4	6	8	10	12
output							