PREREQUISITE SKILLS:
- students must have a clear understanding of signed numbers and their operations
- students must understand meaning of operations and how they relate to one another
- students must be able to make reasonable estimates
- students must know how to write expressions and equations and simplify like terms
- students must have a clear understanding of the definition of absolute value

VOCABULARY:
- equivalent equation: two equations that have the same solution set
- equation: a mathematical statement that two expressions are equivalent
- variable: a symbol used to represent a quantity that can change
- linear equation in one variable- an equation that can be written in the form $ax = b$ where a and b are constants and $a \neq 0$
- term of an expression: the parts of the expression that are added or subtracted
- solution of an equation in one variable: a value or values that make the equation true
- evaluate: to find the value of an algebraic expression by substituting a number for each variable and simplifying by using the order of operations
- formula: a literal equation that states a rule for a relationship among quantities
- literal equation: an equation that contains two or more variables
- solution set: the set of elements from the replacement set that makes an open sentence true
- inequality: a sentence formed when an inequality symbol is placed between two expressions
- absolute value: the distance between the origin and the point representing the real number
- compound inequality: two inequalities connected by *and* or *or*

SKILLS:
- solve and graph linear inequalities in one variable
- solve and graph compound inequalities in one variable
- write, solve and graph absolute value equations and inequalities in one variable
- Create linear inequalities and absolute value equations and inequalities to model real world situations

STANDARDS:

A.REI.A.1 Understand solving equations as a process of reasoning and explain the reasoning.
Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

A.REI.B.3 Understand solving equations as a process of reasoning and explain the reasoning.
Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

A.CED.A.1-1 Create equations that describe numbers or relationships.
Create equations and inequalities in one variable and use them to solve problems.

A.CED.A.3 Create equations that describe numbers or relationships.
Represent constraints by equations or inequalities and interpret solutions as viable or non-viable options in a modeling context.

LEARNING TARGETS:

- 3.1 To solve and graph linear inequalities in one variable
- 3.2 To create linear inequalities in one variable
- 3.3 To solve and graph compound inequalities in one variable
- 3.4 To create linear inequalities in one variable to model real world situations
- 3.5 To solve and graph absolute value equations in one variable
- 3.6 To solve and graph absolute value inequalities in one variable
- 3.7 To create absolute value equations and inequalities in one variable to model real world situations

BIG IDEA: Solving linear equations and inequalities in one variable is a basic algebraic skill. Mastering this skill will enable the students to solve higher-order equations, such as quadratic equations and more advanced equations in future courses.

Note: It is helpful to have the students always check their answers for accuracy and reasonableness.

Notes, Examples and Exam Questions

Unit 3.1 and 3.2 To solve and graph linear inequalities in one variable and To create linear inequalities in one variable.

Now that the students are familiar with solving one-variable linear equations, the transition to inequalities should be seamless. Remind students of the properties of equality and introduce the properties of inequalities. Have students discover what happens when we multiply or divide an inequality by a negative value.

An **inequality** is a statement that compares two expressions that are not strictly equal by using one of the following inequality symbols.

Symbol	Meaning	Graph
$x <$	is less than	Open dot, arrow left
$x \le$	is less than or equal to	Solid dot, arrow left
$x >$	is greater than	Open dot, arrow right
$x \ge$	is greater than or equal to	Solid dot, arrow right

A **solution of an inequality** is any value of the variable that makes the inequality true.

Properties of inequality:

Addition Property of Inequality: if $a > b$, then $a + c > b + c$
if $a < b$, then $a + c < b + c$

Subtraction Property of Inequality: if $a > b$, then $a - c > b - c$
if $a > b$, then $a - c > b - c$

Multiplication Property of Inequality: if $a > b$ and c > 0, then $ac > bc$
if $a < b$ and c > 0, then $ac < bc$
if $a > b$ and c < 0, then $ac < bc$
if $a < b$ and c < 0, then $ac > bc$

Division Property of Inequality: if $a > b$ and c > 0, then $\frac{a}{c} > \frac{b}{c}$
if $a < b$ and c > 0, then $\frac{a}{c} < \frac{b}{c}$
if $a > b$ and c < 0, then $\frac{a}{c} < \frac{b}{c}$
if $a < b$ and c < 0, then $\frac{a}{c} > \frac{b}{c}$

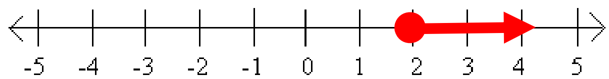
Students should follow the same plan for solving inequalities as was done when solving equations. Use inverse operations to isolate the variable, solve for the variable and then check the solution. Students will also be asked to graph their solutions. The one thing that has to be remembered is that if you multiple or divide by a negative number, then switch the direction of the inequality.

Ex 1: Solve the linear inequality. Write your answer in set notation. Graph the solution on the number line.

$8x-1\ge 15$

$8x-1\ge 15 \quad\Rightarrow\quad 8x\ge 16 \quad\Rightarrow\quad \boxed{x\ge 2}$

set notation: $\{x \mid x\in\mathbb{R} \text{ where } x\ge 2\}$

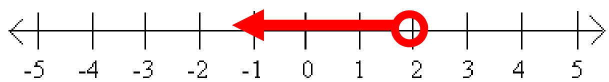


Ex 2: Solve and graph the solution. $-2>n-4$

$-2>n-4 \quad\Rightarrow\quad -2+4>n-4+4 \quad\Rightarrow\quad 2>n$

Using the symmetric property, we get: $\boxed{n<2}$

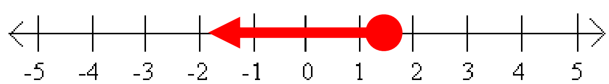
set notation: $\{x \mid x\in\mathbb{R} \text{ where } x<2\}$



Ex 3: Solve and graph the solution. $-4.2m\ge 6.3$

$-4.2m\ge 6.3 \quad\Rightarrow\quad \dfrac{-4.2m}{-4.2}\ge\dfrac{6.3}{-4.2}$ dividing by -4.2 will reverse the inequality symbol $\Rightarrow$ $\boxed{m\le 1.5}$

set notation: $\{x \mid x\in\mathbb{R} \text{ where } x\le 1.5\}$



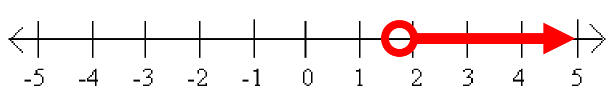
Ex 4: Solve and graph the solution. $-2(m-3)<5(m+1)-12$

$-2(m-3)<5(m+1)-12 \quad\Rightarrow\quad -2m+6<5m+5-12$

$-7m<-13$ dividing by -7 will reverse the inequality symbol

$\Rightarrow \boxed{m>\dfrac{13}{7}>1\dfrac{6}{7}}$

set notation: $\left\{x \,\middle|\, x\in\mathbb{R} \text{ where } x>\dfrac{13}{7}\right\}$



Unit 3.3 and 3.4 To solve and graph compound inequalities in one variable and To create linear inequalities in one variable to model real world situations.

Compound (Double) Inequalities:

A **compound** inequality is a sentence with two inequality statements joined either by the word "or" or by the word "and". "And" indicates that both statements of the compound sentence are true at the same time. It is the overlap or intersection of the two solution sets for the individual statements. The word "and" may not be in the statement and instead it will be an inequality with three expressions. The process for solving compound inequalities is similar in some ways to solving single inequalities and yet very different in other ways. Since there are two inequalities, there isn't any way to get the variables on "one side" of the inequality and the numbers on the other. One simple way is to think of the inequality as having THREE sides and isolate the variable between the two inequality symbols. Any inverse operation that is used to isolate the variable must be done to all THREE parts of the double inequality.

Ex 5: Solve $-2 \le 3x - 8 \le 10$. Graph the solution.

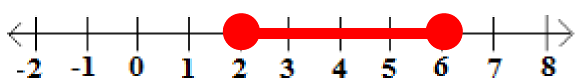
Step One: Write the original inequality $-2 \le 3x - 8 \le 1$

Step Two: Add 8 to each expression $6 \le 3x \le 18$

Step Three: Divide each expression by 3 $2 \le x \le 6$

The solution is all real numbers that are greater than or equal to 2 *and* less than or equal to 6.

set notation: $\{x | x \in \mathbb{R} \text{ where } 2 \le x \le 6\}$



Ex 6: Solve $-14 < -7(3x+2) < 1$. Graph the solution.

Step One: Write the original inequality $-14 < -7(3x+5) < 1$

Step Two: Distribute the negative 7 $-14 < -21x - 35 < 1$

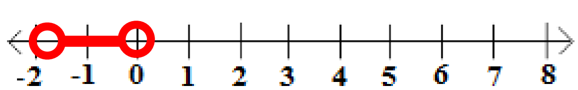
Step Three: Add 14 to each expression $0 < -21x \le 36$

Step Four: Divide each expression by -21 $0 > x > -\frac{12}{7}$

Step Five: Rewrite the inequality $-\frac{12}{7} < x < 0$

The solution is all real numbers that are between $-\frac{12}{7}$ and 0..

set notation: $\left\{x \middle| x \in \mathbb{R} \text{ where } -\frac{12}{7} \le x \le 0\right\}$



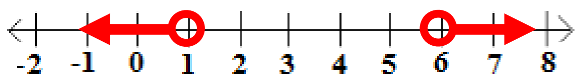
Ex 7: Solve $3x+1<4$ *or* $2x-5>7$. Graph the solution.

A solution of this inequality is a solution of either of its simple parts. You can solve each part separately.

$3x+1<4$	or	$2x-5>7$
$3x<3$	or	$2x>12$
$x<1$	or	$x>6$

The solution is all real numbers that are less than 1 *or* greater than 6.

set notation: $\{x \mid x\in\mathbb{R} \text{ where } x<1 \text{ or } x>6\}$



Ex 8: A full year membership to a gym costs \$325 upfront with no monthly charge. A monthly membership costs \$100 upfront and \$30 per month. Write and solve an inequality to find the number of months it is less expensive to have a monthly membership.

Step One: Set up the inequality — $m=$ months $30m+100<325$

Step Two: Solve the inequality

$30m+100<325$

$30m<225$

$m<7.5$

Step Three: Answer the question

Since we are looking for the number of months it is less expensive to have a monthly membership, we are only interested in counting numbers. The counting numbers less than 7.5 gives us:

$\boxed{m=1,2,3,4,5,6 \text{ or } 7 \text{ months}}$

Ex 9: Jillian is playing in a basketball tournament and scored 24 points in her first game. If she averages over 20 points for both games, she will receive a trophy. How many points can Jillian score in the second game and receive a trophy? Write and solve an inequality to find the number of points.

Step One: Set up the inequality — $p=$ points $\dfrac{p+24}{2}>20$

Step Two: Solve the inequality

$\dfrac{p+24}{2}>20$

$p+24>40$

$p>16$

Step Three: Answer the question

Jillian needs to score 16 points or higher in the second game to receive a trophy.

SAMPLE EXAM QUESTIONS

1. The movie theater donates at least 10% of its sales to charity. From Cassie's purchases, the theater will donate at least \$2.15. Which inequality below shows the amount of money *m* that Cassie spent at the refreshment stand?

A. $m \le 21.50$
B. $m \ge 21.50$
C. $m \le 215$
D. $m \ge 215$

ANS: B

2. Barb has saved \$91 from last year and would like to baby-sit (\$5.50 per hour) to earn enough to buy a mountain bike. She needs a good quality bike to bicycle through Red Rock Canyon. A bike like this costs at least \$300. What numbers of hours *h* can Barb baby-sit to reach her goal?

A. $h \ge 71$
B. $h \ge 38$
C. $h \ge 23$
D. $h \ge 14$

ANS: B

3. A local business has agreed to donate no more than half as much as the senior class raises. Which inequality shows how much money *b* the business will contribute if the seniors raised \$870?

A. $\frac{1}{2}(870) \le b$
B. $870 \le \frac{1}{2}b$
C. $\frac{1}{2}(870) \ge b$
D. $870 \ge \frac{1}{2}b$

ANS: A

4. An internet business sells U.S. flags for \$16.95 each, plus \$2.50 shipping per flag. Shipping is free, however, on orders where more than \$100.00 of flags are purchased. Which correctly shows the number of flags *f* that must be purchased to get free shipping?

A. $16.95f = 100$
B. $16.95f > 100$
C. $19.45f > 100$
D. $16.95f + 2.50 > 100$

ANS: B

5. Sam can spend no more than \$100 on school supplies. He has to spend \$12 on paper and pencils. What is the maximum abount Sam can spend on the graphing calculator he wants?

ANS: let m represent the money left for his graphing calculator

one possible answer: $m+12 \le 100$

6. It costs \$5 to have a tote bag monogrammed with up to 12 letters and \$0.50 for each additional letter. Summer wants her backpack monogrammed, but has a budget of \$8. What is the maximum amount of letters that she can have embroidered into the backpack.

Let l equal the number of letters that she can have monogrammed.

We know the first 12 are free (with the \$5 charge)

We know each additional letter is \$0.50

We know the maximum amount she can spend is \$8

$5 + 0.50l < \$8$

$l+12$ will be the maximum number of letters that can be monogrammed onto her backpack

$x > -9$ or $x < -12$

$\{x \mid x \in \mathbb{R} \text{ where } x < -12 \text{ or } x > -9\}$

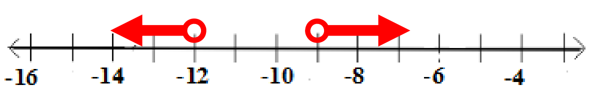
7. Solve the inequality and graph the solution.

$$-3 < x + 6 \quad or \quad \frac{-x}{3} > 4$$

ANS:

$x > -9$ or $x < -12$

$\{x \mid x \in \mathbb{R} \text{ where } x < -12 \text{ or } x > -9\}$



8. Solve the inequality and graph the solution.

$-10 \le -4x - 18 \le 30$

ANS:

$$-10+18 \le -4x-18+18 \le 30+18$$

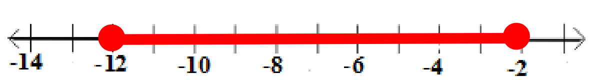
$$8 \le -4x \le 48$$

$$\frac{8}{-4} \ge \frac{-4}{-4}x \ge \frac{48}{-4}$$

$$-2 \ge x \ge -12$$

$$-12 \le x \le -2$$

set notation: $\{x \mid x \in \mathbb{R} \text{ where } -12 \le x \le -2\}$



Unit 3.5, 3.6 and 3.7 To solve and graph absolute value equations in one variable, To solve and graph absolute value inequalities in one variable and To create absolute value equations and inequalities in one variable to model real world situations.

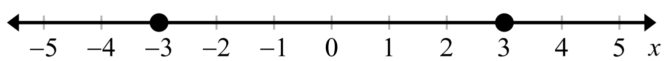
Absolute Value: the distance a number is away from the origin on the number line

Mathematical definition of absolute value: $|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$

Ex 10: Solve the absolute value equation: $|x| = 3$

Note: This means that the distance from the origin is 3. There are only two numbers that have a distance of 3 from the origin, so there are TWO solutions to this equation.

Solution: Two numbers are 3 units away from the origin: $\boxed{x = 3 \text{ and } x = -3}$



When solving an absolute value equation in the form $|ax+b| = c$, you must solve the equations $ax+b=c$ and $ax+b=-c$. Set up the two equations and solve them separately.

Ex 11: Solve the equation $|x+8| = 2$ and graph the solution(s) on a number line.

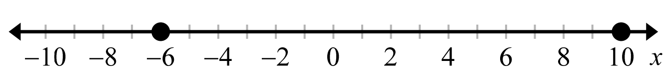
Step One: Rewrite as two equations. $x+8=2 \quad x+8=-2$

Step Two: Solve both equations.

$$\begin{array}{rr} x+8=2 & x+8=-2 \\ -8 \quad -8 & -8 \quad -8 \\ \boxed{x=-6} & \boxed{x=-10} \end{array}$$

Note: The solutions can also be written in set notation as $\{-6, 10\}$.

Step Three: Plot the points on a number line.



On your own: Check your solutions by substituting them back into the original equation.

Ex 12: Solve the equation $|2x-7|=9$.

This equation states that $2x-7$ is 9 spaces from zero. So, that means that $2x-7$ is either equal to 9 or -9 because both values are 9 spaces from zero. To find the solution set, we solve both equations.

Step One: Rewrite as two equations $\quad 2x-7=9 \qquad 2x-7=-9$

Step Two: Solve both equations

$$\begin{array}{cc} 2x-7=9 & 2x-7=-9 \\ +7 \quad +7 & +7 \quad +7 \\ 2x=16 & 2x=-2 \\ \boxed{x=8} & \boxed{x=-1} \end{array}$$

Step Three: Write the solution $\quad$ set notation: x = {-1, 8}

A second way to solve absolute value equations is to directly translate what the statement tells us and graph it. Then, write the solution set based on the graph.

Ex 13: Solve the absolute value equation: $|5x-1|+3=14$

Step One: Put the equation in $|ax+b|=c$ form. The absolute value must be isolated before it can be rewritten as two equations. $\quad |5x-1|=11$

Step Two: Rewrite as two equations. $\quad 5x-1=11 \qquad 5x-1=-11$

Step Three: Solve both equations. $\quad 5x=12 \qquad 5x=-10$

Solutions: $\boxed{x=\frac{12}{5}}$ or $\boxed{x=-2}$

One your own: Check your solutions by substituting them back into the original equation.

Ex 14: Ice cream should be stored at 12° F with an allowance for 3°F. Write and solve an equation to find the maximum and minimum temperatures at which the ice cream should be stored.

To solve we will need to know this is an absolute value equation because of the allowance of a 3°F difference. Therefore, $|t-12|$ will be the first part of the equation. The allowance of temperature is what it will be equal to, which is 3 degrees.

$|t-12|=3$

Students should be able to identify that there could be two possible answers and so we write the original equation (case 1) where the expression inside the absolute value symbol is positive or zero and we write the opposite (case 2) where the expression inside the absolute value symbol is negative.

Solve each equation.

Case 1: $t-12=3$ — Case 2: $-(t-12)=3$

$\Rightarrow \boxed{t=15}$ — $\Rightarrow -t+12=3 \quad \Rightarrow -t=-9 \quad \Rightarrow \boxed{t=9}$

Therefore, the minimum temperature the ice cream could be stored at is 9° F and the maximum temperature it should be stored at is 15°F.

A third way to solve absolute value equations is to make each case the same distance away from the given point, therefore, making one solution positive and one solution negative.

Special Cases:

Absolute Value Equations with One Solution

Ex 15: Solve the equation: $|3n+4|+2=2$

Step One: Put the equation in $|ax+b|=c$ form.

$$\begin{aligned} |3n+4|+2&=2 \\ -2\ \ &\ \ -2 \\ |3n+4|\quad &=0 \end{aligned}$$

Step Two: Rewrite as two equations. $\quad 3n+4=0 \quad 3n+4=-0$

(Note: +0 and −0 both equal 0!) $\quad 3n+4=0$

Step Three: Solve the equation.

$$\begin{aligned} 3n+4&=0 \\ -4\ \ &\ -4 \\ \frac{3n}{3} &= \frac{-4}{3} \\ \boxed{n=-\frac{4}{3}} \end{aligned}$$

Absolute Value Equations with No Solution

Ex 16: Solve the equation $3|x-8|=-6$.

Step One: Put the equation in $|ax+b|=c$ form.

$$\frac{3|x-8|}{3}=\frac{-6}{3}$$
$$|x-8|=-2$$

⚠ Note: This reads that the absolute value of the quantity x minus 8 equals −2. This can never be true! The absolute value of any number or expression must be a non-negative number.

Therefore, this equation has NO SOLUTION.

You Try: Solve the equation $10 = |2x - 4| - 8$ and graph the solution(s) on a number line.

QOD: Explain why an absolute value equation can have two solutions.

SAMPLE EXAM QUESTIONS

1. What is the solution set of $|6x + 4| = 2$?

(A) $\left\{-\frac{1}{3}\right\}$

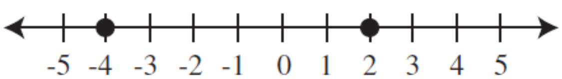
(B) $\{-1\}$

(C) $\left\{-1, -\frac{1}{3}\right\}$

(D) $\left\{\frac{1}{3}, 1\right\}$

ANS: C

The graph below represents the solution set of an equation.



2. Which of these is the equation?

(A) $|x| = 2$

(B) $|x| = 4$

(C) $|x + 1| = 3$

(D) $|x + 4| = 0$

ANS: C

3. Solve the equation. Then graph the solution set.

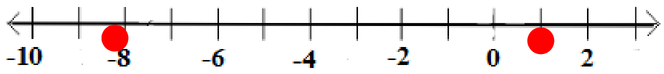
$|p - 4| = 6$

ANS: $\{-2, 10\}$

4. Solve the absolute value equation and graph the solution. $|x+3.6|=4.6$

ANS: $x+3.6=4.6$ $\qquad$ $x+3.6=-4.6$

$x=1.0$ $\qquad$ $x=-8.2$

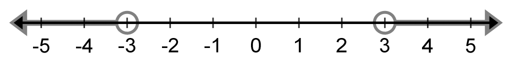


Solving Absolute Value Inequalities

The inequality $|x|<4$ means that we are looking for all numbers, *x*, whose distance from 0 on the number line is less than 4. Graphically, the solutions are:

Algebraically, we would use the compound inequality $-4<x<4$ to represent the solutions to the absolute value inequality.

The inequality $|x|>3$ means that we are looking for all numbers, *x*, whose distance from 0 the number line is greater than 3. Graphically, the solutions are:



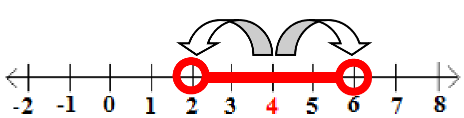
Algebraically, we would use the compound inequality $x<-3$ or $x>3$ to represent the solutions to the absolute value inequality.

Summary: Inequality Solutions

$\lvert ax+b\rvert<c$	$ax+b<c$	AND	$ax+b>-c$
$\lvert ax+b\rvert\le c$	$ax+b\le c$	AND	$ax+b\ge -c$
$\lvert ax+b\rvert>c$	$ax+b>c$	OR	$ax+b<-c$
$\lvert ax+b\rvert\ge c$	$ax+b\ge c$	OR	$ax+b\le -c$
$\lvert ax+b\rvert=c$	$ax+b=c$	OR	$ax+b=-c$

**Use the acronym GOAL to help students remember when to use AND and when to use OR. GOAL stands for "Greater than, Or, And, Less than".

Ex 17: $|x-4|<2$ This statement tells us that the difference between a number (*x*) and 4 is less than 2. So, the distance between *x* and 4 is less than 2. Let's graph that:

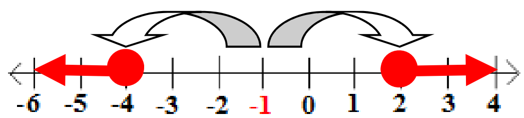


All of the solutions to this inequality will be values that fall between 2 and 6, because all of these values are within 2 spaces from 4.

$x>2$ and $x<6$ or $\boxed{2<x<6}$ $\qquad$ set notation: $\{x \mid x\in\mathbb{R} \text{ where } 2<x<6\}$

Ex 18: $|x+1| \geq 3$ This statement tells us about the sum of a number (x) and 1, so let's rewrite the statement so that it tells us about the difference between x and 1.

$|x-(-1)| \geq 3$ Now the statement tells us that the difference between a number (x) and -1 is greater than or equal to 3. So, the distance between x and -1 is greater than or equal to 3. Let's graph that:



All of the solutions to this inequality will be values that fall outside of and including -4 and 2, because all of these values are greater than or equal to 3 spaces from -1.

$\boxed{x \leq -4 \text{ or } x \geq 2}$ set notation: $\{x \mid x \in \mathbb{R} \text{ where } x \leq -4 \text{ or } x \geq 2\}$

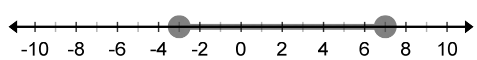
Ex 19: Solve the absolute value inequality $|x-2| \leq 5$ and graph the solution on a number line.

Step One: Rewrite as two inequalities. $x-2 \leq 5$ AND $x-2 \geq -5$

Step Two: Solve each inequality. $x \leq 7$ $x \geq -3$

Step Three: Write the solution as a compound inequality. $\boxed{-3 \leq x \leq 7}$

Step Four: Graph the solution on a number line.



Ex 20: Solve the absolute value inequality $|3x-3|+4>10$ and graph the solution on a number line.

Step One: Rewrite the inequality in $|ax+b|>c$ form.

$$|3x-3|+4>10$$
$$-4 \quad -4$$
$$|3x-3|>6$$

Step Two: Rewrite as two inequalities. $3x-3>6$ OR $3x-3<-6$

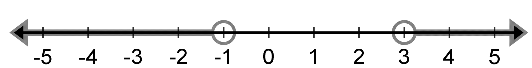
Step Three: Solve each inequality.

$3x>9$ $3x<-3$

$x>3$ $x<-1$

Step Four: Write the solution as a compound inequality. $\boxed{x<-1 \text{ or } x>3}$

Step Five: Graph the solution on a number line.



Ex 21: Solve the absolute value inequality $3|x+2|+4<16$ and graph the solution on a number line.

Step One: Rewrite the inequality in $|ax+b|<c$ form.

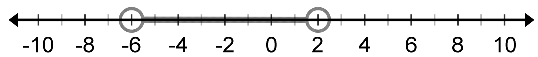
$$\begin{aligned} 3|x+2|+4 &< 16 \\ -4 \quad & \;\; -4 \\ 3|x+2| \quad &< 12 \end{aligned} \quad \Rightarrow \quad \begin{aligned} \frac{3|x+2|}{3} &< \frac{12}{3} \\ |x+2| &< 4 \end{aligned}$$

Step Two: Rewrite as two inequalities. $x+2<4$ AND $x+2>-4$

Step Three: Solve each inequality. $x<2$ $x>-6$

Step Four: Write the solution as a compound inequality. $\boxed{-6<x<2}$

Step Five: Graph the solution on a number line.



Ex 22: Solve the absolute value inequality $|3-x|<5$ and graph the solution on a number line.

Step One: Rewrite as two inequalities. $3-x<5$ AND $3-x>-5$

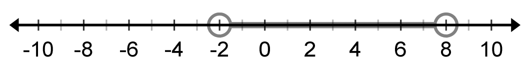
Step Two: Solve each inequality.

$$\frac{-x}{-1}<\frac{2}{-1} \qquad \frac{-x}{-1}<\frac{-8}{-1} \qquad \text{FLIP!}$$

$$x>-2 \qquad x<8$$

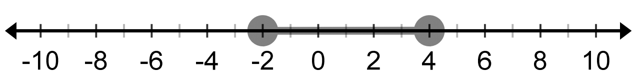
Step Three: Write the solution as a compound inequality. $\boxed{-2<x<8}$

Step Four: Graph the solution on a number line.



<u>Writing an Absolute Value Inequality</u>

Ex 23: Write an absolute value inequality that has the solution shown on the number line below.



Note: This compound inequality is an "and" statement with closed circles, so we will use an absolute value inequality of the form $|ax+b|\le c$.

The endpoints are −2 and 4. The half-way point between the two endpoints is 1. So, all of the possible values of the variable (x) must lie within 3 units of the half-way point. (i.e.: The difference between the values of x and 1 must be less than or equal to 3.)

Solution: $\boxed{|x-1|\le 3}$ or $\boxed{|1-x|\le 3}$

Application Problem with Absolute Value Inequalities

Ex 24: At a bottling company, the machine accepts bottles only if the number of fluid ounces is between $17\frac{8}{9}$ and $18\frac{1}{9}$ ounces. Let *n* be the number of fluid ounces that the machine accepts. Write an absolute value inequality that describes all possible values of *n*.

> ⚠ Note: The number of fluid ounces must be "between" two values, therefore we are looking for an "and" statement. So we will use an absolute value inequality of the form $|ax+b|<c$.

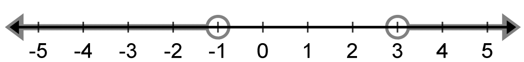
The half-way point between $17\frac{8}{9}$ and $18\frac{1}{9}$ is 18. All of the possible values of *n* must lie within $\frac{1}{9}$ of the half-way point. (i.e.: The difference between the values of *n* and 18 must be less than $\frac{1}{9}$.

Solution: $\boxed{|n-18|<\frac{1}{9}}$ or $\boxed{|18-n|<\frac{1}{9}}$

On Your Own: Solve the absolute value inequality above and verify that is satisfies the problem.

<u>You Try:</u>

1. Solve the absolute value inequality $|5x+3|-4\le 9$ and graph the solution on a number line.
2. Write an absolute value inequality that has the solution shown on the number line below.



<u>QOD:</u> Explain how to determine if the solution to an absolute value inequality is an "and" or "or" compound inequality.

SAMPLE EXAM QUESTIONS

1. Solve the inequality: $|4x-2|<3$

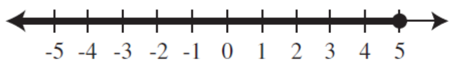
(A) $-\frac{1}{4}<x<\frac{5}{4}$ (B) $-\frac{5}{4}<x<\frac{5}{4}$

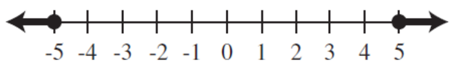
(C) $x<-\frac{1}{4}$ or $x>\frac{5}{4}$ (D) $x<-\frac{5}{4}$ or $x>\frac{5}{4}$

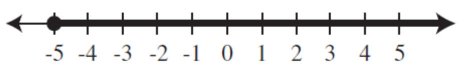
ANS: A

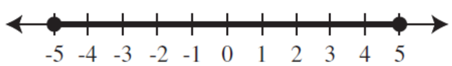
2. Look at the inequality at the right: $|x| \le 5$

Which graph represents all of the solutions of the inequality?

A


B


C


D


ANS: D

3. What are the solutions of the inequality $|3x-4|>5$**?**

A $x<-3$ or $x>\frac{1}{3}$

B $x<-\frac{1}{3}$ or $x>3$

C $-3<x<\frac{1}{3}$

D $-\frac{1}{3}<x<3$

ANS: B

4. A panda has a lifespan of 14 to 20 years. The inequality $|x-17|\le c$ **gives the number of years a panda may live. What is the value of** c**?**

A. 20
B. 14
C. 6
D. 3

ANS: D

5. Solve the inequality. Then graph the solution. $|9+x|\le 7$

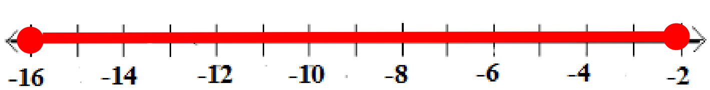
ANS:

Case 1: $9+x\le 7$ *Case* 2: $9+x\ge -7$

$x\le -2$ $x\ge -16$

$$-16\le x\le -2$$

$\{x \mid x\in \mathbb{R} \text{ where } x\ge -16 \text{ and } x\le -2\}$



6. **Solve the inequality. Then graph the solution.** $|3x-9|>5$

ANS:

$Case\ 1:$ $3x-9>5$

$3x>14$

$\frac{3x}{3}>\frac{14}{3}$

$x>\frac{14}{3}$

$Case\ 2:$ $3x-9<-5$

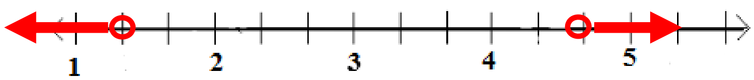
$3x<4$

$\frac{3x}{3}<\frac{4}{3}$

$x<\frac{4}{3}$

$$x>\frac{14}{3} \quad or \quad x<\frac{4}{3}$$

$$\left\{x \middle| x \in \mathbb{R} \text{ where } x<\frac{4}{3}\ or\ x>\frac{14}{3}\right\}$$



7. **Test scores in your class range from 60 to 100. Write an absolute value inequality describing the range of the test scores.**

ANS:

$\frac{100+60}{2}=80 \qquad \Rightarrow |x-80|$

$\frac{100-60}{2}=20 \qquad \Rightarrow 20$

$|x-80|\le 20$ would be the equation for the range of test scores.

8. **Some fire extinguishers contain pressurized water. The water pressure should be 162.5 psi (pounds per square inch), but it is acceptable for the pressure to differ from this value by at most 12.5 psi. Write and solve an absolute-value inequality to find the range of acceptable pressures.**

(A) $|p-12.5|\le 162.5$

$-150.0\le p\le 175.0$

(B) $|p-12.5|\le 162.5$

$p\le -150.0$ or $p\ge 175.0$

(C) $|p-162.5|\le 12.5$

$p\le 150.0$ or $p\ge 175.0$

(D) $|p-162.5|\le 12.5$

$150.0\le p\le 175.0$

ANS: D