



## Math Background

### Prior knowledge and skills

- Translate phrases into expressions
- Simplifying and evaluating expressions
- Combining like terms and distributive property

### In this unit you will study

- Solve equations by using addition and subtraction
- Solve equations by using multiplication and division
- Solve equations with fractions or decimals
- Solving literal equations for a given variable

### You can use the skills in this unit to

- Solve and interpret more complicated equations
- Solve equations that can be used for prediction or interpretation of applications
- Create linear equations in one variable to model real world situations
- Solve formulas

### Overall Big Ideas

- Solving equations give all the values that make the equation true.

### Essential Questions

- How do the arithmetic operations become useful in solving linear equations?
- Why is it important to be able to solve linear equations?

### STANDARDS:

#### **A.REI.A.1 Understand solving equations as a process of reasoning and explain the reasoning.**

Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

**A.REI.B.3 Solve equations and inequalities in one variable.** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

**A.CED.A.1 Create equations in one variable and use them to solve problems.** Only linear equations.

**A.CED.A.3 Represent constraints by equations and interpret solutions as viable or non-viable options in a modeling context.**

**A.CED.A.4 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.** For example, rearrange Ohm's law  $V = IR$  to highlight resistance,  $R$ .



### Notes, Examples and Exam Questions

**Equation:** an algebraic “sentence” that consists of two equal algebraic expressions

**Linear Equation:** an equation that can be written in the form  $ax + b = c$

Note: the variable has an exponent of one and does not occur inside absolute value symbols or roots or in the denominator of a fraction.

**Ex 1:** Which of the following are linear equations?      **Solution: A and C**

- A.  $3x = 6$                       Yes, the variable is raised to the first power (exponent is one).
- B.  $2 + \sqrt{x} = 4$                   No, the variable is inside a square root.
- C.  $3 - (x + 8) = 2x$               Yes, the variable is raised to the first power (exponent is one).
- D.  $3 = |x|$                       No, the variable is inside absolute value symbols.
- E.  $\frac{1}{x + 6} = 2x$                       No, the variable is in the denominator of a fraction.
- F.  $3x^2 - 1 = 4$                       No, the variable is raised to the second power (exponent is two).

**Solution of an Equation:** the value(s) of the variable that make the equation true

**Equivalent Equations:** equations with the same solution

**Inverse Operations:** operations that undo each other

Addition  $\Leftrightarrow$  Subtraction              Multiplication  $\Leftrightarrow$  Division

**▲Note:** We will use inverse operations to isolate the variable and solve the equation.

**Properties of Equality:**

Addition Property of Equality	If $a = b$ , then $a + c = b + c$ .
Subtraction Property of Equality	If $a = b$ , then $a - c = b - c$ .
Multiplication Property of Equality	If $a = b$ , then $a \cdot c = b \cdot c$
Division Property of Equality	If $a = b$ , then $\frac{a}{c} = \frac{b}{c}$ . ( $c \neq 0$ )
Symmetric Property of Equality	If $a = b$ , then $b = a$ .



Solving One-Step Equations:

**Ex 2:** Solve the equation  $x + 6 = -8$ .

Because 6 is being **added** to  $x$ , we will use the inverse operation (**subtraction**) to isolate the variable.

$$\begin{array}{l}
 x + 6 = -8 \\
 \text{Use the Subtraction Property of Equality} \quad -6 \quad -6 \\
 \boxed{x = -14} \\
 \\
 x + 6 = -8 \\
 \text{Check your solution in the original equation:} \quad (-14) + 6 = -8 \\
 -8 = -8
 \end{array}$$


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**Ex 3:** Solve the equation  $-4 = x - 3$ .

Because 3 is being **subtracted** from  $x$ , we will use the inverse operation (**addition**) to isolate the variable.

$$\begin{array}{l}
 -4 = n - 3 \\
 \text{Use the Addition Property of Equality} \quad +3 \quad +3 \\
 \boxed{-1 = n}
 \end{array}$$

You can write the solution as  $\boxed{n = -1}$  by the symmetric property.

$$\begin{array}{l}
 -4 = n - 3 \\
 \text{Check your solution in the original equation:} \quad -4 = (-1) - 3 \\
 -4 = -4
 \end{array}$$


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**Ex 4:** Solve the equation  $5 = -3 + x$

$$\begin{array}{l}
 5 = -3 + x \\
 \text{Use the Subtraction Property of Equality} \quad -(-3) \quad -(-3) \\
 \boxed{8 = x}
 \end{array}$$

You can write the solution as  $\boxed{x = 8}$  by the symmetric property.

$$\begin{array}{l}
 5 = -3 + x \\
 \text{Check your solution in the original equation:} \quad 5 = -3 + (8) \\
 5 = 5
 \end{array}$$


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**Ex 5:** Solve the equation  $15x = 3$ .

Because  $x$  is being **multiplied** by 15, we will use the inverse operation (**division**) to isolate the variable.

$$\frac{15x}{15} = \frac{3}{15}$$

Use the Division Property of Equality

$$x = \frac{3}{15}$$

$$x = \frac{1}{5}$$

$$15x = 3$$

Check your solution in the original equation:

$$15\left(\frac{1}{5}\right) = 3$$

$$3 = 3$$


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**Ex 6:** Solve the equation:  $\frac{t}{-2} = 9$ .

Because  $t$  is being **divided** by  $-2$ , we will use the inverse operation (**multiplication**) to isolate the variable.

$$\frac{t}{-2} = 9$$

Use the Multiplication Property of Equality

$$-2 \cdot \frac{t}{-2} = -2 \cdot 9$$

$$t = -18$$

$$\frac{t}{-2} = 9$$

Check your solution in the original equation:

$$\frac{(-18)}{-2} = 9$$

$$9 = 9$$

**Teacher Note:** Review with students that sign placement within a fraction is irrelevant. For example,

$\frac{t}{-2} = \frac{-t}{2} = -\frac{t}{2}$ . Use examples that illustrate all of these within student practice.

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**Ex 7:** Solve the equation  $-x = 10$ .

Another way to write the equation above is  $-1x = 10$ . Because  $x$  is being **multiplied** by  $-1$ , we will use the inverse operation (**division**) to isolate the variable.



$$-1x = 10$$

Use the Division Property of Equality

$$\frac{-1x}{-1} = \frac{10}{-1}$$

$$x = -10$$

Check your solution in the original equation:

$$-x = 10$$

$$-(-10) = 10$$

$$10 = 10$$


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**Ex 8:** Solve the equation  $\frac{3}{4}b = 27$ .

Because  $b$  is being **multiplied** by  $\frac{3}{4}$ , we will use the inverse operation (**division**) to isolate the variable.

Remember, to divide by a fraction we must **multiply by the reciprocal**.

$$\frac{3}{4}b = 27$$

Use the Multiplication Property of Equality

$$\frac{4}{3} \cdot \frac{3}{4}b = \frac{4}{3} \cdot 27$$

$$b = 36$$

Check your solution in the original equation:

$$\frac{3}{4}b = 27$$

$$\frac{3}{4}(36) = 27$$

$$27 = 27$$


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**▲Note:** You may have to simplify within an equation first before isolating the variable.

**Ex 9:** Solve the equation  $x - (-8) = 10$ .

Rewrite the equation first by simplifying the parentheses.

$$x + 8 = 10$$

Use the Subtraction Property of Equality

$$x + 8 = 10$$

$$-8 \quad -8$$

$$x = 2$$

Check your solution in the original equation:

$$x - (-8) = 10$$

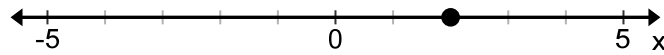
$$(2) - (-8) = 10$$

$$2 + 8 = 10$$

$$10 = 10$$



\*\*\*We can graph the solution of this equation on the number line.  $x = 2$



**Ex 10:** Solve the equation  $|-5| = -\frac{3}{2}y$ .

Rewrite the equation first by simplifying the absolute value.  $5 = -\frac{3}{2}y$

$$5 = -\frac{3}{2}y$$

Use the Multiplication Property of Equality

$$-\frac{2}{3} \cdot 5 = -\frac{2}{3} \cdot -\frac{3}{2}y$$

$$\boxed{-\frac{10}{3} = y}$$

$$|-5| = -\frac{3}{2}y$$

Check your solution in the original equation:

$$5 = -\frac{3}{2}\left(-\frac{10}{3}\right)$$

$$5 = 5$$

You Try:

1. Solve the equation  $4 = \left|\frac{1}{3}\right|x$ .

2. Solve the equation  $(-3)^2 + p = 5$ .

3. Solve the equation  $-\frac{7}{2} = -x$ . Graph your solution on the number line.

QOD: Why are inverse operations used to isolate the variable in an equation?



## SAMPLE EXAM QUESTIONS

1. Solve the equation  $\frac{x}{4} = -20$ .
- A.  $x = -80$
  - B.  $x = -5$
  - C.  $x = 5$
  - D.  $x = 80$

ANS: A

For questions 2 and 3, use the solution to the equation  $2x - 3 = 11$  below.

Start:  $2x - 3 = 11$

Step 1:  $2x - 3 + 3 = 11 + 3$

Step 2:  $2x = 14$

Step 3:  $\frac{1}{2}(2x) = \frac{1}{2}(14)$

Step 4:  $x = 7$

2. In Step 1, the addition property of equality was applied.
- A. True
  - B. False

ANS: A

3. In Step 3, the symmetric property of equality was applied.
- A. True
  - B. False

ANS: B



**Solving an Equation in the Form:**  $ax + b = c$

▲Note: To solve an equation, we use the properties of equality to isolate the variable on one side of the equation. When isolating the variable, we will **“undo” addition/subtraction first**, then “undo” multiplication/division using inverse operations. **\*\*This is the order of operations in reverse!**

**Ex 11:** Solve the equation  $-8x + 6 = 5$ .

$$-8x + 6 = 5$$

Step One: Undo addition (using subtraction).

$$\begin{array}{r} -6 \quad -6 \\ -8x \quad = -1 \end{array}$$

Step Two: Undo multiplication (using division).

$$\frac{-8x}{-8} = \frac{-1}{-8}$$

$$x = \frac{1}{8}$$

Step Three: Check your solution.

$$\begin{array}{l} -8x + 6 = 5 \\ -8\left(\frac{1}{8}\right) + 6 = 5 \end{array} \Rightarrow \begin{array}{l} -1 + 6 = 5 \\ 5 = 5 \end{array}$$

**Ex 12:** Solve the equation  $-4 = -8 + 3x$ .

Step One: Rewrite in the form  $ax + b = c$ .

Symmetric Property of Equality:  $-8 + 3x = -4$

Commutative Property of Addition:  $3x - 8 = -4$

Step Two: Undo subtraction (using addition).

$$\begin{array}{r} 3x - 8 = -4 \\ +8 \quad +8 \\ 3x \quad = 4 \end{array}$$

Step Three: Undo multiplication (using division).

$$\frac{3x}{3} = \frac{4}{3}$$

$$x = \frac{4}{3}$$

Step Four: Check your solution.

$$\begin{array}{l} -4 = -8 + 3x \\ -4 = -8 + 3\left(\frac{4}{3}\right) \end{array} \Rightarrow \begin{array}{l} -4 = -8 + 4 \\ -4 = -4 \end{array}$$

▲For more complicated equations, you may have to **simplify both sides of the equation first** using the distributive property and combining like terms. Note: Our goal is to write the equation in  $ax + b = c$  form.





**Ex 13:** Solve the equation:  $6(x - 4) - 2x = 24$

Step One: Use the distributive property.

$$6x - 24 - 2x = 24$$

Step Two: Combine like terms.

$$4x - 24 = 24 \quad (\text{This is in } ax + b = c \text{ form.})$$

Step Three: Undo subtraction (using addition).

$$\begin{aligned} 4x - 24 &= 24 \\ + 24 &+ 24 \\ 4x &= 48 \end{aligned}$$

Step Four: Undo multiplication (using division).

$$\frac{4x}{4} = \frac{48}{4}$$

$$x = 12$$

Step Five: Check your solution.

$$\begin{aligned} 6(x - 4) - 2x &= 24 \\ 6((12) - 4) - 2(12) &= 24 \quad \Rightarrow \quad 48 - 24 = 24 \\ 6(8) - 2(12) &= 24 \quad \quad \quad 24 = 24 \end{aligned}$$

**Ex 14:** Solve the equation:  $20 = 5y - 3(4 - y)$

Step One: Use the distributive property.

$$20 = 5y - 12 + 3y$$

Step Two: Combine like terms.

$$20 = 8y - 12$$

▲Note: We will write this in  $ax + b = c$  form using the symmetric property.

$$8y - 12 = 20$$

Step Three: Undo subtraction (using addition).

$$\begin{aligned} 8y - 12 &= 20 \\ + 12 &+ 12 \\ 8y &= 32 \end{aligned}$$

Step Four: Undo multiplication (using division).

$$\frac{8y}{8} = \frac{32}{8}$$

$$y = 4$$

Step Five: Check your solution.

$$\begin{aligned} 20 &= 5y - 3(4 - y) \\ 20 &= 5(4) - 3(4 - (4)) \quad \Rightarrow \quad 20 = 20 - 0 \\ 20 &= 20 - 3(0) \quad \quad \quad 20 = 20 \end{aligned}$$

You Try:

1.  $\frac{x}{3} + 6 = -15$

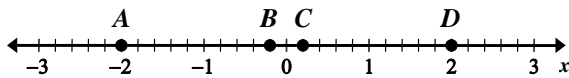
2.  $-12 = x + \frac{1}{3}x$

3.  $4x - 2(3x + 6) = 10$

QOD: When solving an equation in the form  $ax + b = c$ , how are the steps related to the order of operations?

**SAMPLE EXAM QUESTIONS**

1. Which point on the number line represents the solution of  $1.8x + 1.9 = -1.7$ ?



- A. Point A
- B. Point B
- C. Point C
- D. Point D

**ANS: A**

**Solve a Linear Equation with Variables on Both Sides**

▲If there are variables on both sides, you will need to first simplify both sides of the equation and then bring all variables to the same side using inverse operations.

**Ex 15:** Solve the equation  $3x - 2 = 4(x + 6)$

Step One: Use the distributive property.

$$3x - 2 = 4x + 24$$

Step Two: Bring all variables to the same side.

$$3x - 2 = 4x + 24$$

$$-4x \quad -4x$$

$$-1x - 2 = 24$$

Note: This is now in  $ax + b = c$  form.

$$-1x - 2 = 24$$

Step Three: Undo subtraction (using addition).

$$+2 \quad +2$$

$$-1x \quad = 26$$



$$\frac{-1x}{-1} = \frac{26}{-1}$$

Step Four: Undo multiplication (using division).

$$x = -26$$

Step Five: Check your solution.

$$3x - 2 = 4(x + 6) \Rightarrow -78 - 2 = 4(-20)$$

$$3(-26) - 2 = 4((-26) + 6) \Rightarrow -80 = -80$$

▲Note: We can also write the answer in set notation. The solution set to the equation above is  $\{-26\}$ .

Alternate Method: In the example above, we collected the variables on the left side. We could have also collected the variables on the right side.

$$3x - 2 = 4(x + 6) \Rightarrow 3x - 2 = 4x + 24 \Rightarrow -2 = x + 24$$

$$3x - 2 = 4x + 24 \Rightarrow -3x - 3x \Rightarrow -24 - 24$$

$$3x - 2 = 4x + 24 \Rightarrow -2 = x + 24 \Rightarrow \boxed{-26 = x}$$

### Solve Linear Equations with Fractions or Decimals

When solving an equation with **fractions**, it saves time to “wipe-out” (or clear) the fractions by multiplying both sides by the LCD (lowest common denominator). This means rewriting an equivalent equation with integer terms and coefficients.

**Ex 16:** Solve the equation:  $\frac{3}{5}x - \frac{9}{10} = \frac{1}{3} + 2x$

The LCD of the fractions is 30. Multiply every term in the equation by 30:

$$30 \cdot \frac{3}{5}x - 30 \cdot \frac{9}{10} = 30 \cdot \frac{1}{3} + 30 \cdot 2x \Rightarrow 18x - 27 = 10 + 60x$$

Now use the steps illustrated in the previous examples to solve the equation.

When solving an equation with **decimals**, it saves time to “wipe-out” (or clear) the decimals by multiplying both sides by a power of 10. This means rewriting an equivalent equation with integer terms and coefficients.

**Ex 17:** Solve the equation:  $3.2 + 5x = 4.3x - 1.7$

This equation has decimals with digits in the tenth place. Therefore, we will multiply both sides 10 to wipe out the decimals.

$$10 \cdot 3.2 + 10 \cdot 5x = 10 \cdot 4.3x - 10 \cdot 1.7 \Rightarrow 32 + 50x = 43x - 17$$

Now use the steps illustrated in the previous examples to solve the equation.



**Ex 18:** Solve the equation:  $4.23 = 5.8x - 6$

This equation has decimals with digits in the hundredths place. Therefore, we will multiply both sides by 100 to wipe out the decimals.

$$100 \cdot 4.23 = 100 \cdot 5.8x - 100 \cdot 6 \quad \Rightarrow \quad 423 = 580x - 600$$

$$423 = 580x - 600$$

$$\text{Isolate the variable: } +600 \quad +600 \quad \Rightarrow \quad \frac{1023}{580} = \frac{580x}{580}$$

$$1023 = 580x \quad \boxed{1.76 \approx x}$$

Note: Using the calculator, we rounded (approximated) the solution. Therefore, we use the symbol for APPROXIMATELY EQUAL TO:  $\approx$

**Identity:** an equation that is true for all values of the variable. An identity has infinitely many solutions (all real numbers).

### Solving an Equation with Infinitely Many Solutions

**Ex 19:** Solve the equation:  $4(x - 1) + 3 = -(1 - 4x)$

$$\begin{array}{ll} \text{Step One: Use the distributive property.} & 4x - 4 + 3 = -1 + 4x \\ \text{Step Two: Combine like terms on both sides.} & 4x - 1 = -1 + 4x \\ & 4x - 1 = -1 + 4x \\ \text{Step Three: Move the variables to one side.} & -4x \quad -4x \\ & -1 = -1 \end{array}$$

$\Delta$ Note: The variable was eliminated! Because  $-1 = -1$  is a TRUE statement, this equation is an IDENTITY. Therefore, the solutions to this equation are **ALL REAL NUMBERS**.

### Solving an Equation with No Solution

**Ex 20:** Solve the equation:  $3x + 2 - 5x = 4 - 2x$

$$\begin{array}{ll} \text{Step One: Combine like terms on both sides.} & -2x + 2 = 4 - 2x \\ & -2x + 2 = 4 - 2x \\ \text{Step Two: Move the variables to one side.} & +2x \quad +2x \\ & 2 = 4 \end{array}$$

$\Delta$ Note: The variable was eliminated! Because  $2 = 4$  is a FALSE statement, this equation has **NO SOLUTION**.

You Try:

1. Solve the equation  $\frac{2}{3}(p - 6) + 4 = \frac{1}{6}$ .
2. Solve the equation  $4x - 5(3x + 1) = 25 - x$ .
3. Solve the equation  $\frac{1}{3}(60 + 18t) = 6t + 24$ .
4. Solve the equation  $0.2 - x = 0.1(2 - 10x)$ .

QOD: When rounding a solution of an equation, what special symbol do we need to use instead of an equal (=) sign, and why?

Sample CCSD Common Exam Practice Question(s):

**1. Solve the equation  $8y - 3 = 5(2y + 1)$ .**

- A.  $y = -4$
- B.  $y = -2$
- C.  $y = 1$
- D.  $y = 3$

ANS: A

**2. Solve for  $x$ .  $6x = -14(10 + x)$**

- A.  $x = -28$
- B.  $x = -20$
- C.  $x = -7$
- D.  $x = 7$

ANS: C



**Using Equations in Real-Life:**

**Problem Solving Model**

- I. Write a verbal model.
- II. Assign labels.
- III. Write an algebraic model.
- IV. Solve the algebraic model and label answer appropriately.

**Ex 21:** About one-eighth of the population is left-handed. If 5 students in a math class are left-handed, about how many students would you expect to be in the class?

I. Fraction of Left-Handed Students · # of Students in the Class = # of Left-Handed Students

II. Fraction of Lefties =  $\frac{1}{8}$       # of Students in the Class =  $s$       # of Lefties = 5

III.  $\frac{1}{8}s = 5$

IV.  $8 \cdot \frac{1}{8}s = 8 \cdot 5$       Solution: You would expect there to be about 40 students in the class.

$$s = 40$$

**Ex 22:** Ann earns \$8 per hour for every 40 hours worked per week, and time-and-a-half for every hour over 40 hours. If Ann earned \$380 last week, how many hours did she work?

I. Hourly Pay · 40 + Overtime Pay · # of Hours Over 40 = Total Earnings

II. Hourly Pay = 8    Overtime Pay =  $1.5(8) = 12$     # of Hours over 40 =  $h$     Total Earnings = 380

III.  $8 \cdot 40 + 12h = 380$

$$320 + 12h = 380$$

IV.  $12h = 60$       Note:  $h$  is the number of hours Ann worked over 40.

$$h = 5$$

Solution: Ann worked 45 hours last week.



**Ex 23:** The temperature within Earth's crust increases about  $86^\circ$  Fahrenheit for each kilometer of depth beneath the surface. If the temperature at Earth's surface is  $72^\circ$  F, at what depth would you expect the temperature to be  $244^\circ$  F?

I. Temp. Inside Earth = Temp. at Earth's Surface + Rate of Temp. Increase  $\cdot$  Depth Below Surface

II. Temp. Inside Earth = 244      Temp. at Earth's Surface = 72

Rate of Temp. Increase = 86      Depth Below Surface =  $d$

III.  $244 = 72 + 86 \cdot d$

$$244 = 72 + 86d$$

$$\begin{array}{r} -72 \\ -72 \end{array}$$

IV.  $172 = 86d$

Solution: The temp. will be  $244^\circ$  F at 2 kilometers deep.

$$\boxed{2 = d}$$

**Ex 24:** In 2005, 176 students bought non-spiral notebooks, and this number is increasing by 24 per year. That same year, 331 students bought spiral notebooks, and that number is decreasing by 15 per year. When will the number of spiral notebooks be half of the non-spirals?

I.  $\frac{1}{2} (\# \text{ of Non-Spiral in } 2005 + \text{Rate of Non-Spiral Increase} \cdot \# \text{ of Years after } 2005) =$

$$\# \text{ of Spiral in } 2005 - \text{Rate of Spiral Decrease} \cdot \# \text{ of Years after } 2005$$

II.  $\# \text{ of Non-Spiral in } 2005 = 176$       Rate of Non-Spiral Increase = 24

$\# \text{ of Spiral in } 2005 = 331$       Rate of Spiral Decrease = 15       $\# \text{ of Years after } 2005 = y$

III.  $\frac{1}{2}(176 + 24y) = 331 - 15y$

$$\frac{1}{2}(176 + 24y) = 331 - 15y$$

$$\begin{array}{r} 88 + 27y = 331 \\ -88 \quad -88 \end{array}$$

IV.  $88 + 12y = 331 - 15y$        $\Rightarrow$        $\frac{27y}{27} = \frac{243}{27}$

$$\begin{array}{r} +15y \quad +15y \end{array}$$

$$88 + 27y = 331$$

$$\boxed{y = 9}$$

Solution: The number of spiral notebooks will be half of the non-spirals 9 years after 2005, which is the year 2014.



**Ex 25:** Find three consecutive integers whose sum is 75.

I. First Integer + Second Integer + Third Integer = Sum

II. First Integer =  $n$       Second Integer =  $n + 1$       Third Integer =  $n + 2$       Sum = 75

III.  $n + (n + 1) + (n + 2) = 75$

IV. 
$$\begin{array}{r} n + n + 1 + n + 2 = 75 \\ 3n + 3 = 75 \\ -3 \quad -3 \\ \hline 3n = 72 \end{array} \Rightarrow \frac{3n}{3} = \frac{72}{3} \quad \text{Solution: The three integers are 24, 25, and 26.}$$

$n = 24$

You Try: A wireless internet provider charges \$9 for the first hour and then \$0.95 per hour for every subsequent hour. If Charlie was charged \$14.70 for his wireless internet use, how many total hours did he use the internet?

QOD: List the steps to the problem-solving plan.

### SAMPLE EXAM QUESTIONS

**1. Sara uses the equation  $P = 8h + 40$  to figure her weekly pay,  $P$ , in dollars, for  $h$  hours. How many hours should Sara work to earn exactly \$320 this week?**

- A. 35
- B. 40
- C. 45
- D. 50

ANS: A

**2. The \$189 selling price of an MP3 player is \$51 more than 3 times the wholesale cost. What is the wholesale cost?**

- A. \$36
- B. \$46
- C. \$80
- D. \$138

ANS: B





### Solve a literal equation or formula for a given variable

\*The equation  $3x + 2y = 5$  is an example of an equation that is **implicitly defined**. Sometimes it is useful to rewrite the equation so that it is **explicitly defined**, which means it is solved for one of the variables.

**Ex 26:** Solve the equation  $3x + 2y = 5$  for  $y$ .

Step One: Subtract  $3x$  from both sides.  $2y = 5 - 3x$

Step Two: Divide both sides by 2.

$$y = \frac{5}{2} - \frac{3}{2}x$$

▲Note: There are many ways to write the correct answer. This equation is equivalent to the following:

$$y = -\frac{3}{2}x + \frac{5}{2} \quad \text{and} \quad y = \frac{5 - 3x}{2}$$

**Sometimes it is useful to solve formulas for one of the variables.**

**Ex 27:** Solve the formula for the area of a trapezoid for  $b_1$ .

$$A = \frac{1}{2}h(b_1 + b_2)$$

Step One: Multiply both sides by 2 (LCD).  $2A = h(b_1 + b_2)$

Step Two: Use the distributive property.  $2A = hb_1 + hb_2$

Step Three: Subtract  $hb_2$  from both sides.  $2A - hb_2 = hb_1$

Step Four: Divide both sides by  $h$ .  $\frac{2A - hb_2}{h} = b_1$  or  $b_1 = \frac{2A}{h} - b_2$

**Extension:** Given that the area of a trapezoid is  $42 \text{ in}^2$ , the height is 6 in., and one of the bases is 8 in., find the length of the other base.

Use the equation from the example above.  $\frac{2A}{h} - b_2 = b_1$

Substitute the given values into the new equation.  $\frac{2(42)}{6} - 8 = b_1$   $14 - 8 = b_1$   
 $b_1 = 6$

Solution: The other base is 6 in. long.



**Ex 28:** Alana has \$2000 to invest in a savings account that pays simple interest. She would like to leave the money in the account for 3 years and earn an additional \$150. What interest rate must the bank pay in order for Alana to earn this amount? (Use the simple interest formula  $I = Prt$ .)

**△Note:** Be sure that time ( $t$ ) is in years and the rate ( $r$ ) is written as a decimal when using this formula.

Solve the interest formula for  $r$ .

$$\frac{I}{Pt} = \frac{Prt}{Pt}$$

$$\frac{I}{Pt} = r$$

Substitute the values given in the problem.

$$\frac{150}{2000 \cdot 3} = r$$

$$\boxed{0.025 = r}$$

Solution: Alana's bank will have to pay 2.5% interest.

**You Try:** Solve the temperature conversion formula for  $C$ :  $F = \frac{9}{5}C + 32$

Use the new equation to find the temperature in Celsius when it is  $83^\circ F$ .

**QOD:** Describe a real-life situation where you would need to be able to solve the area formula of a circle for  $r$ .

### SAMPLE EXAM QUESTIONS

1. Solve for  $y$  in the equation  $w = \frac{1}{4}xy$ .

A.  $y = \frac{xw}{4}$

B.  $y = \frac{w}{4x}$

C.  $y = 4w - x$

D.  $y = \frac{4w}{x}$

ANS: D



2. An equation is shown below.

$$T = 2\pi\sqrt{\frac{m}{K}}$$

Which shows the equation correctly solved for  $m$ ?

**A**  $m = \frac{KT^2}{4\pi^2}$

**B**  $m = \frac{K\sqrt{T}}{2\pi}$

**C**  $m = \frac{KT^2}{2\pi}$

**D**  $m = K\sqrt{\frac{T}{4\pi}}$

ANS: A