Modeling Data with Radical Functions

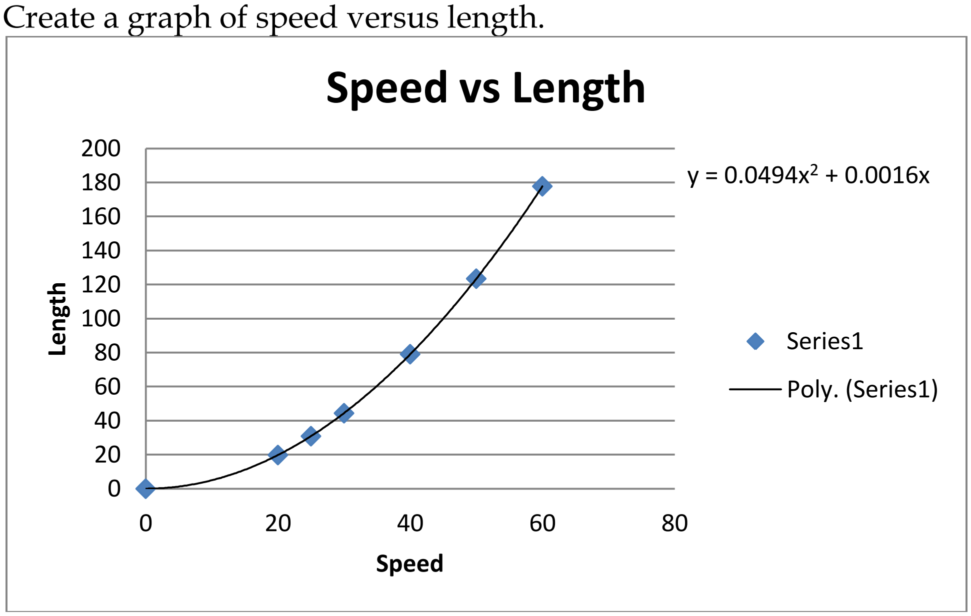
The speed a car is travelling at is proportional to the square root of the length of the skid mark it creates during an automobile accident. This can be expressed as $V = k\sqrt{D}$ where V represents speed, D represents distance, and k is the constant of proportionality.

In order to determine how fast a vehicle was travelling before an accident, police are able to measure the length of the skid mark. However, in order to use the above formula, they need to find the constant of proportionality. To do this, they have acquired the following data about the speed of a vehicle, in miles per hour, and the corresponding length of its skid mark, in feet, when stopped suddenly.

Speed (miles per hour)	**Length (feet)**
0	0
20	19.8
25	30.9
30	44.4
40	79.1
50	123.5
60	177.8

In order to determine how fast a vehicle must have been travelling in order to create a skid mark that was 130 feet long, you need to complete the following steps.

i. Create a graph of speed versus length.



ii. What type of function is created from the data? (example: linear, quadratic, radical)

Quadratic

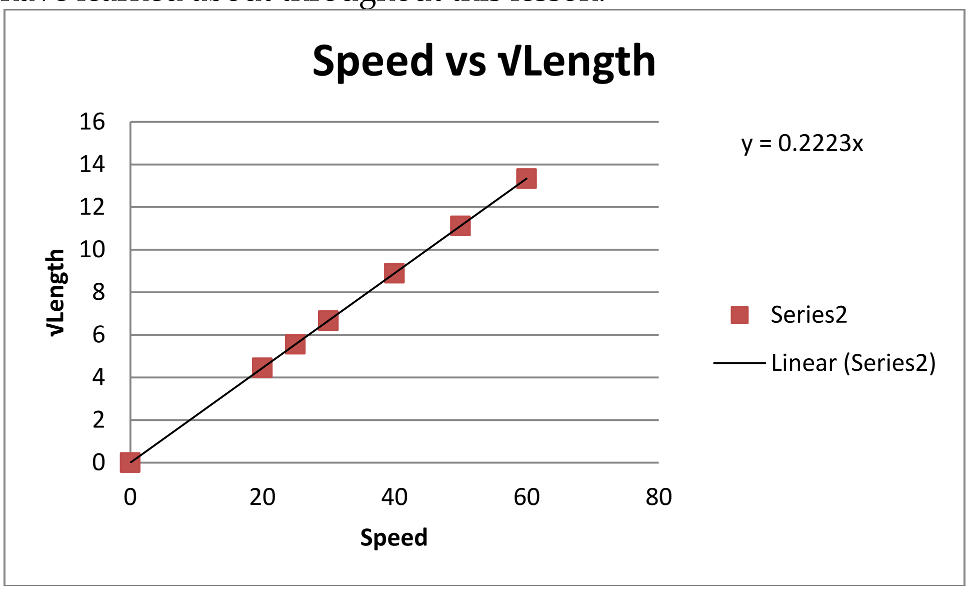
This is a quadratic function as it resembles a parabola.

iii. What type of function would be created from the square root of the graph you just created? (example: linear, quadratic, radical)

Linear

The square root of a quadratic function whose vertex is on the x-axis (as the vertex of this quadratic function seems to be) is an absolute value function. However, note how it is impossible to have a negative speed and thus this graph does not continue for negative values of x. Therefore, only half of the absolute value function would exist. This can also be called a linear function.

iv. Take the square root of this graph using the transformation properties you have learned about throughout this lesson.



The one easily identifiable invariant point is $(0, 0)$. The majority of the square root of this graph will be somewhere below the Speed vs. Length graph. To get a more accurate picture of what the square root graph looks like, create a table of values.

Speed (miles)	Length (feet)	$\sqrt{Length}$ (feet)
0	0	0
20	19.8	4.45
25	30.9	5.56
30	44.4	6.66
40	79.1	8.89
50	123.5	11.11
60	177.8	13.33

v. What is the slope of this graph?

0.222

To determine the slope of this graph, use the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$. Note: You may want to use this formula twice with two different sets of values in order to get an accurate value for the slope.

Using the points $(0, 0)$ and $(20, 4.45)$:

$m = \frac{4.45-0}{20-0} = \frac{4.45}{20} = 0.2225$

Using the points $(50, 11.11)$ and $(25, 5.56)$:

$m = \frac{5.56-11.11}{25-50} = \frac{-5.55}{-25} = 0.222$

An approximate value for the slope of this graph is 0.222.

vi. What is the relationship between the slope of the graph and the constant k in the equation $V = k\sqrt{D}$?

They are reciprocals of each other.

The graph represents $Speed$ versus $\sqrt{Length}$. As $speed$ is graphed on the x-axis, let $speed$ or V be represented by x. As $\sqrt{length}$ is graph on the y-axis, let $\sqrt{length}$ or $\sqrt{D}$ be represented by y. This will allow you to identify the slope, m, when the equation is in the form $y = mx$.

Therefore, $V = k\sqrt{D}$ becomes:

$x = ky$ or $y = \frac{x}{k}$

The slope of this line is $\frac{1}{k}$.

Thus, $m = \frac{1}{k}$ and $= \frac{1}{m} = \frac{1}{0.222} = 4.50$.

vii. What is the value of k in the equation $V = k\sqrt{D}$?

4.50

From the above reasoning in (vi), $k = 4.50$.

viii. Using the equation you found in (vii), determine how fast the vehicle must have been travelling in order to create a 130 foot skid mark.

$V = 4.5\sqrt{D}$

$V = 4.5\sqrt{130}$

$V = 51.31$ miles per hour