DIRECT, INVERSE AND JOINT VARIATION WORKSHEET

Direct Variation: \( y = kx \)  
Inverse Variation: \( y = \frac{k}{x} \)  
Joint Variation: \( y = kxz \)

Combined Variation: Combining any of the three types of variation listed above within a single problem.

Four Steps to Solve a Variation Problem
1. Write the general variation formula for the problem.
2. Use the formula to find the constant of variation, \( k \). 
3. Rewrite the formula, including the value of \( k \).
4. Answer the question.

State whether each equation represents a direct, inverse, or joint variation. Name the constant of variation.

1) \( y = 2x \)  
2) \( \frac{x}{5} = y \)  
3) \( xy = 12 \)  
4) \( D = \frac{3}{4}gh \)

Translate each statement into a formula. Use \( k \) as the constant of variation.

5) \( E \) varies jointly as \( M \) and the square of \( V \).

6) The volume, \( V \), of a gas varies directly as the temperature, \( T \), and inversely as the pressure \( P \).

7) The mass, \( M \), of a cement block varies jointly as the length, \( L \), width, \( W \), and thickness, \( T \), of the block.

8) \( P \) varies directly as the square of \( V \) and inversely as \( R \).

Write an equation for each statement. Then, solve the equation.

9) If \( y \) varies inversely as \( x \) and \( y = 2 \) when \( x = 8 \), find \( x \) when \( y = 14 \).

10) Suppose \( y \) varies jointly with \( x \) and \( z \). If \( y = 20 \) when \( x = 2 \) and \( z = 5 \), find \( y \) when \( x = 14 \) and \( z = 8 \).

11) If \( y \) varies inversely as \( x \) and \( x = 7 \) when \( y = 21 \), find \( y \) when \( x = 42 \).

12) Find \( y \) when \( x = 1.5 \), if \( y \) varies directly as \( x \) and \( y = -16 \) when \( x = 6 \).
13) The frequency of a vibrating string varies inversely as its length. A string 3 feet long vibrates 175 cycles per second. Find the frequency of a 5 foot string.

14) The force of the wind blowing on a vertical surface varies jointly as the area of the surface and the square of the velocity. If a wind blowing at 50 mph exerts a force of 75 pounds on a surface of 500 $ft^2$, how much force will a wind of 75 mph place on a surface of 10 $ft^2$?

15) The volume of a can varies jointly as the height of the can and the square of its radius. A can with an 8 inch height and 4 inch radius has a volume of 402.12 $in^3$. What is the volume of a can that has a 2 inch radius and a 10 inch height?

16) The time required to process a shipment of goods at Wal-Mart varies directly with the number of items in the shipment and inversely with the number of workers assigned. If 15,000 items can be processed by 8 workers in 10 hours, then how long would it take 12 workers to process 20,000 items?

17) A person’s level of fatness is measured using the Body Mass Index, or BMI. A BMI (rounded to the nearest whole number) in the low 20’s is desirable. BMI varies directly as a person’s weight in pounds and inversely as the square of the person’s height in inches. A person who weighs 140 pounds and is 70 inches tall has a BMI of 20. Find the BMI of a person who weighs 165 pounds and is 71 inches tall.

18) Disregarding wind resistance, the distance a body falls from rest varies directly as the square of the time it falls. If a skydiver falls 64 $ft$ in 2 seconds, how far will he fall in 10 seconds?

19) Albertson’s found that the demand for Coke products varies inversely as the price of the product. When the price of a Coke product is $2.75, the weekly demand is 1250. Find the weekly demand if the price is raised to $4.00.

20) The maximum load of a horizontal beam that is supported at both ends varies jointly as the width and the square of the height and inversely as the length between the supports. A beam 6 m long, 0.1 m wide, and 0.06 m high supports a load of 360 kg. What is the maximum load supported by a beam 16 m long, 0.2 m wide, and 0.08 m high?