

FACTORING REVIEW



To factor an expression, we basically rewrite it as two (or more) expressions that are MULTIPLIED TOGETHER

FIRST RULE OF FACTORING:

Your first step anytime you factor any problem is to look for a common factor to “pull out.” This is known as **FACTORING OUT THE GREATEST COMMON FACTOR (GCF)**.

Ex. In $8x^3 - 14x^2 + 4x - 2$, you should be able to see that there is a 2 that is common to all 4 terms. So, you “factor out” the 2:

$$8x^3 - 14x^2 + 4x - 2 \rightarrow 2(4x^3 - 7x^2 + 2x - 1)$$

AFTER APPLYING THE FIRST RULE:

At this point there are 3 *special cases* that you should look for, and they will drastically reduce the amount of work you will have to do if they follow the following patterns:

2 TERMS: If you have a 2 term equation, you automatically check to see if you have a **DIFFERENCE OF SQUARES (DOS)**. You have a difference of squares if the first term is a perfect square, the last term is a perfect square, and there is a subtraction sign between them. If these situations occur, it will follow the following pattern: $a^2 - b^2 = (a + b)(a - b)$

Ex. $9x^2 - 25 \rightarrow (3x + 5)(3x - 5)$

3 TERMS: If you have 3 terms, you may have a **PERFECT SQUARE TRINOMIAL (PST)**. In a PST, if the first term is a perfect square, and the last term is a perfect square, and the MIDDLE term is twice the product of the square of the first and last, then you follow this form: $a^2 \pm 2ab + b^2 = (a \pm b)^2$

Ex. $4x^2 - 28x + 49 \rightarrow (2x - 7)^2$

4 TERMS: When this occurs, you **FACTOR BY GROUPING**. To do this, you “group” the first two terms and the last two terms. Then ignore the last two terms, and take the first group and factor out the GCF. After that, look at the second group, and factor out whatever it takes so that what is left over is exactly what was left over from the first group. The last step is to “factor” out the common group.

Ex. In $3x^3 - 9x^2 + 4x - 12$, group like this: $(3x^3 - 9x^2)(+4x - 12)$. Notice that the GCF of the first term is $3x$, so that becomes $3x(x - 3)$. Now, we need to look at the second group, and factor out something so that what is left over is $(x - 3)$. Do you see a $+4$ can be factored out so that becomes $+4(x - 3)$.

So, $3x^3 - 9x^2 + 4x - 12 \rightarrow 3x(x - 3) + 4(x - 3) \rightarrow (x - 3)(3x + 4)$

AFTER CHECKING FOR SPECIAL CASES:

If none of the above have occurred, or have not got you completely factored, you will have an expression in the standard form $ax^2 \pm bx \pm c$, where a, b , and c are constants (numbers).

WHEN $a = 1$: In this case, we will factor using what I call the TWO PARENTHESES METHOD. There are some things we can look for in order to guide us through the factoring process.

- 1) Put down 2 sets of parentheses, and put an x in each one
- 2) Check the sign of c , if it is positive, the signs in the parenthesis will be THE SAME.
 - a. Look at the sign of b , if it is positive, both signs will + signs.
 - b. Look at the sign of b , if it is negative, both signs will - signs.
 - c. Now, find the two numbers that MULTIPLY to c , and ADD to b . Put them in the parentheses.
- 3) Check the sign of c , if it is negative, the signs in the parenthesis will be DIFFERENT
 - a. This one is a little different, because we need to find the two numbers that MULTIPLY to c , and SUBTRACT to b .
 - b. Now we look at the sign of b . The bigger of the two numbers will keep the sign of b , and the smaller of the two numbers will get the opposite sign.

Exs. Factor $x^2 + 6x + 5 \rightarrow (x + \quad)(x + \quad) \rightarrow (x + 5)(x + 1)$
Factor $x^2 - 12x + 27 \rightarrow (x - \quad)(x - \quad) \rightarrow (x - 3)(x - 9)$
Factor $x^2 - 7x - 18 \rightarrow (x + \quad)(x - \quad) \rightarrow (x + 2)(x - 9)$
Factor $x^2 + 3x - 18 \rightarrow (x + \quad)(x - \quad) \rightarrow (x + 6)(x - 3)$

WHEN $a \neq 1$: The most *reliable, quickest, and dependable* method to factor this is to use the ac method!

THE AC METHOD:

When factoring $ax^2 \pm bx \pm c$ when $a \neq 1$, you start the process by multiplying the coefficient of the x^2 term (a) and the constant term (c) together to get ac . Then, list what the coefficient of the x term is (b) is, and find the two numbers that multiply to ac and add to b . Now the cool part: use the two numbers that that multiply to ac and add to b to re-write the original 3 term quadratic to one with 4 terms by replacing the single middle term with the two numbers a new coefficients of x . Then use FACTORING BY GROUPING to complete the factoring process!

Ex. Factor $3x^2 + 7x + 2$

- 1) Find ac : $ac = (3)(2) \quad ac = 6$
- 2) List b : $b = 7$
- 3) Find the two numbers that multiply to 6 and add to 7: 6, 1
- 4) Now, replace the middle term $7x$ with 6 and 1 as the new coefficients of x :
 $3x^2 + 6x + x + 2$
- 5) Factor by grouping!

$$3x^2 + 6x + x + 2 \rightarrow (3x^2 + 6x)(+x + 2) \rightarrow 3x(x + 2) + 1(x + 2) \rightarrow (3x + 1)(x + 2)$$

Ex. Factor $8x^2 - 8x - 6$

$$ac = -48$$

$$b = -8$$

$$-12, 4$$

$$\begin{aligned} &\rightarrow 8x^2 - 12x + 4x - 6 \\ &(8x^2 - 12x)(+4x - 6) \\ &4x(2x - 3) + 2(2x - 3) \\ &(2x - 3)(4x + 2) \end{aligned}$$