Lesson 18: Determining the Surface Area of Three-Dimensional Figures

Student Outcomes

- Students determine that a right rectangular prism has six faces: top and bottom, front and back, and two sides. They determine that surface area is obtained by adding the areas of all the faces and develop the formula $SA = 2lw + 2lh + 2wh$.
- Students develop and apply the formula for the surface area of a cube as $SA = 6s^2$.

Lesson Notes

In order for students to complete this lesson, each student will need a ruler and the shape template that is attached to the lesson. To save time, teachers should have the shape template cut out for students.

Classwork

Opening Exercise (5 minutes)

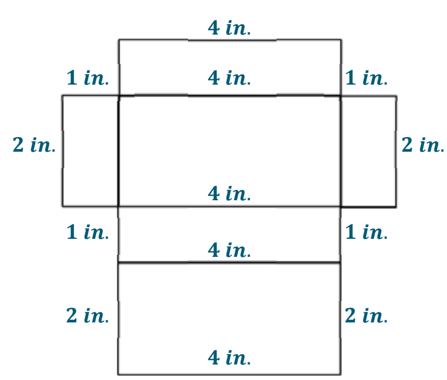
In order to complete the Opening Exercise, each student needs a copy of the shape template that is already cut out.

Opening Exercise

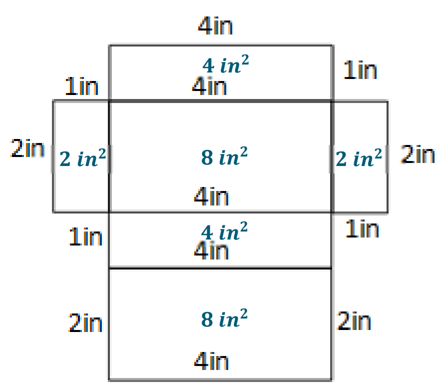
a. **What three-dimensional figure will the net create?**

Rectangular Prism

b. **Measure (in inches) and label each side of the figure.**



c. **Calculate the area of each face, and record this value inside the corresponding rectangle.**



d. **How did we compute the surface area of solid figures in previous lessons?**

To determine surface area, we found the area of each of the faces then added those areas.

e. **Write an expression to show how we can calculate the surface area of the figure above.**

$(4\ in. \times 1\ in.) + (4\ in. \times 2\ in.) + (4\ in. \times 1\ in.) + (4\ in. \times 2\ in.) + (2\ in. \times 1\ in.) + (2\ in. \times 1\ in.)$

$2(4\ in. \times 1\ in.) + 2(4\ in. \times 2\ in.) + 2(2\ in. \times 1\ in.)$

f. **What does each part of the expression represent?**

Each part of the expression represents an area of one face of the given figure. We were able to write a more compacted form because there are three pairs of two faces that are identical.

g. **What is the surface area of the figure?**

$(4\ in. \times 1\ in.) + (4\ in. \times 2\ in.) + (4\ in. \times 1\ in.) + (4\ in. \times 2\ in.) + (2\ in. \times 1\ in.) + (2\ in. \times 1\ in.)$

$2(4\ in. \times 1\ in.) + 2(4\ in. \times 2\ in.) + 2(2\ in. \times 1\ in.)$

$28\ in^2$

Example 1 (8 minutes)

- Fold the net used in the Opening Exercise to make a rectangular prism. Have the two faces with the largest area be the bases of the prism.
- Fill in the second row of the table below.

Example 1

Fold the net used in the Opening Exercise to make a rectangular prism. Have the two faces with the largest area be the bases of the prism. Fill in the second row of the table below.

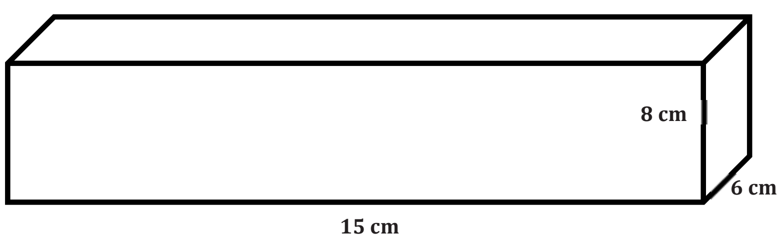
Area of Top (base)	Area of Bottom (base)	Area of Front	Area of Back	Area of Left Side	Area of Right Side
$8\ in^2$	$8\ in^2$	$4\ in^2$	$4\ in^2$	$2\ in^2$	$2\ in^2$

- What do you notice about the areas of the faces?
  - *Pairs of faces have equal areas.*
- What is the relationship between the faces having equal area?
  - *The faces that have the same area are across from each other. The bottom and top have the same area, the front and the back have the same area, and the two sides have the same area.*
- How do we calculate the area of the two bases of the prism?
  - $length \times width$
- How do we calculate the area of the front and back faces of the prism?
  - $length \times height$
- How do we calculate the area of the right and left faces of the prism?
  - $width \times height$
- Using the name of the dimensions, fill in the third row of the table.

Area of Top (base)	Area of Bottom (base)	Area of Front	Area of Back	Area of Left Side	Area of Right Side
$4\ in. \times 2\ in.$	$4\ in. \times 2\ in.$	$2\ in. \times 2\ in.$	$2\ in. \times 2\ in.$	$1\ in. \times 2\ in.$	$1\ in. \times 2\ in.$
$8\ in^2$	$8\ in^2$	$4\ in^2$	$4\ in^2$	$2\ in^2$	$2\ in^2$
$l \times w$	$l \times w$	$l \times h$	$l \times h$	$w \times h$	$w \times h$

- Calculate the surface area.
- Examine the rectangular prism below. Complete the table.

Examine the rectangular prism below. Complete the table.



Area of Top (base)	Area of Bottom (base)	Area of Front	Area of Back	Area of Left Side	Area of Right Side
$15\ cm \times 6\ cm$	$15\ cm \times 6\ cm$	$15\ cm \times 8\ cm$	$15\ cm \times 8\ cm$	$8\ cm \times 6\ cm$	$8\ cm \times 6\ cm$
$90\ cm^2$	$90\ cm^2$	$120\ cm^2$	$120\ cm^2$	$48\ cm^2$	$48\ cm^2$
$l \times w$	$l \times w$	$l \times h$	$l \times h$	$w \times h$	$w \times h$

MP.8

- When comparing the methods to finding surface area of the two rectangular prisms, can you develop a general formula?
  - $SA = l \times w + l \times w + l \times h + l \times h + w \times h + w \times h$
- Since we use the same expression to calculate the area of pairs of faces, we can use the distributive property to write an equivalent expression for the surface area of the figure that uses half as many terms.

Scaffolding:

Students may benefit from a poster or handout highlighting the length, width, and height of a three-dimensional figure. This poster may also include that $l =$ length, $w =$ width, and $h =$ height.

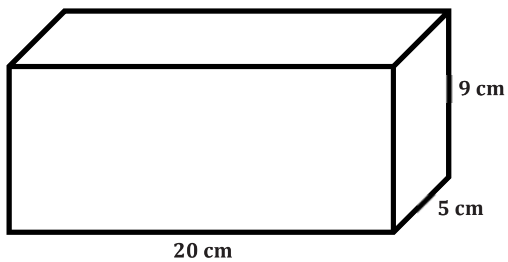
MP.8

- We have determined that there are two $l \times w$ dimensions. Let's record that as 2 times l times w, or simply $2(l \times w)$. How can we use this knowledge to alter other parts of the formula?
  - *We also have two* $l \times h$, *so we can write that as* $2(l \times h)$, *and we can write the two* $w \times h$ *as* $2(w \times h)$.
- Writing each pair in a simpler way, what is the formula to calculate the surface area of a rectangular prism?
  - $SA = 2(l \times w) + 2(l \times h) + 2(w \times h)$
- Knowing the formula to calculate surface area makes it possible to calculate the surface area without a net.

Example 2 (5 minutes)

Work with students to calculate the surface area of the given rectangular prism.

Example 2



- What are the dimensions of the rectangular prism?
  - *The length is* 20 *cm, the width is* 5 *cm, and the height is* 9 *cm*.
- We will use substitution in order to calculate the area. Substitute the given dimensions into the surface area formula.
  - $SA = 2(20)(5) + 2(20)(9) + 2(5)(9)$
- Solve the equation. Remember to use order of operations.
  - $SA = 200 + 360 + 90$
    $SA = 650\ cm^2$

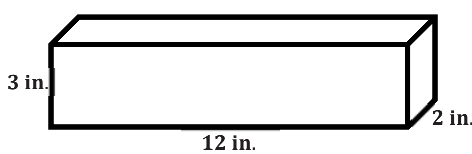
Exercises (17 minutes)

Students work individually to answer the following questions.

Exercises

1. **Calculate the surface area of each of the rectangular prisms below.**

   a.

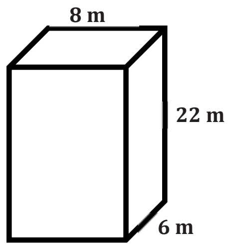


$SA = 2(12)(2) + 2(12)(3) + 2(2)(3)$

$SA = 48 + 72 + 12$

$SA = 132\ in^2$

b.

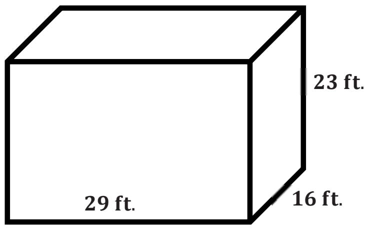


$SA = 2(8)(6) + 2(8)(22) + 2(6)(22)$

$SA = 96 + 352 + 264$

$SA = 712\ m^2$

c.

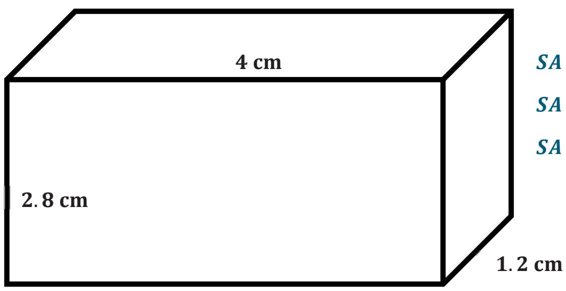


$SA = 2(29)(16) + 2(29)(23) + 2(16)(23)$

$SA = 928 + 1334 + 736$

$SA = 2998\ ft^2$

d.

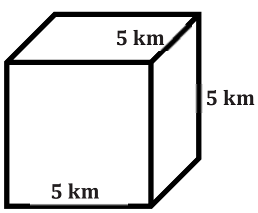


$SA = 2(4)(1.2) + 2(4)(2.8) + 2(1.2)(2.8)$

$SA = 9.6 + 22.4 + 6.72$

$SA = 38.72\ cm^2$

2. **Calculate the surface area of the cube.**



$SA = 2(5)(5) + 2(5)(5) + 2(5)(5)$

$SA = 50 + 50 + 50$

$SA = 150\ km^2$

MP.3

3. **All the edges of a cube have the same length. Tony claims that the formula $SA = 6s^2$, where s is the length of each side of the cube, can be used to calculate the surface area of a cube.**

   a. **Use the dimensions from the cube in Problem 2 to determine if Tony's formula is correct.**

   Tony's formula is correct because $SA = 6(5)^2 = 150\ km^2$, which is the same surface area when we use the surface area formula for rectangular prisms.

   b. **Why does this formula work for cubes?**

   Each face is a square, and to find the area of a square, you multiply the side lengths together. However, since the side lengths are the same, you can just square the side length. Also, a cube has six identical faces, so after calculating the area of one face, we can just multiply this area by 6 to determine the total surface area of the cube.

MP.3

c. **Becca doesn't want to try to remember two formulas for surface area so she is only going to remember the formula for a cube. Is this a good idea? Why or why not?**

Becca's idea is not a good idea because the surface area formula for cubes will only work for cubes because rectangular prisms do not have six identical faces. Therefore, Becca also needs to know the surface area formula for rectangular prisms.

Closing (5 minutes)

- Use two different ways to calculate the surface area of a cube with side lengths of 8 cm.
- If you had to calculate the surface area of twenty different sized cubes, which method would you prefer to use and why?

Lesson Summary

Surface Area Formula for a Rectangular Prism: $SA = 2lw + 2lh + 2wh$

Surface Area Formula for a Cube: $SA = 6s^2$

Exit Ticket (5 minutes)

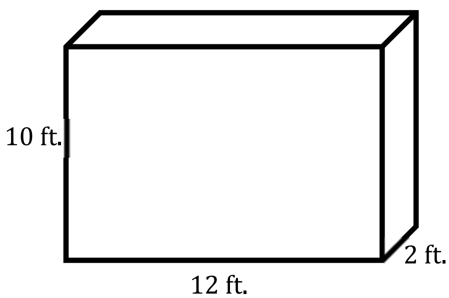
Name ____ Date ________________

Lesson 18: Determining Surface Area of Three-Dimensional Figures

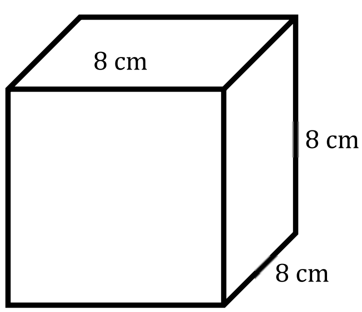
Exit Ticket

Calculate the surface area of each figure below. Figures are not drawn to scale.

1.



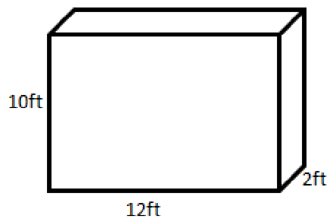
2.



Exit Ticket Sample Solutions

Calculate the surface area of each figure below. Figures are not drawn to scale.

1.



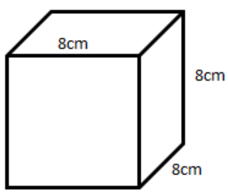
$SA = 2lw + 2lh + 2wh$

$SA = 2(12\ ft.)(2\ ft.) + 2(12\ ft.)(10\ ft.) + 2(2\ ft.)(10\ ft.)$

$SA = 48\ ft^2 + 240\ ft^2 + 40\ ft^2$

$SA = 328\ ft^2$

2.



$SA = 6s^2$

$SA = 6(8\ cm)^2$

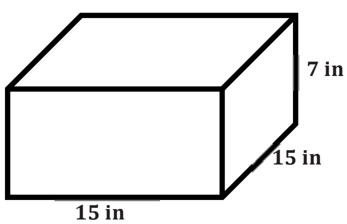
$SA = 6(64\ cm^2)$

$SA = 384\ cm^2$

Problem Set Sample Solutions

Calculate the surface area of each figure below. Figures are not drawn to scale.

1.

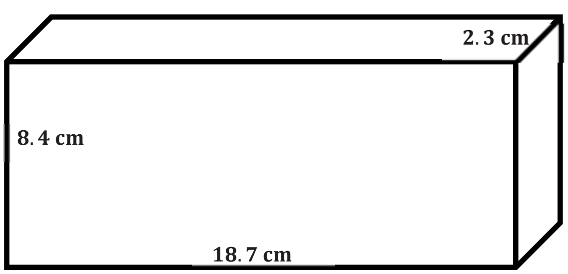


$SA = 2(15\ in.)(15\ in.) + 2(15\ in.)(7\ in.) + 2(15\ in.)(7\ in.)$

$SA = 450\ in^2 + 210\ in^2 + 210\ in^2$

$SA = 870\ in^2$

2.

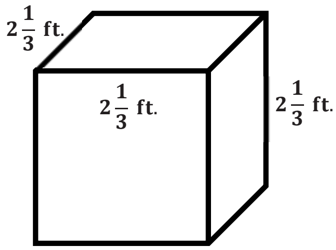


$SA = 2(18.7\ cm)(2.3\ cm) + 2(18.7\ cm)(8.4\ cm) + 2(2.3\ cm)(8.4\ cm)$

$SA = 86.02\ cm^2 + 314.16\ cm^2 + 38.64\ cm^2$

$SA = 438.82\ cm^2$

3.



$SA = 6\left(2\frac{1}{3}\ ft.\right)^2$

$SA = 6\left(\frac{7}{3}\ ft.\right)^2$

$SA = 6\left(\frac{49}{9}\ ft^2\right)$

$SA = \frac{294}{9} = 32\frac{2}{3}\ ft^2$

4.

$SA = 2(32.3\ m)(24.7\ m) + 2(32.3\ m)(7.9\ m) + 2(24.7\ m)(7.9\ m)$

$SA = 1595.62\ m^2 + 510.34\ m^2 + 390.26\ m^2$

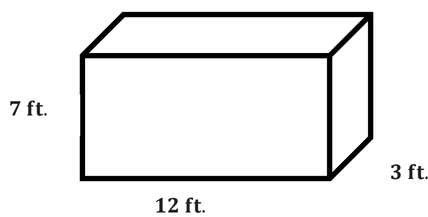
$SA = 2496.22\ m^2$

5. Write a numerical expression to show how to calculate the surface area of the rectangular prism. Explain each part of the expression.

$2(12\ ft. \times 3\ ft.) + 2(12\ ft. \times 7\ ft.) + 2(7\ ft. \times 3\ ft.)$

The first part of the expression shows the area of the top and bottom of the rectangular prism. The second part of the expression shows the area of the front and back of the rectangular prism. The third part of the expression shows the area of the two sides of the rectangular prism.

The surface area of the figure is $282\ ft^2$.



6. When Louie was calculating the surface area for Problem 4, he identified the following:

length $= 24.7$ m, width $= 32.3$ m, height $= 7.9$ m.

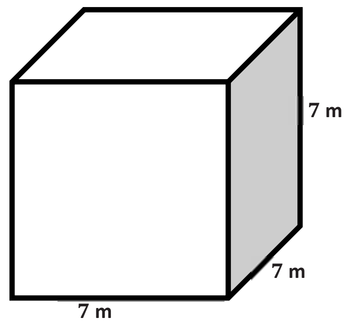
However, when Rocko was calculating the surface area for the same problem, he identified the following:

length $= 32.3$ m, width $= 24.7$ m, height $= 7.9$ m.

Would Louie and Rocko get the same answer? Why or why not?

Louie and Rocko would get the same answer because they are still finding the correct area of all six faces of the rectangular prism.

7. Examine the figure below.



a. What is the most specific name of the three-dimensional shape?

Cube

b. Write two different expressions for the surface area.

$(7\ m \times 7\ m) + (7\ m \times 7\ m) + (7\ m \times 7\ m) + (7\ m \times 7\ m) + (7\ m \times 7\ m) + (7\ m \times 7\ m)$

$6 \times (7\ m)^2$

c. Explain how these two expressions are equivalent.

The two expressions are equivalent because the first expression shows $7\ m \times 7\ m$*, which is equivalent to* $(7\ m)^2$*. Also, the* 6 *represents the number of times the product* $7\ m \times 7\ m$ *is added together.*