Lesson 15: Representing Three-Dimensional Figures Using Nets

Student Outcomes

- Students construct three-dimensional figures through the use of nets. They determine which nets make specific solid figures and determine if nets can or cannot make a solid figure.

Lesson Notes

Using geometric nets is a topic that has layers of sequential understanding as students progress through the years. For Grade 6, specifically in this lesson, the working description of a net is this: If the surface of a three-dimensional solid can be cut along enough edges so that the faces can be placed in one plane to form a connected figure, then the resulting system of faces is called a net of the solid.

A more student-friendly description used for this lesson is the following: Nets are two-dimensional figures that can be folded to create three-dimensional solids.

Solid figures and the nets that represent them are necessary for this lesson. These three-dimensional figures include a cube, a right rectangular prism, a triangular prism, a tetrahedron, a triangular pyramid (equilateral base and isosceles triangular sides), and a square pyramid.

There are reproducible copies of these nets included with this lesson. The nets of the cube and right rectangular prism are sized to wrap around solid figures made from wooden or plastic cubes, having 2 cm edges. Assemble these two solids prior to the lesson in enough quantities to allow students to work in pairs. The nets should be reproduced on card stock if possible and pre-cut and pre-folded before the lesson. One folded and taped example of each should also be assembled before the lesson.

The triangular prism has a length of 6 cm and has isosceles right triangular bases with identical legs that are 2 cm in length. Two of these triangular prisms can be arranged to form a rectangular prism.

The rectangular prism measures 4 cm $\times$ 6 cm $\times$ 8 cm, and its net will wrap around a Unifix cube solid that has dimensions of $2 \times 3 \times 4$ cubes.

The tetrahedron has an edge length of 6 cm. The triangular pyramid has a base edge length of 6 cm and isosceles sides with a height of 4 cm.

The square pyramid has a base length 6 cm and triangular faces which have a height of 4 cm.

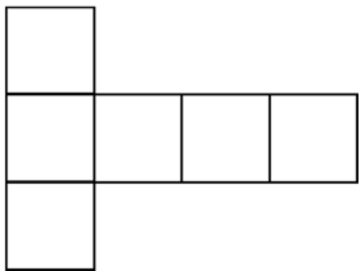
Also included is a reproducible sheet that contains 20 unique arrangements of 6 squares. Eleven of these can be folded to a cube, while $nine$ cannot. These should also be prepared before the lesson, as indicated above. Make enough sets of nets to accommodate the number of groups of students.

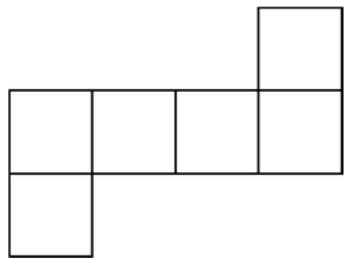
Prior to the lesson, cut a large cereal box into its net which will be used for the Opening Exercise. Tape the top flaps thoroughly so this net will last through several lessons. If possible, get two identical boxes and cut two different nets like the graphic patterns of the cube nets below. Add a third uncut box to serve as a right rectangular solid model.

Classwork

Mathematical Modeling Exercise (10 minutes)

Display the net of the cereal box with the unprinted side out, perhaps using magnets on a whiteboard. Display the nets below as well (images or physical nets).





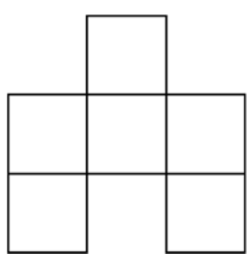
- What can you say about this cardboard (the cereal box)?
  - *Accept all correct answers, such as it is irregularly shaped; it has three sets of identical rectangles; all vertices are right angles; it has fold lines; it looks like it can be folded into a 3-D shape (box), etc.*
- How do you think it was made?
  - *Accept all plausible answers, including the correct one.*
- Compare the cereal box net to these others that are made of squares.
  - *Similarities: There are* 6 *sections in each; they can be folded to make a 3-D shape; etc.*
  - *Differences: One is made of rectangles; others are made of squares; there is a size difference; etc.*

Turn over the cereal box to demonstrate how it was cut. Reassemble it to resemble the intact box. Then direct attention to the six square arrangements.

- What do you think the six square shapes will fold up into?
  - *Cubes*
- If that were true, how many faces would it have?
  - *Six*

Fold each into a cube.

- Consider this six-square arrangement:



- Do you think it will fold to a cube?

Encourage a short discussion, inviting all views. As students make claims, ask for supporting evidence of their position. Use the cut-out version to demonstrate that this arrangement will not fold into a cube. Then define the term *net*.

- Today we will work with some two-dimensional figures that can be folded to create a three-dimensional solid. These are called geometric nets or just nets.

Ask students if they are able to visualize folding the nets without touching them. Expect a wide variety of spatial visualization abilities necessary to do this. Those that cannot readily see the outcome of folding will need additional time to handle and actually fold the models.

Example 1 (10 minutes): Cube

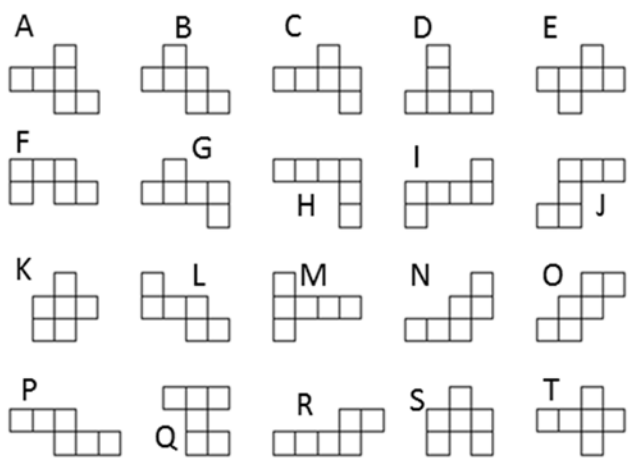
Use the previously cut out six square arrangements. Each pair or triad of students will need a set of 20 with which to experiment. These are sized to wrap around a cube with side lengths of 4 cm, which can be made from eight Unifix cubes. Each group needs one of these cubes.

- There are some six square arrangements on your student page. Sort each of the six square arrangements into one of two piles, those that are nets of a cube (can be folded into a cube) and those that are not.

Exercise 1

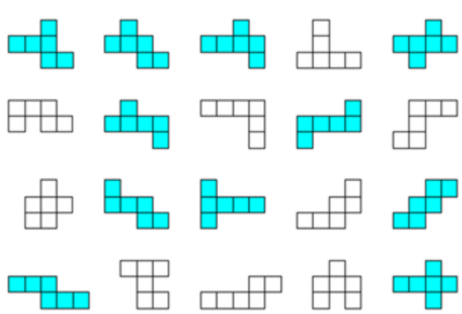
Exercise 1

1. **Nets are two-dimensional figures that can be folded up into three-dimensional solids. Some of the drawings below are nets of a cube. Others are not cube nets; they can be folded but not into a cube.**



MP.1

   a. **Experiment with the larger cut out patterns provided. Shade in each of the figures above that will fold into a cube.**



MP.1

b. **Write the letters of the figures that can be folded up into a cube.**

A, B, C, E, G, I, L, M, O, P, T

c. **Write the letters of the figures that cannot be folded up into a cube.**

D, F, H, J, K, N, Q, R, S

Example 2 (10 minutes): Other Solid Figures

Provide student pairs with a set of nets for each of the following: right rectangular prism, triangular prism, tetrahedron, triangular pyramid (equilateral base and isosceles triangular sides), and square pyramid.

Define prism: A prism is a solid geometric figure whose two bases are parallel to identical polygons and whose sides are parallelograms.

Define pyramid: A pyramid is a solid geometric figure formed by connecting a polygonal base and a point forming triangular lateral faces. (Note: The point is sometimes referred to as the *apex*.)

Display one of each solid figure. Assemble them so the grid lines are hidden (inside).

Allow time to explore the nets folding around the solids.

- Why are the faces of the pyramid triangles?
  - *The base of the triangle matches the edge of the base of the pyramid. The top vertex of the lateral face is at the apex of the pyramid. Further, each face has two vertices that are the endpoints of one edge of the pyramid's base, and the third vertex is the apex of the pyramid.*
- Why are the faces of the prism parallelograms?
  - *The two bases are identical polygons on parallel planes. The lateral faces are created by connecting each vertex of one base with the corresponding vertex of the other base, thus forming parallelograms.*
- How are these parallelograms related to the shape and size of the base?
  - *The length of the base edges will match one set of sides of the parallelogram. The shape of the base polygon will determine the number of lateral faces the prism has.*
- If the bases are hexagons does this mean the prism must have six faces?
  - *No, there are six sides on the prism, plus two bases, for a total of eight faces.*
- What is the relationship between the number of sides on polygonal base and the number of faces on the prism?
  - *The total number of faces will be two more than the number of sides in the polygonal bases.*
- What additional information do you know about a prism if its base is a regular polygon?
  - *All sides of the prism will be identical.*

Scaffolding:

Assembled nets of each solid figure should be made available to students who might have difficulty making sharp, precise folds.

Scaffolding:

- All students may benefit from a working definition of the word lateral. In this lesson, the word side can be used (as opposed to the base).
- ELLs may hear similarities to the words ladder or literal, neither of which are related nor make sense in this context.

Example 3 (10 minutes): Tracing Nets

If time allows, or as an extension, ask students to trace the faces of various solid objects (i.e., wooden or plastic geo-solids, paperback books, packs of sticky notes, or boxes of playing cards). After tracing a face, the object should be carefully rolled so one edge of the solid matches one side of the polygon that has just been traced. If this is difficult for students because they lose track of which face is which as they are rolling, the faces can be numbered or colored differently to make this easier. These drawings should be labeled "Net of a [*Name of Solid*]*"*. Challenge students to make as many different nets of each solid as they can.

Closing (3 minutes)

- What kind of information can be obtained from a net of a prism about the solid it creates?
  - *We can identify the shape of the bases and the number and shape of the lateral faces (sides). The surface area can be more easily obtained since we can see all faces at once.*
- When looking at a net, how can you determine which faces are the bases?
  - *If the net is a pyramid, there will be multiple, identical triangles that will form the lateral faces of the pyramid, while the remaining face will be the base (and will identify the type of pyramid it is). Examples are triangular, square, pentagonal, and hexagonal pyramids.*
- How do the nets of a prism differ from the nets of a pyramid?
  - *If the pyramid is not a triangular pyramid, the base will be the only polygon that is not a triangle. All other faces will be triangles. Pyramids have one base and triangular lateral faces while prisms have two identical bases, which could be any type of polygon, and lateral faces that are parallelograms.*
- Constructing solid figures from their nets helps us see the "suit" that fits around it. We can use this in our next lesson to find the surface area of these solid figures as we wrap them.

> Lesson Summary
>
> **Nets are two-dimensional figures that can be folded to create three-dimensional solids.**
>
> **A prism is a solid geometric figure whose two bases are parallel identical polygons and whose sides are parallelograms.**
>
> **A pyramid is a solid geometric figure formed by connecting a polygonal base and a point and forming triangular lateral faces. (Note: The point is sometimes referred to as the apex.)**

Exit Ticket (4 minutes)

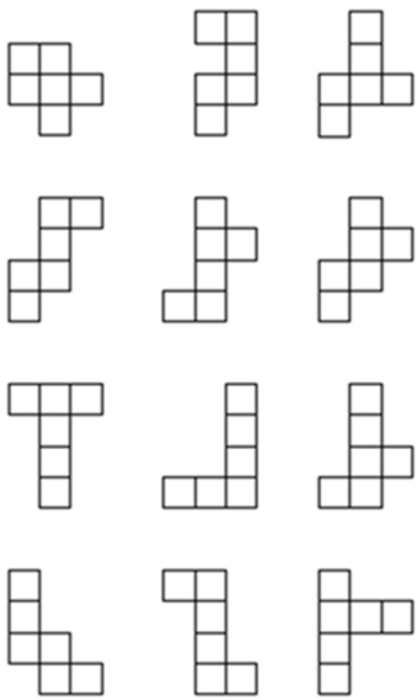
Name ______________________________ Date ______________

Lesson 15: Representing Three-Dimensional Figures Using Nets

Exit Ticket

1. What is a net? Describe it in your own words.

2. Which of the following will fold to make a cube? Explain how you know.



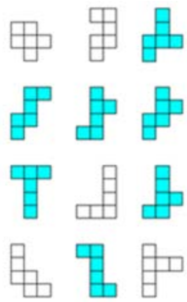
Exit Ticket Sample Solutions

1. **What is a net? Describe it in your own words.**

   Answers will vary but should capture the essence of the definition used in this lesson. A net is a two-dimensional figure that can be folded to create a three-dimensional solid.

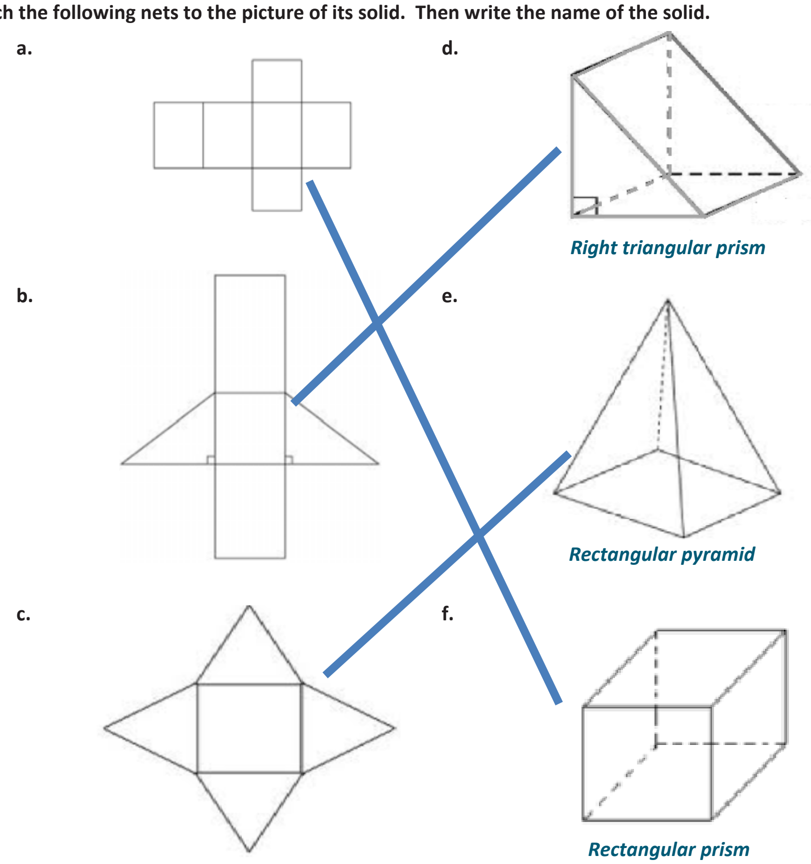
2. **Which of the following will fold to make a cube? Explain how you know.**

   Evidence for claims will vary.



Problem Set Sample Solutions

1. **Match the following nets to the picture of its solid. Then write the name of the solid.**



Right triangular prism

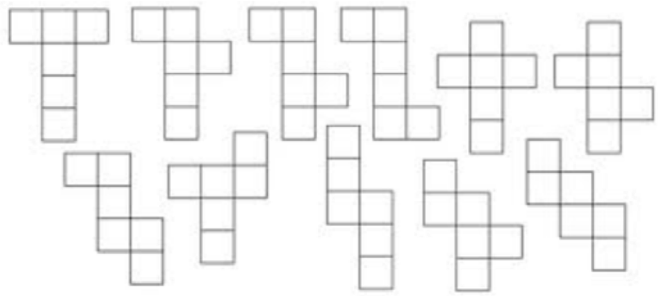
Rectangular pyramid

Rectangular prism

2. **Sketch a net that will fold into a cube.**

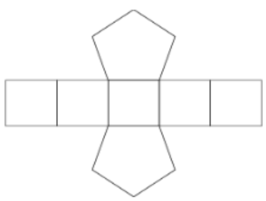
Sketches will vary but will match one of the shaded ones from earlier in the lesson.

Here are the 11 possible nets for a cube.



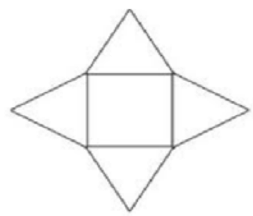
3. **Below are the nets for a variety of prisms and pyramids. Classify the solids as prisms or pyramids, and identify the shape of the base(s). Then write the name of the solid.**

a.



Prism, bases are pentagons
Pentagonal Prism

b.

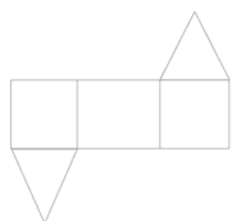


Pyramid, the base is a rectangle
Rectangular Pyramid

c.

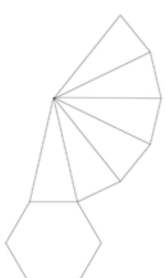
Pyramid, the base is a triangle
Triangular Pyramid

d.



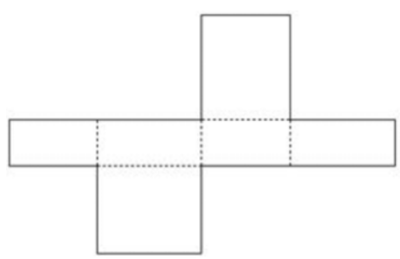
Prism, bases are triangles
Triangular Prism

e.



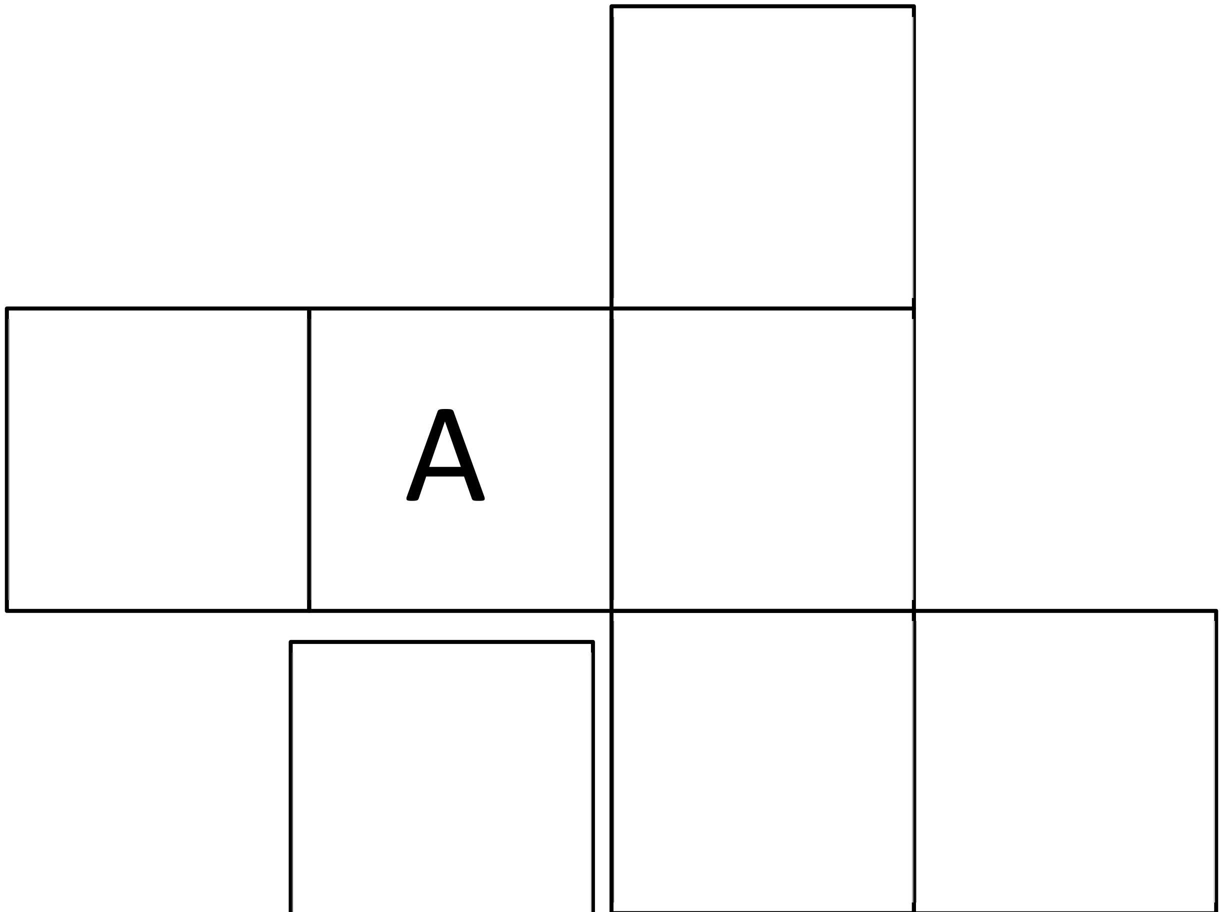
Pyramid, base is a hexagon
Hexagonal Pyramid

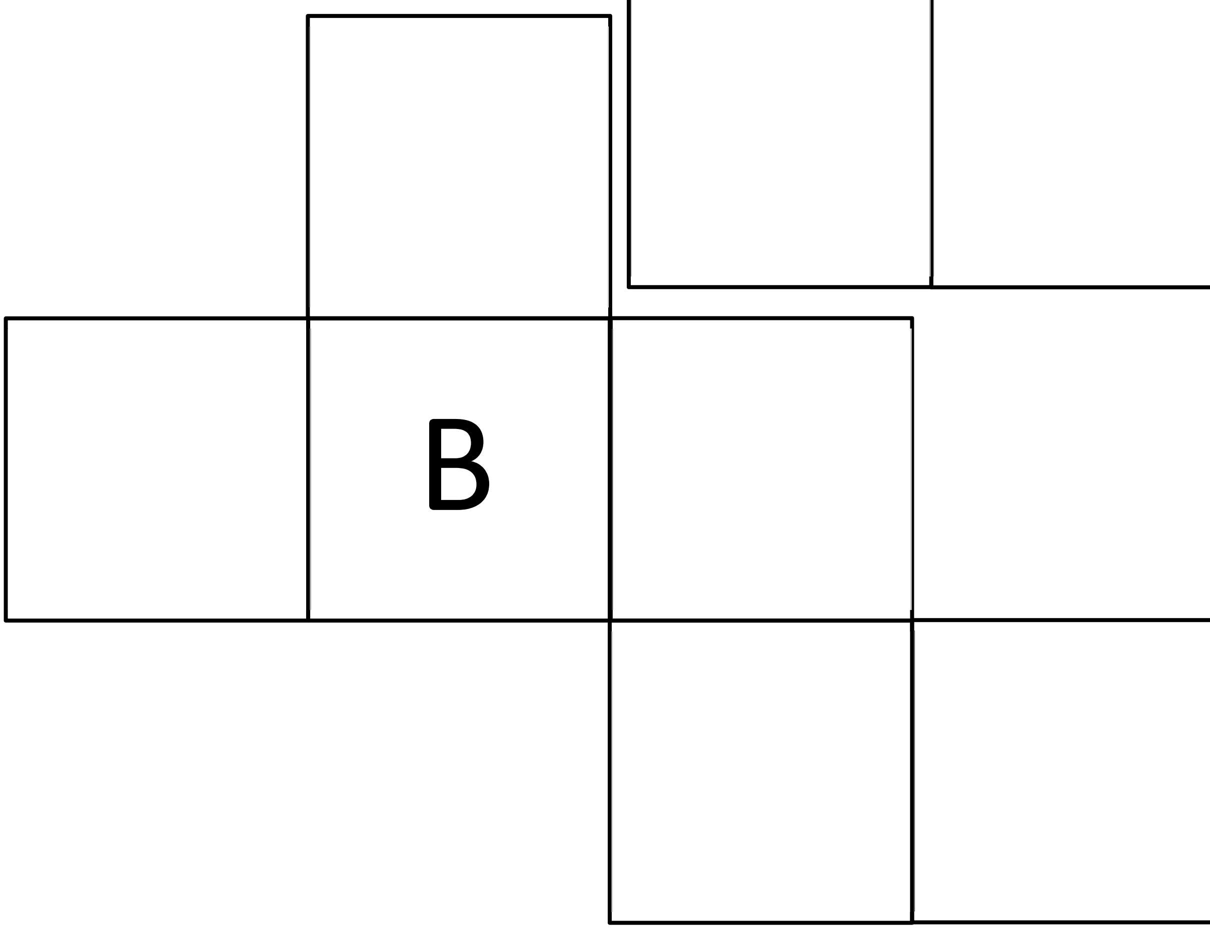
f.



Prism, bases are rectangles
Rectangular Prism

Below are graphics needed for this lesson. The graphics should be printed at 100% scale to preserve the intended size of figures for accurate measurements. Adjust your copier or printer settings to actual size and set page scaling to none.


A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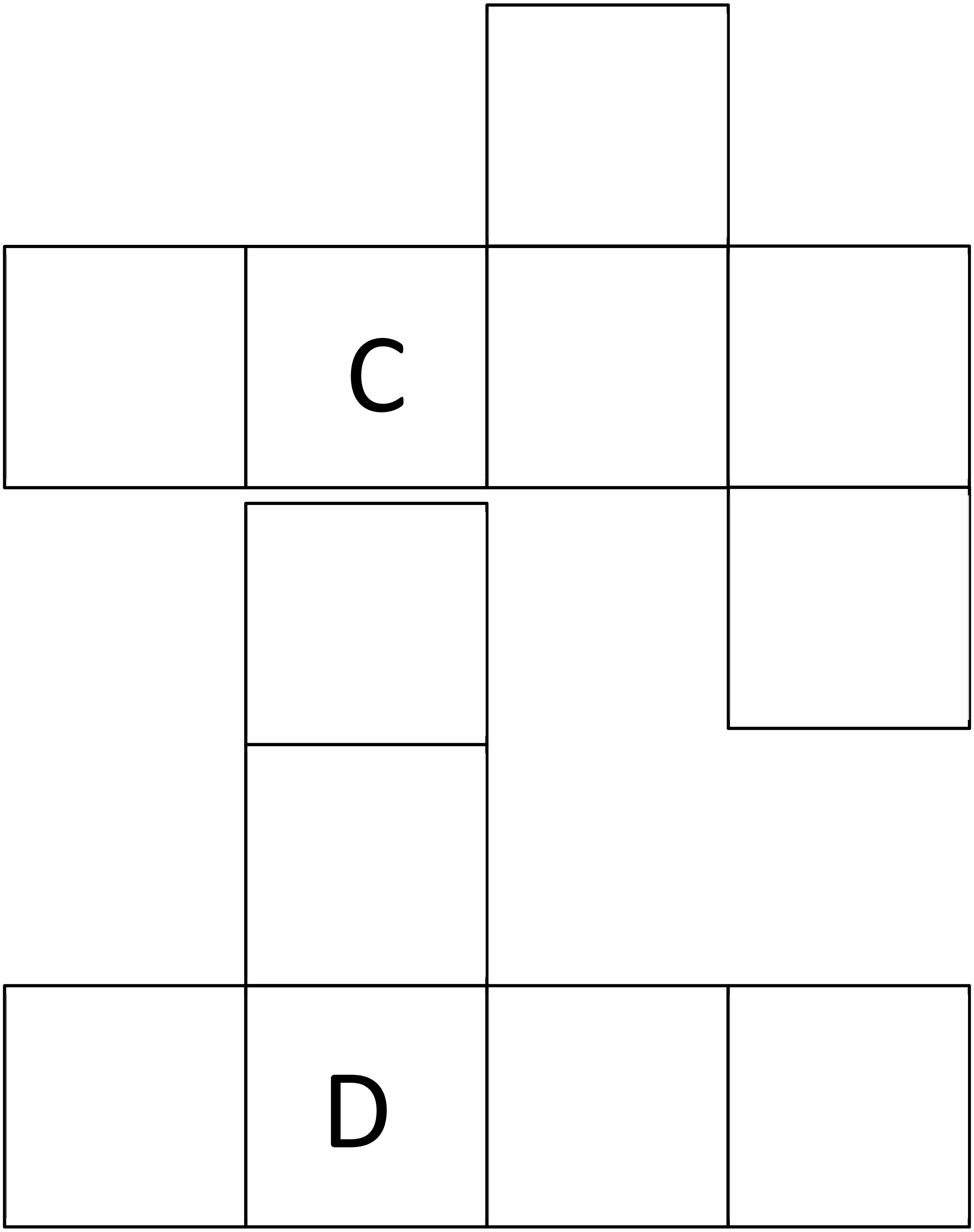
C
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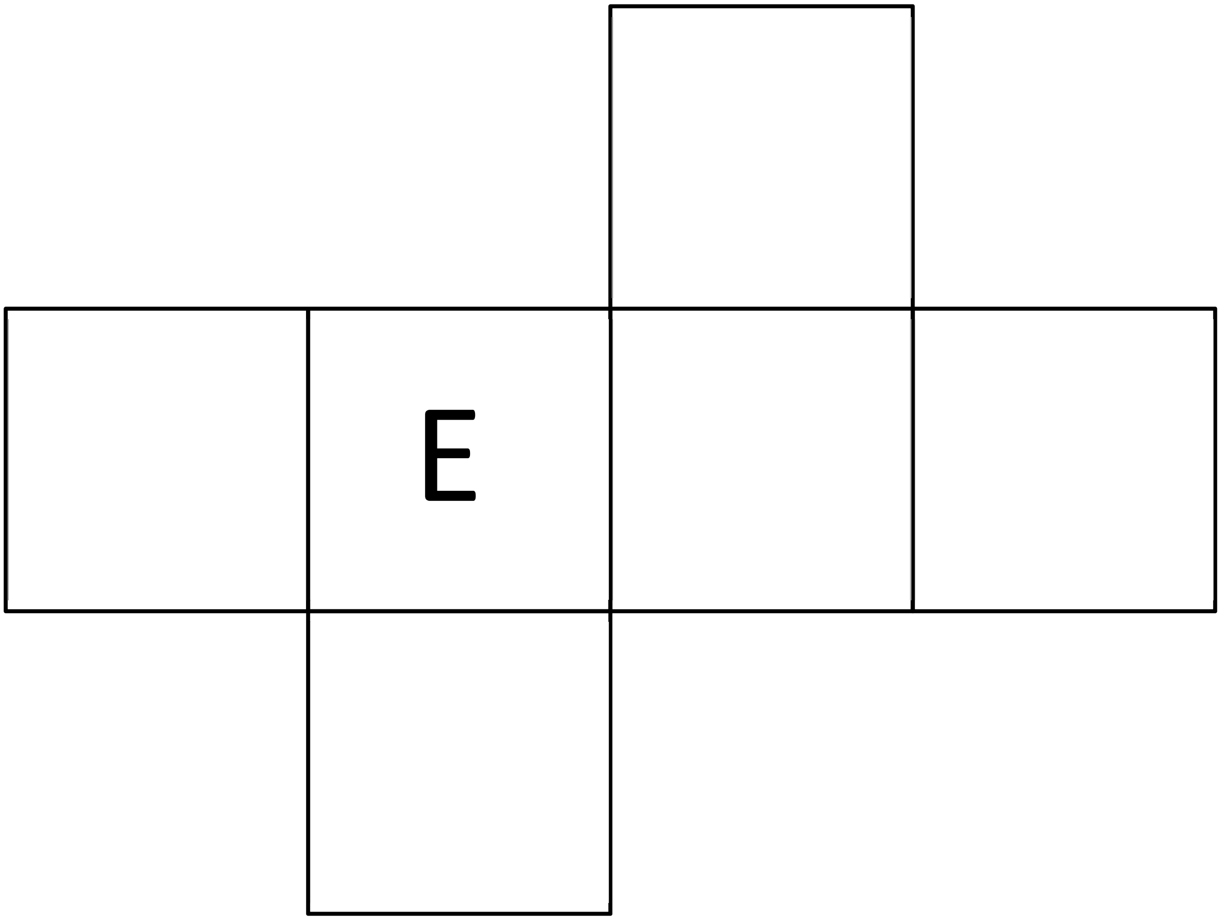
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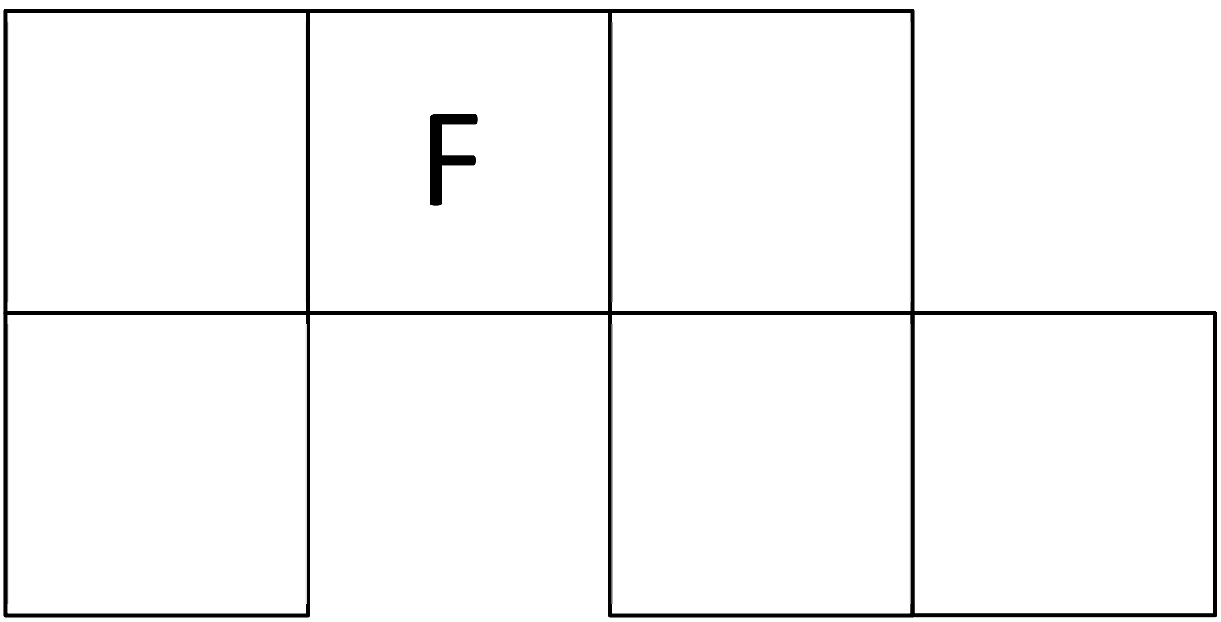
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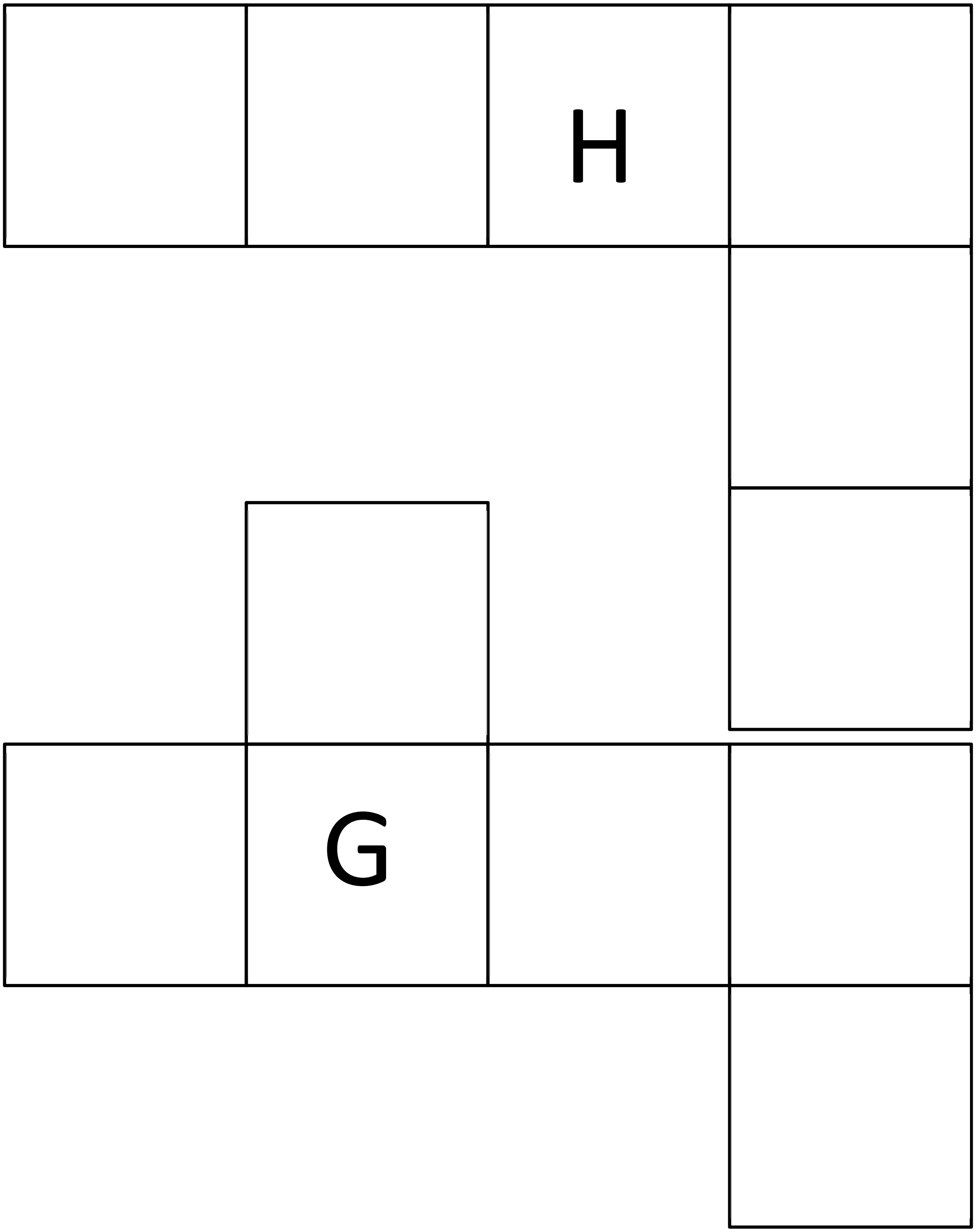
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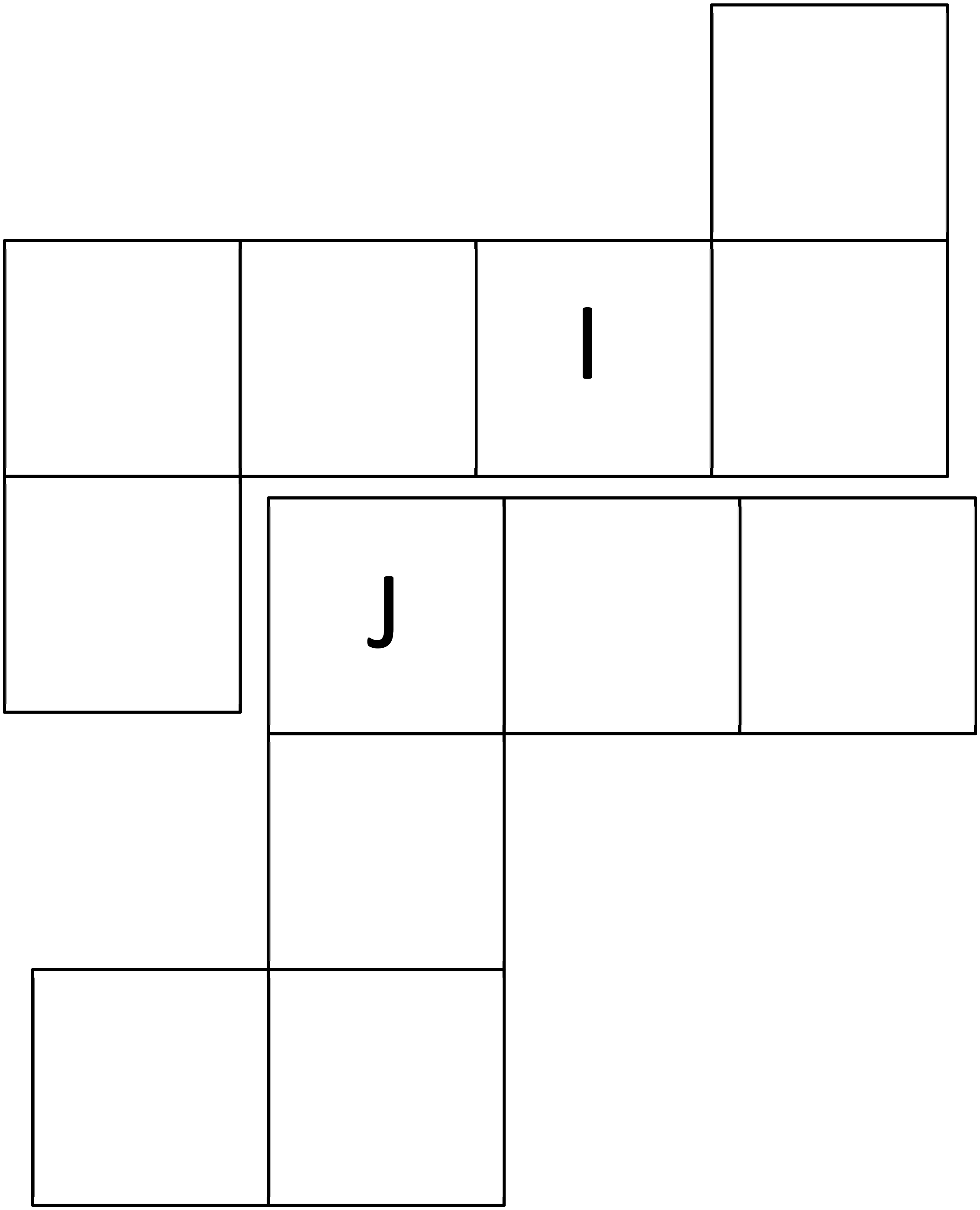
G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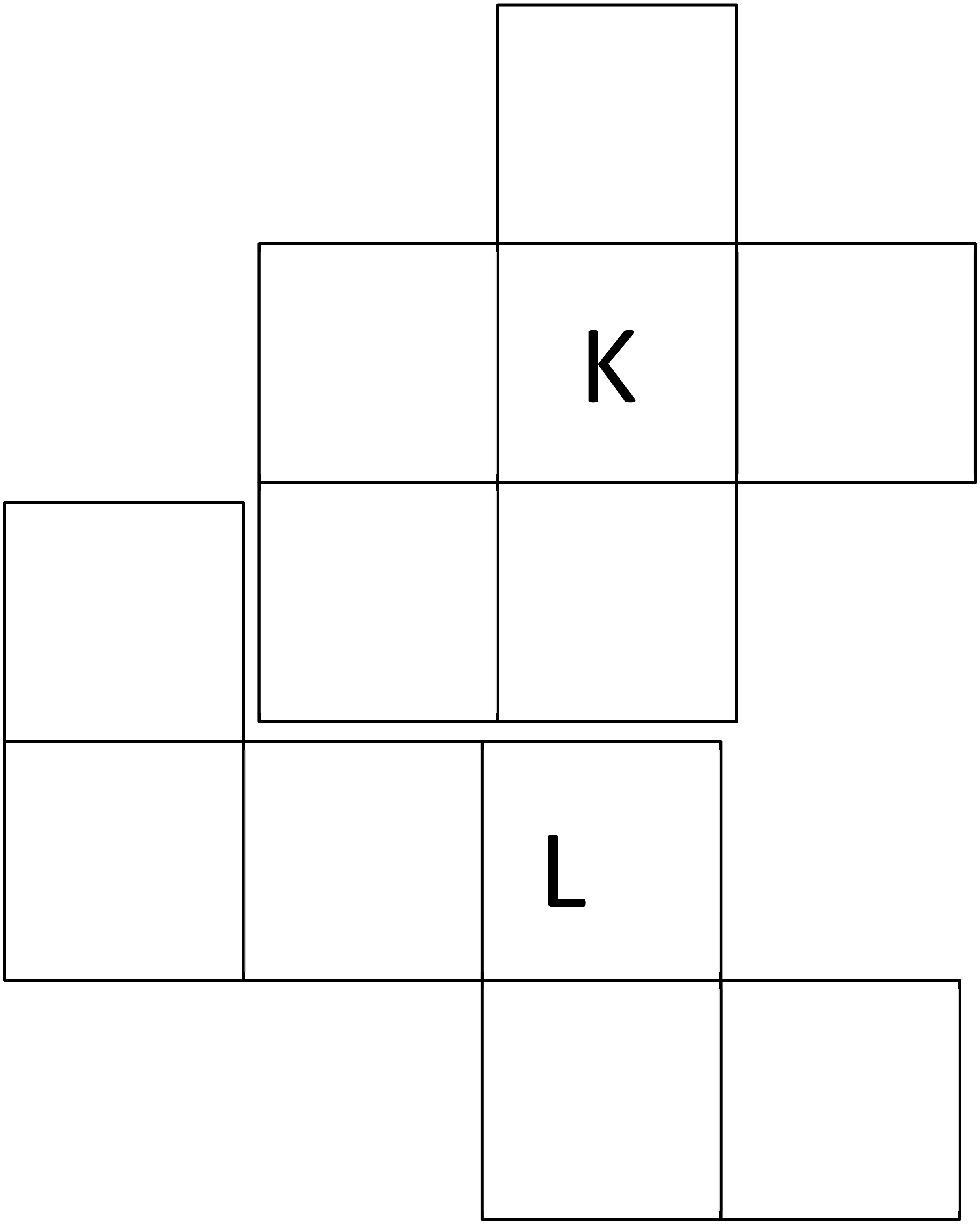
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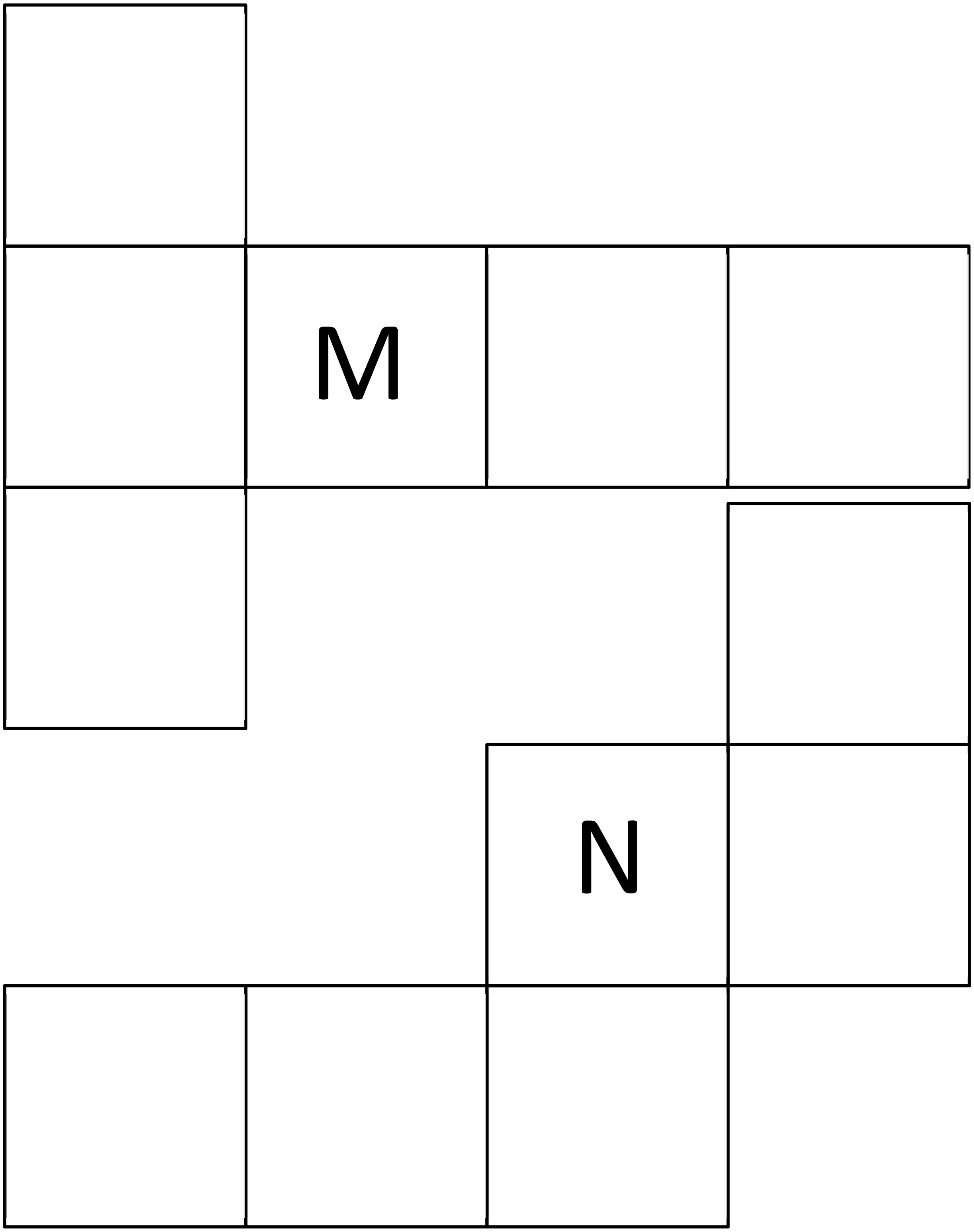
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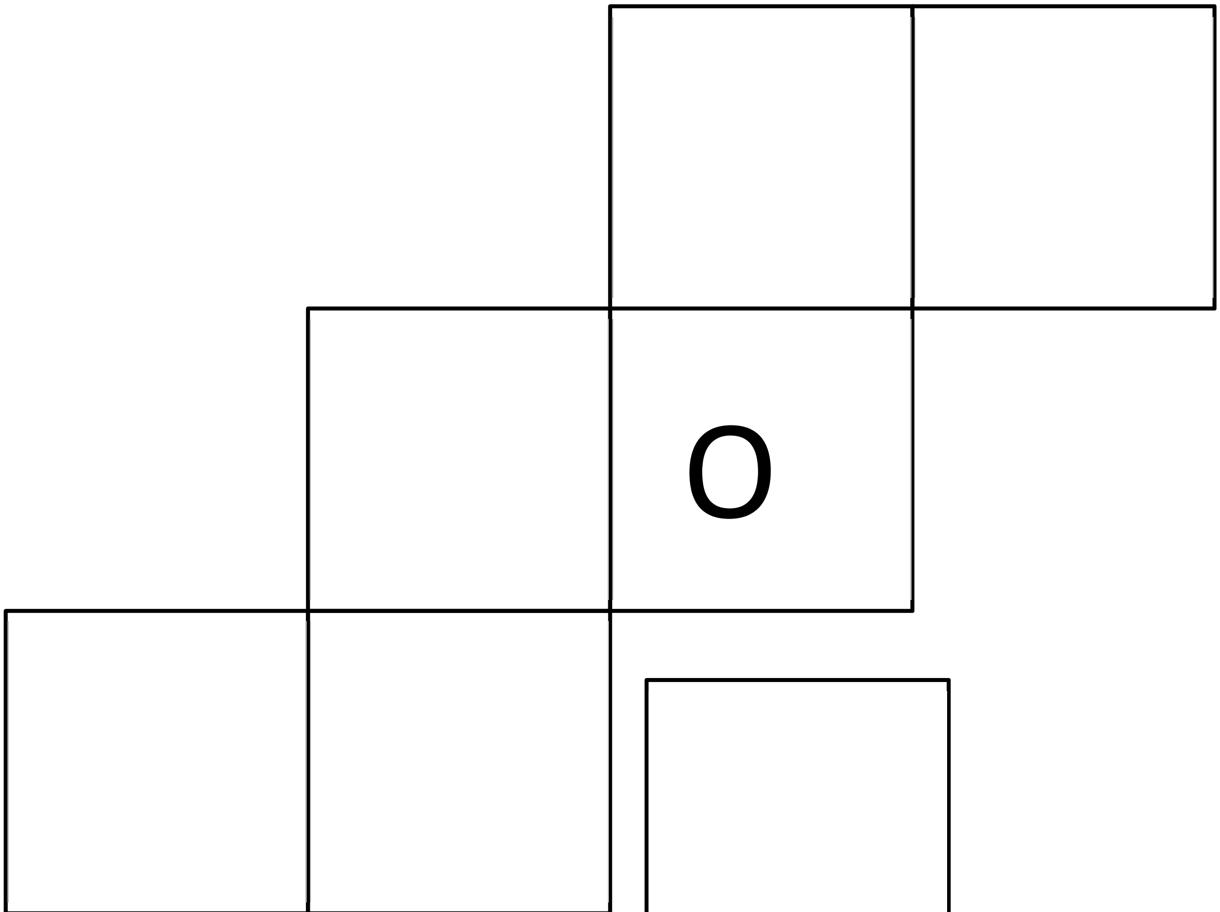
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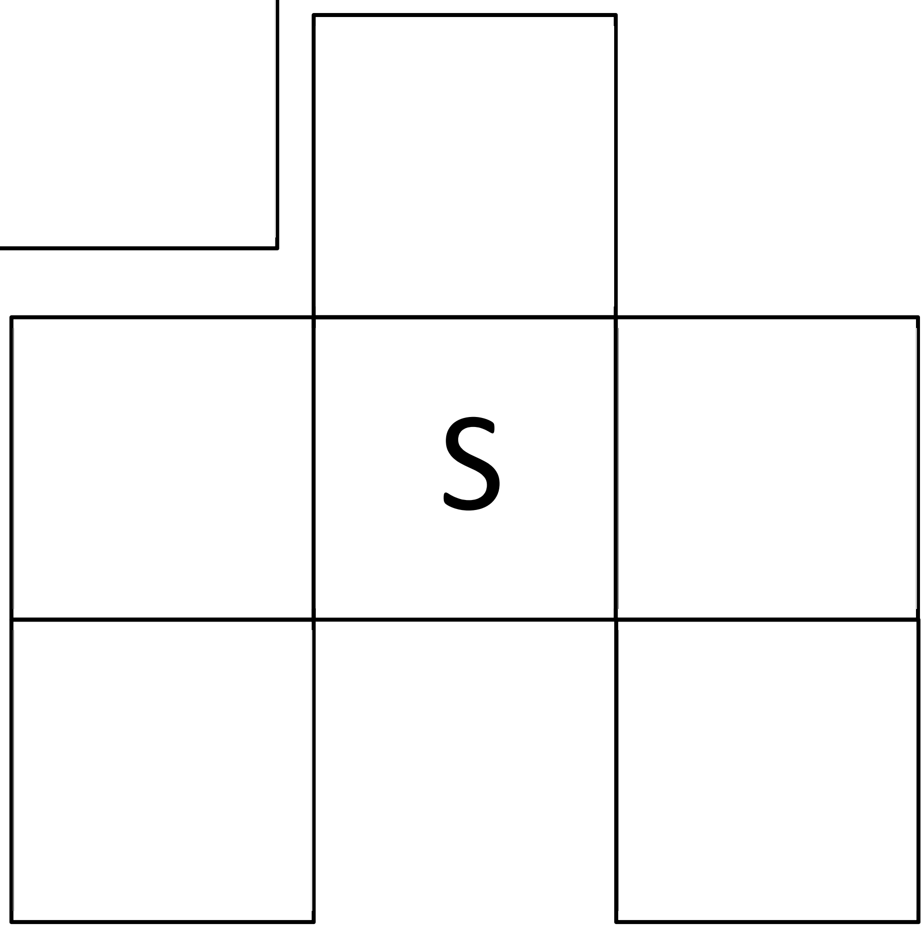
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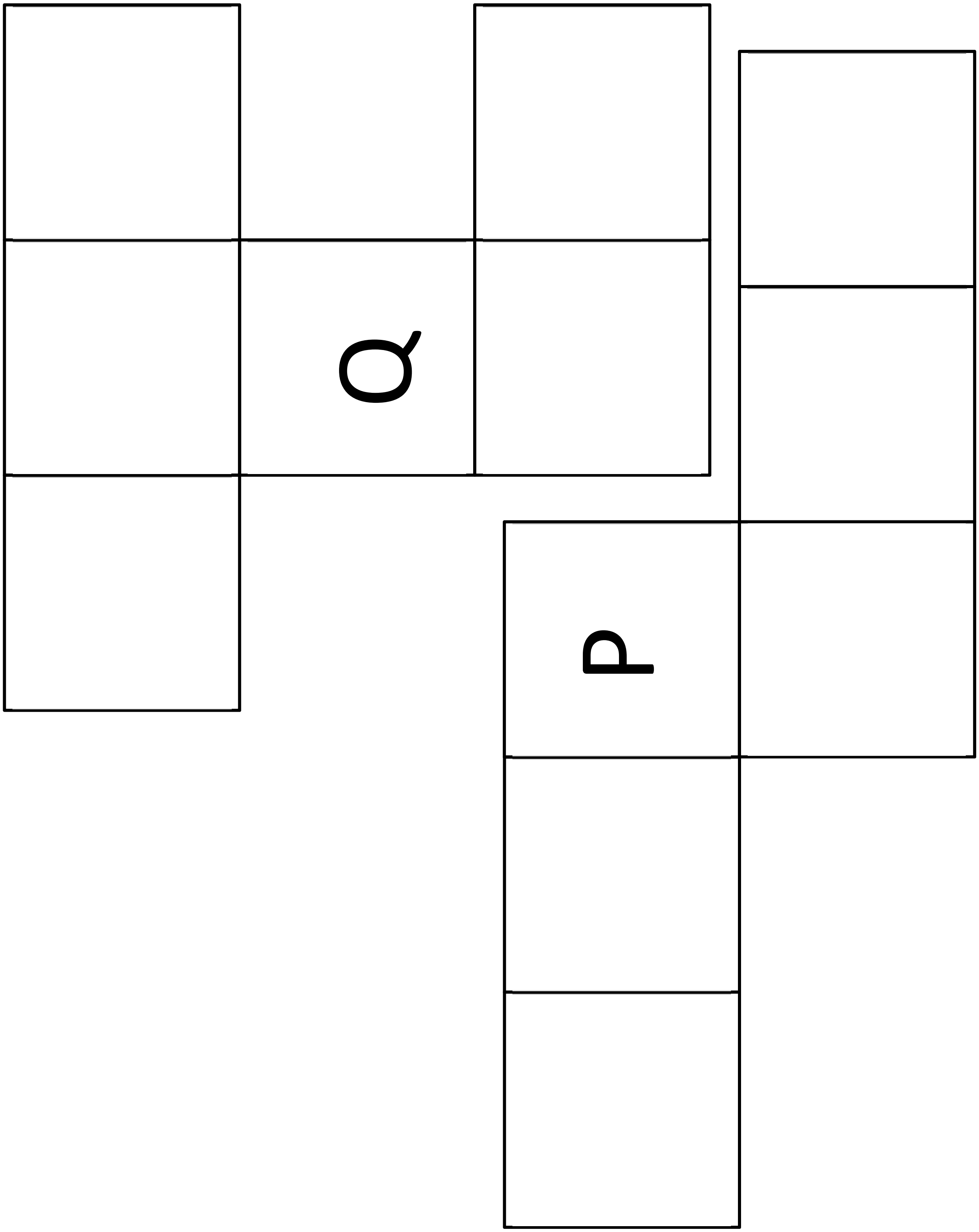

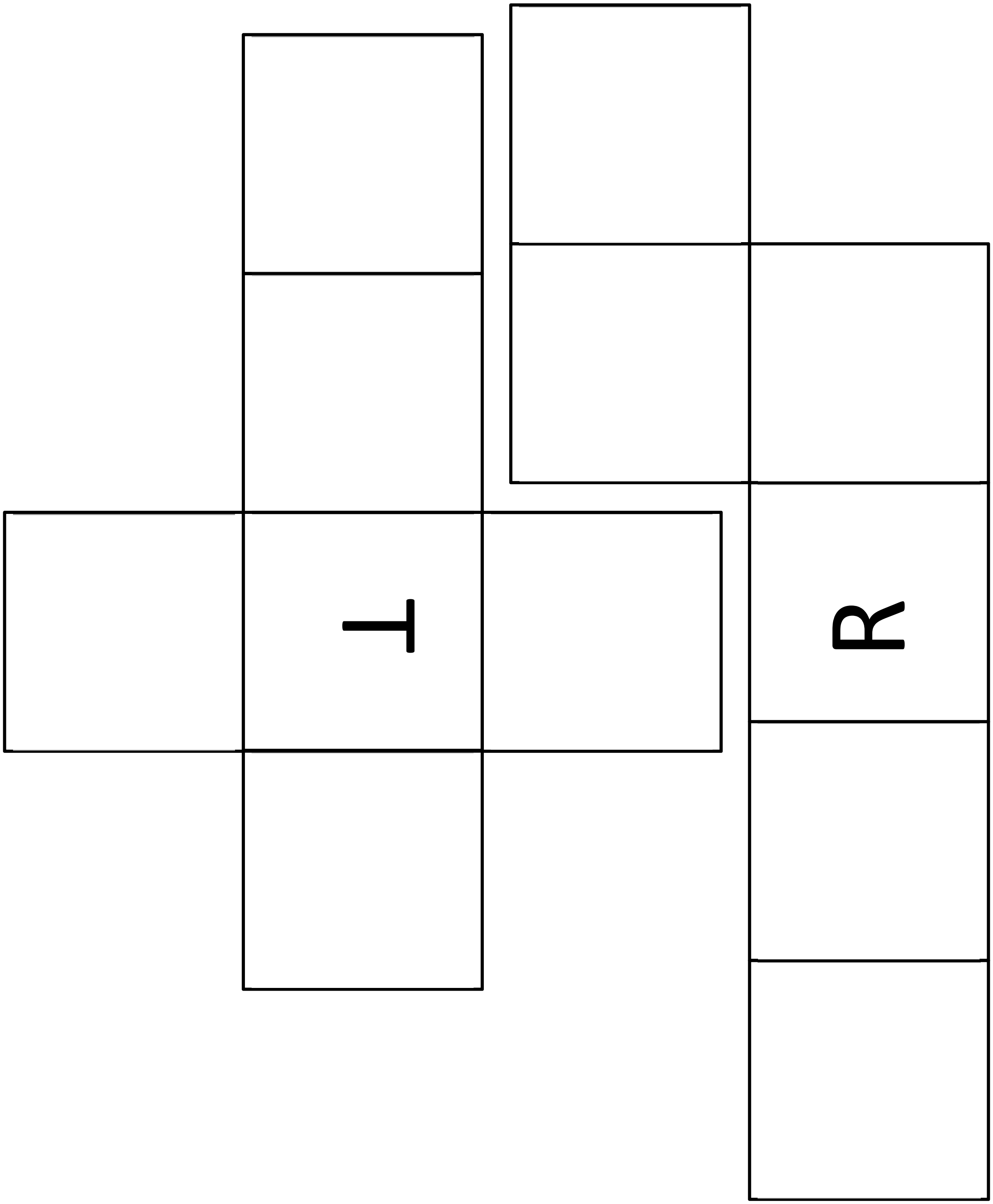
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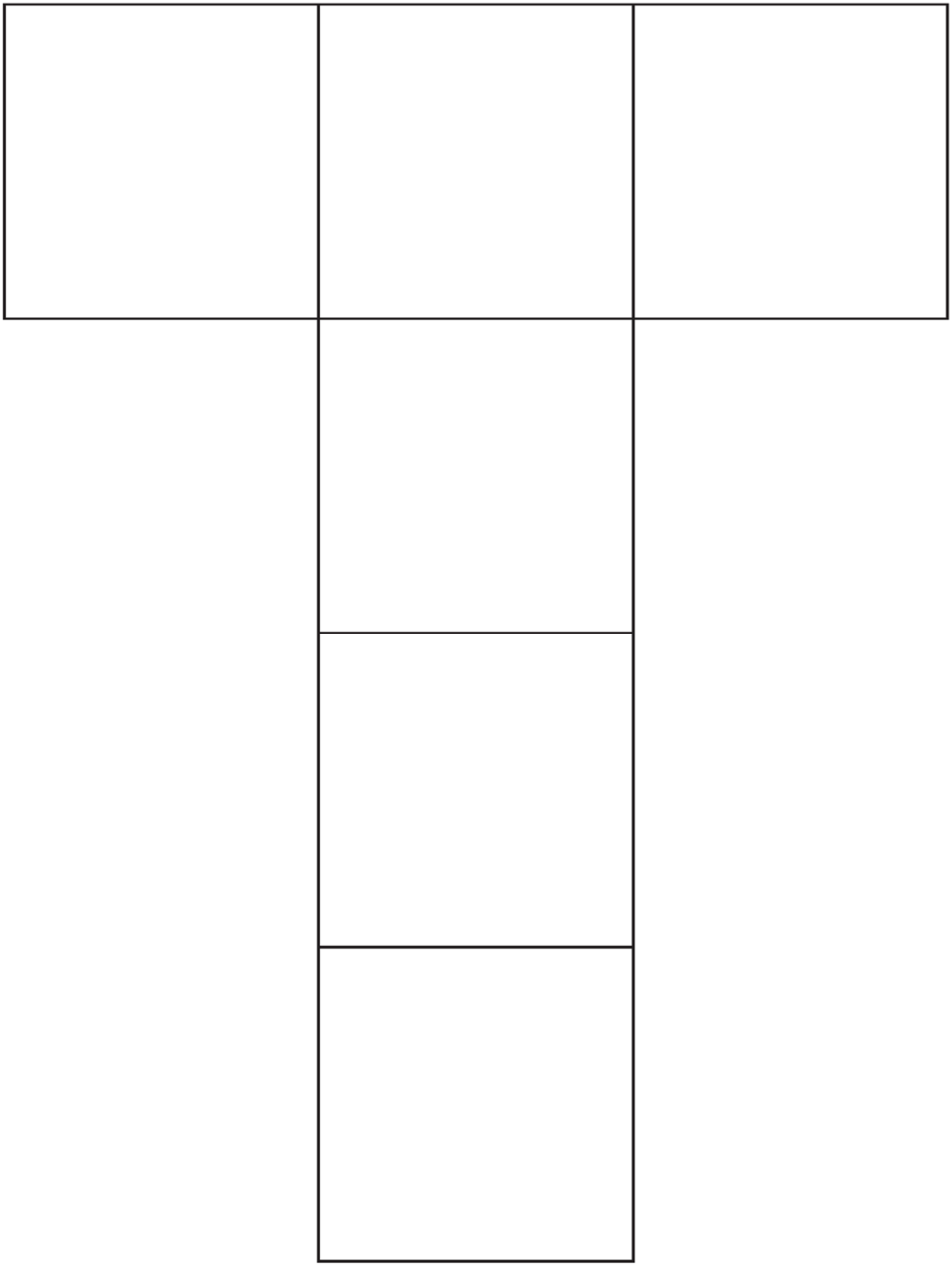
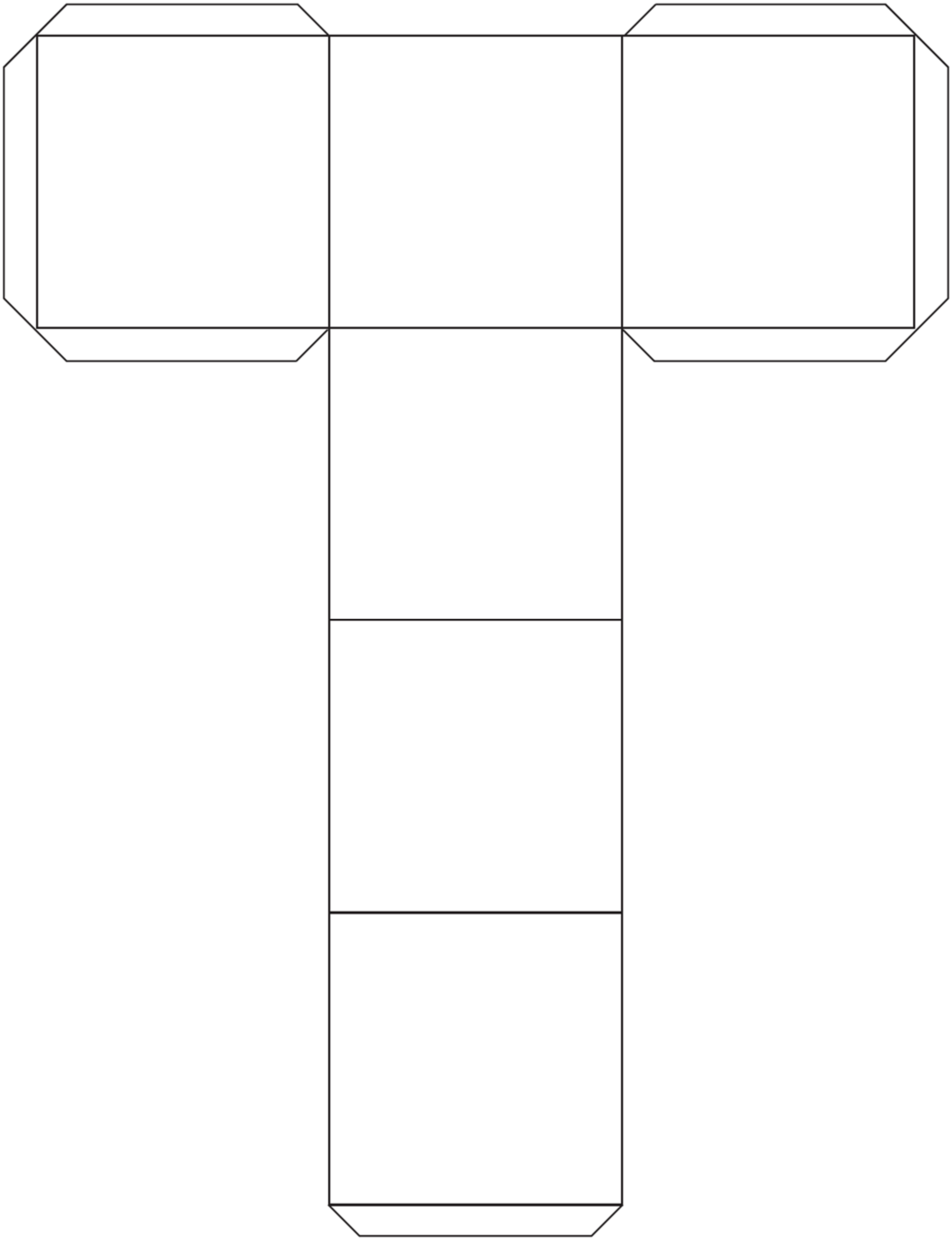
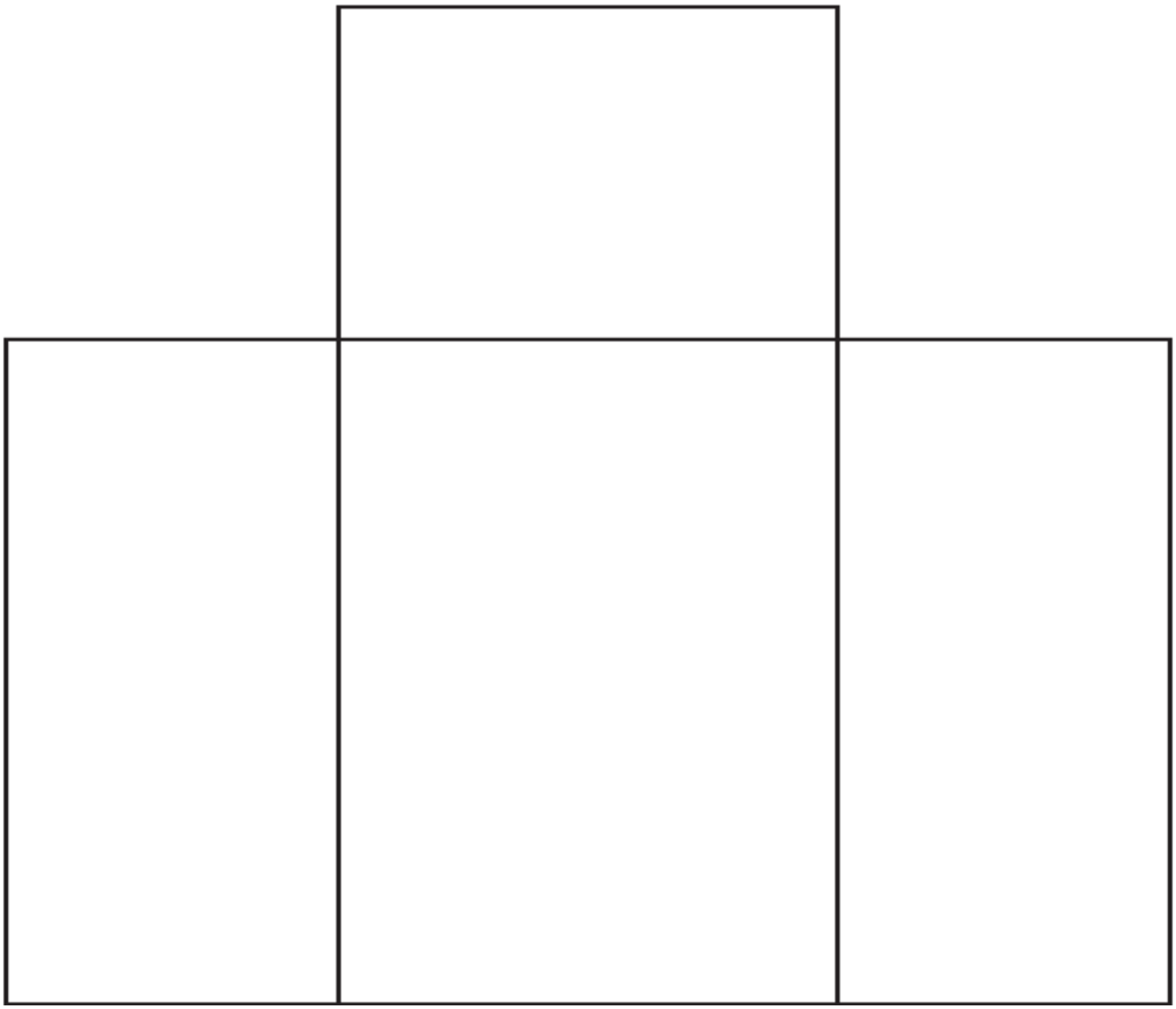
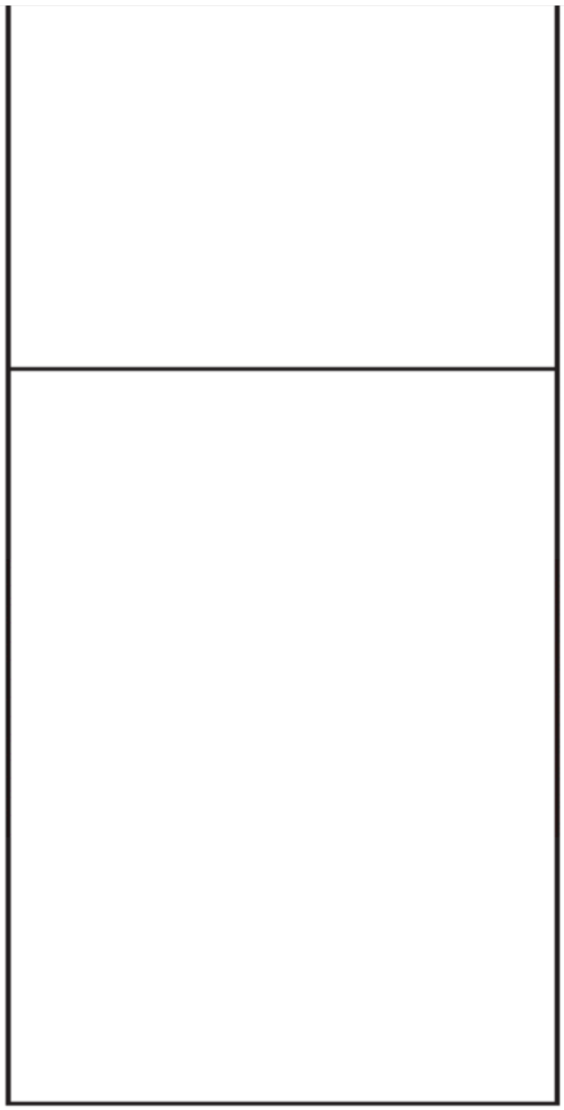




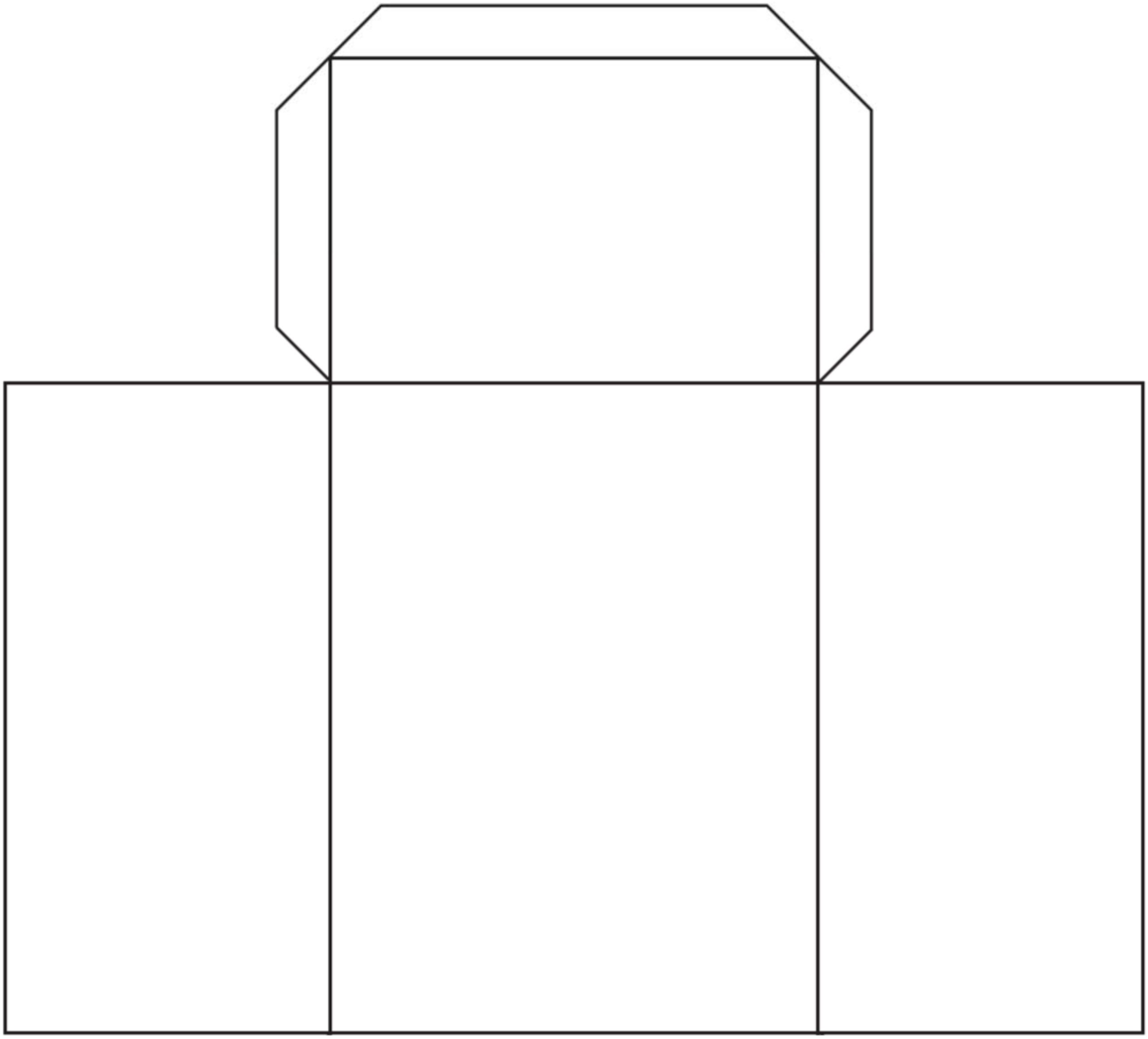
Part 1 of 2



Part 2 of 2



Part 1 of 2



Part 2 of 2