Polynomial Functions

Math Background

Previously, you

- Graphed quadratic functions with and without technology
- Used factoring to solve quadratic equations
- Worked with properties of integers
- Identified the zeros or roots of a quadratic function
- Graphed one and two variable quadratic inequalities

In this unit you will

- Identify key features of the graphs of polynomial functions, including zeroes and end behavior
- Graph polynomial functions using function characteristics
- Divide polynomials
- Use the Factor Theorem and Remainder Theorem to factor polynomials completely

You can use the skills in this unit to

- Factor a polynomial using synthetic or long division.
- State the end behavior of given polynomials.
- Identify zeros based on graphs of polynomials.
- Identify relative maximums and minimums of the graphs.
- Solve and graph solutions of single variable and two variable polynomial inequalities

Vocabulary

- **End behavior** – The behavior of the graph of a polynomial function as x approaches positive or negative infinity.
- **Even Function –** A function $f(x)$ which satisfies the property that $f(-x) = f(x)$. The functions with even degree are even functions.
- **Fundamental Theorem of Algebra** – Every equation which can be put in the form with zero on one side of the equal sign and a polynomial of degree greater than or equal to one with real or complex coefficients on the other has at least one root which is a real or complex number.
- **Global (or Absolute) Maximum –** A value of a given function that is greater than or equal to any value of the given function. An absolute maximum is the greatest of all values.
- **Global (or Absolute) Minimum –** A value of a given function that is less than or equal to any value of the given function. An absolute minimum is the lowest of all values.
- **Factor Theorem** – It states that if $f(a) = 0$ in which $f(x)$ represents a polynomial in x, then $(x - a)$ is one of the factors of $f(x)$.
- **Local (or Relative) Maximum –** A value of a function that is greater than those values of the function at the surrounding points, but is not the greatest of all values.
- **Local (or Relative) Minimum** – A value of a function that is less than those values of the function at the surrounding points, but is not the lowest of all values.
- **Multiplicity** – It is how often a certain root or zero is part of the factoring. It is the number of times the root is a zero of the function.

- **Odd Function –** Any function $f(x)$ that satisfies the property that $f(-x) = -f(x)$. The functions with odd degree are odd functions.
- **Parent Polynomial Function –** The simplest function of a family of functions, $f(x) = x^n$, where n is a positive integer.
- **Polynomial** – An algebraic expression that consists of two or more terms.
- **Rational Root (Zero) Theorem –** A theorem that provides a complete list of possible rational roots of the polynomial equation $a_n x^n + a_{n-1} x^{n-1} + ... + a_2 x^2 + a_1 x + a_0 = 0$ where all coefficients are integers.
- **Remainder Theorem –** The theorem stating that if a polynomial in x, $f(x)$, is divided by $(x-a)$, where a is any real or complex number, then the remainder is $f(a)$.
- **Root** – The value(s) of a variable that makes the equation true.
- **Synthetic Division** – A method of performing polynomial long division, with less writing and fewer calculations.
- **Zeros of a function** – An input value that produces an output of zero. It is also known as a root and is where the graph meets the x-axis, also known as an x-intercept.

Essential Questions

- What does a polynomial function look like? How do I identify the zeros and the end behavior?
- Why are graphs of polynomials important?
- How does the Fundamental Theorem of Algebra apply to the solutions of a polynomial function?

Overall Big Ideas

The sketch of a polynomial is a continuous graph with possible changes in shape and direction. The zeros are located where the graph crosses the x-axis. The end behavior is determined by the leading coefficient and the degree of the polynomial.

Polynomial equations provide some of the most classic problems in all of algebra. Finding zeros and extrema have many real-world applications. Real-life situations are modeled by writing equations based on data and using those equations to determine or estimate other data points (speed, volume, time, profits, patterns, etc.).

The degree of a polynomial determines the number of roots a function possesses.

Skill

To use long division and synthetic division to divide polynomials.

To analyze graphs of polynomial functions determining the characteristics of the graph, including zeroes and end behavior.

To graph polynomial functions using characteristics determined by the equation of the function.

To use transformations to sketch graphs of quadratic and polynomial functions.

To use the Fundamental Theorem of Algebra to determine the number of zeroes of polynomial functions.

To find all zeroes of a polynomial function using the Remainder Theorem and the Rational Roots Theorem.

To solve and graph solutions of single variable and two variable polynomial inequalities.

Related Standards

A.APR.D.6

Rewrite simple rational expressions in different forms; write $\frac{a(x)}{b(x)}$ in the form $q(x)+\frac{r(x)}{b(x)}$, where $a(x), b(x), q(x),$ and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

A.APR.B.3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

F.IF.A.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

F.IF.B.4-2

For any function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. *(Modeling Standard)

F.IF.C.7c

Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. *(Modeling Standard)

G.CO.A.2

Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).

F.BF.B.3-2

Identify the effect on the graph of replacing $f(x)$ by $f(x)+k$, $kf(x)$, $f(kx)$, and $f(x+k)$ for specific values of k (both positive and negative) and find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Include simple radical, rational, an exponential functions, note the effect of multiple transformations on a single graph, and emphasize common effects of transformations across function types.

N.CN.C.9

Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

A.APR.B.2

Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x-a$ is $p(a)$, so $p(a)=0$ if and only if $(x-a)$ is a factor of $p(x)$.

F.IF.C.8a

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

A.CED.A.1-2

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from all types of functions, including simple rational and radical functions. *(Modeling Standard)

A.CED.A.2-2

Create equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. Use all types of equations. *(Modeling Standard)

Notes, Examples, and Exam Questions

<u>Unit 2.1 To use long division and synthetic division to divide polynomials.</u>

<u>Polynomial Function:</u> a function of the form $f\left(x\right)=a_nx^n+a_{n-1}x^{n-1}+\ldots+a_2x^2+a_1x+a_0$, $a_n\neq 0$

Note: Exponents are whole numbers and coefficients are real numbers.

Leading Coefficient: a_n Constant Term: a_0 Degree: n, the largest exponent of x

<u>Long Division of Polynomials</u>

The process works just like the long (numerical) division students did back in elementary school, except that now there is division with variables. Make sure to subtract the polynomials. If the division does not come out even, the remainder is turned into a fraction (the remainder divided by the original divisor) and it is added to the quotient.

Ex 1: Find the quotient. $\left(3x^3-5x^2+10x-3\right)\div\left(3x+1\right)$

$x^2-2x\ +4$

$3x+1\overline{)3x^3-5x^2+10x-3}$

$\underline{3x^3+1x^2}$ (make sure to subtract)

$-6x^2+10x$ (bring down the next term)

$\underline{-6x^2-\ 2x}$

$12x-3$

$\underline{12x+4}$

-7 (remainder)

We place the remainder, -7, over the divisor. Just like long division with natural numbers.

Solution: $x^2-2x+4-\dfrac{7}{3x+1}$

Ex 2: Find the quotient of y^4+2y^2-y+5 and y^2-y+1.

y^2+y+2

$y^2-y+1\overline{)\ y^4+0y^3+2y^2-y+5}$

$\underline{-y^4+y^3\quad -y^2}$ (add the opposite)

$y^3+\quad y^2-y$ (bring down the next term)

$\underline{-y^3+\ y^2-y}$

$2y^2-2y+5$

$-\underline{2y^2+2y-2}$

3 (remainder)

Remember to put a place for the missing term.

Solution: $y^2+y+2+\dfrac{3}{y^2-y+1}$

Ex 3: Find the quotient. $\left(x^4-8x^3+11x-6\right)\div\left(x+3\right)$

Note: Every term of the polynomial in the dividend must be represented. Since this polynomial is missing an x^2 term, we must include the term $0x^2$.

$$\begin{array}{r}
x^3-11x^2+33x-88 \\
x+3\,\big)\,\overline{x^4-8x^3+0x^2+11x-6} \\
\underline{x^4+3x^3} \\
-11x^3+0x^2 \\
\underline{-11x^3-33x^2} \\
33x^2+11x \\
\underline{33x^2+99x} \\
-88x-6 \\
\underline{-88x-264} \\
258
\end{array}$$

Solution: $x^3-11x^2+33x-88+\dfrac{258}{x+3}$

Dividing Polynomials Using Synthetic Division: (Note: This procedure can only be used when the divisor is in the form $x-k$ (a linear binomial).)

Ex 4: Divide the polynomial x^3-3x^2-7x+6 by $x+3$.

With the polynomial in standard form, write the coefficients in a row. If a term is missing, make sure to put a zero in the row.

Put the *k* value (-3) to the upper left in the "box".

Bring down the first coefficient, then multiply by the *k* value. Add straight down the columns and repeat.

$$\begin{array}{r|rrrr}
-3 & 1 & -3 & -7 & 6 \\
 & & -3 & 18 & -33 \\
\hline
 & 1 & -6 & 11 & \text{R}\ -27
\end{array}$$

Note for graphing: This means that (-3,-27) is an ordered pair that is on the graph of the function.

The coefficients of the quotient and remainder appear in synthetic substitution.

Quotient: $1x^2-6x+11-\dfrac{27}{x+3}$

**Notice the quotient is now one degree lower than the dividend (the original polynomial). We call this the "depressed polynomial". Note: Every term of the polynomial in the dividend must be represented. If the polynomial is missing a term, we must include a "0" in its place.

Ex 5: Find $\left(2x^4-3x^3-5x^2+3x+8\right)\div(x-2)$ using synthetic substitution.

Using the polynomial in standard form, write the coefficients in a row. Put the *x*-value to the upper left.

$$\underline{2|}\; 2 \quad -3 \quad -5 \quad 3 \quad 8$$

Bring down the first coefficient, then multiply by the *x*-value.

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & & & \\ \hline & 2 & & & & \end{array}$$

Add straight down the columns, and repeat.

$$\begin{array}{r|rrrrr} 2 & 2 & -3 & -5 & 3 & 8 \\ & & 4 & 2 & -6 & -6 \\ \hline & 2 & 1 & -3 & -3 & \boxed{2} \end{array} \text{ (remainder is 2)}$$

Quotient: $\boxed{2x^3+x^2-3x-3+\dfrac{2}{x-2}}$

SAMPLE EXAM QUESTIONS

1. Divide $(x^4+2x^3-7)\div(x^2-1)$ **using long division.**

(A) $x^2+2x+1+\dfrac{6}{x^2-1}$

(B) $x^2+2x-1+\dfrac{2x-6}{x^2-1}$

(C) $x^2+2x-1+\dfrac{2x+6}{x^2-1}$

(D) $x^2+2x+1+\dfrac{2x-6}{x^2-1}$

Ans: D

2. What is x^3-3x^2-9x+2 **divided by** $x+2$**?**

A. x^2-x-11

B. $x^2-2x-11$

C. x^2-5x+1

D. x^2-4x+5

Ans: C

3. Write an expression that represents the width of a rectangle with length $x+5$ and area $x^3+12x^2+47x+60$.

(A) x^3+7x^2+12x

(B) $x^2+7x+12$

(C) $x^2+17x-38-\frac{50}{x+5}$

(D) $x^2+17x+132+\frac{720}{x+5}$

Ans: B

4. Divide $\left(2x^4-x^3-15x^2+3x\right)\div(x+3)$ using synthetic division.

(A) $2x^3-7x^2+6x-15+\dfrac{45}{x+3}$

(B) $2x^3+5x^2+3+\dfrac{9}{x+3}$

(C) $2x^3-5x^2+3-\dfrac{9}{x+3}$

(D) $2x^3-7x^2+6x-15-\dfrac{45}{x+3}$

Ans: A

5. Divide by using long division: $\left(5x+6x^3-8\right)\div(x-2)$.

(A) $6x^2+12x+29$

(B) $6x^2-12x+29-\frac{64}{(x-2)}$

(C) $6x^2+12x+29+\frac{50}{(x-2)}$

(D) $6x^2+5-\frac{8}{(x-2)}$

Ans: C

6. Divide by using synthetic division. $\left(x^2-9x+10\right)\div(x-2)$

(A) $x-9+\frac{6}{x-2}$

(B) $x-7+\frac{-4}{x-2}$

(C) $x-11+\frac{32}{x-2}$

(D) $2x-18+\frac{10}{x-2}$

Ans: B

NOTE: This unit is a review unit. Unit 4, Polynomial Functions, is a six week unit covered in the first quarter of Algebra II. Most of these topics are not new and should be treated as review. Graphing single variable and two variable polynomial inequalities are the only new topics in this unit. Students identified polynomials, performed operations with polynomials (including division), graphed polynomials, solved polynomials finding the zeros with the Fundamental Theorem of Algebra and Rational Roots Theorem and used polynomial functions to model data in Algebra II Unit 4.

Unit 2.2 To analyze graphs of polynomial functions determining the characteristics of the graph, including zeros and end behavior.

Unit 2.3 To graph polynomial functions using characteristics determined by the equation of the function.

Unit 2.4 To use transformations to sketch graphs of quadratic and polynomial functions.

Graphs of Polynomials:

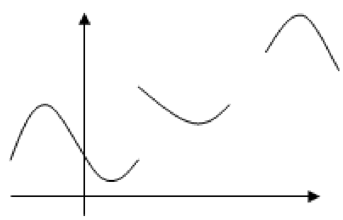
*The graphs of polynomials of degree 0 or 1 are lines.

*The graphs of polynomials of degree 2 are parabolas.

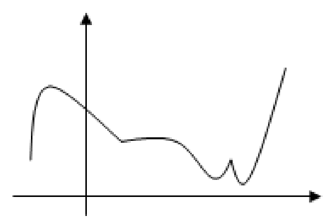
*The greater the degree of the polynomial, the more complicated its graph can be.

*The graph of a polynomial function is always a smooth curve; that is it has no breaks or corners.

Examples of graphs of non-polynomial functions:

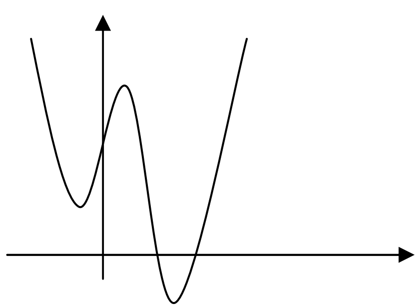


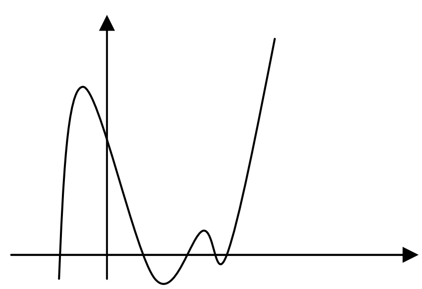
There is a break and a hole.



There is a corner and a sharp turn.

Examples of graphs of polynomial functions:





Graphing Polynomial Functions: To graph a polynomial function, make a table of values using synthetic substitution, plot the points, and determine the *end behavior* to draw the rest of the graph.

End Behavior: the behavior of the graph as x gets very large (approaches positive infinity $(+\infty)$) OR as x gets very small (or approaches negative infinity $(-\infty)$).

Notation: $x \to +\infty$ (x approaches positive infinity) (The very far right end of a graph).

$x \to -\infty$ (x approaches negative infinity) (The very far left end of a graph).

Exploration Activity: Graph each function on the calculator. Determine the end behavior of $f(x)$ as x approaches negative and positive infinity. Fill in the table and write your conclusion regarding the degree of the function and the end behavior. (Teacher Note: Answers are in red.)

$f(x)$	Degree	Sign of Leading Coefficient	$x \to$ ___	$f(x) \to$ ___
$f(x)=x^2$	2	+	$+\infty$ $-\infty$	$+\infty$ $+\infty$
$f(x)=-x^2$	2	−	$+\infty$ $-\infty$	$-\infty$ $-\infty$
$f(x)=x^3$	3	+	$+\infty$ $-\infty$	$+\infty$ $-\infty$
$f(x)=-x^3$	3	−	$+\infty$ $-\infty$	$-\infty$ $+\infty$
$f(x)=x^4$	4	+	$+\infty$ $-\infty$	$+\infty$ $+\infty$
$f(x)=-x^4$	4	−	$+\infty$ $-\infty$	$-\infty$ $-\infty$
$f(x)=x^5$	5	+	$+\infty$ $-\infty$	$+\infty$ $-\infty$
$f(x)=-x^5$	5	−	$+\infty$ $-\infty$	$-\infty$ $+\infty$
$f(x)=x^6$	6	+	$+\infty$ $-\infty$	$+\infty$ $+\infty$
$f(x)=-x^6$	6	−	$+\infty$ $-\infty$	$-\infty$ $-\infty$

Conclusion: The graph of a polynomial function

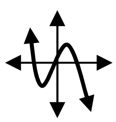
$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

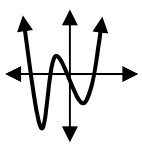
has the following end behavior. These patterns are very predictable.

Think of end behavior as what happens on either end of the graph. There can be a lot of curves, etc. in the middle, but polynomial functions either increase or decrease at the far ends (as $x \to \pm\infty$, $f(x) \to \pm\infty$).

Degree	Leading Coefficient	End Behavior
Even	Positive	as $x \to -\infty,\ f(x) \to +\infty$ as $x \to +\infty,\ f(x) \to +\infty$
Even	Negative	as $x \to -\infty,\ f(x) \to -\infty$ as $x \to +\infty,\ f(x) \to -\infty$
Odd	Positive	as $x \to -\infty,\ f(x) \to -\infty$ as $x \to +\infty,\ f(x) \to +\infty$
Odd	Negative	as $x \to -\infty,\ f(x) \to +\infty$ as $x \to +\infty,\ f(x) \to -\infty$

Ex 6: Indicate if the degree of the polynomial function shown in the graph is odd or even and indicate the sign of the leading coefficient.

a)  Odd degree; Negative leading coefficient

b)  Even degree; Positive leading coefficient

Analyzing polynomial graphs

Concept Summary:

Let $f(x) = a_n x^n + a_{n-1}x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$ be a polynomial function.

The following statements are equivalent:

Zero: ***k* is a zero of the function *f*.**

Factor: ***x* − *k* is a factor of polynomial *f(x)*.**

Solution: ***k* is a solution of the polynomial function *f(x)=0*.**

***x* - Intercept:** ***k* is an *x*-intercept of the graph of the polynomial function *f*.**

<u>Using Zeros to Graph a Polynomial:</u> We find the values of *x* that make the polynomial equal to zero.

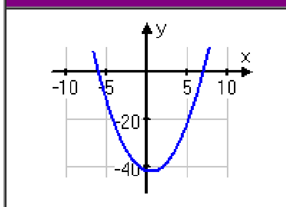
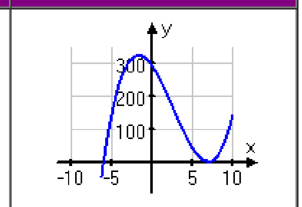
If $P(x) = (x-2)(x+3)$ then the zeros of the polynomials are 2 and -3.

This means that the graph of this polynomial crosses the x-axis at 2 and -3 (2 and -3 are the x intercepts)

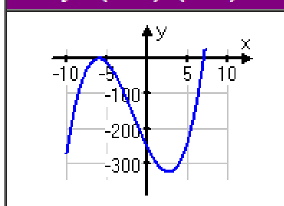
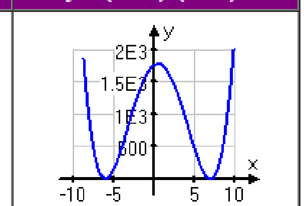
<u>Zeros of Polynomial Functions</u>

<u>Multiplicity</u>: This refers to the number of times the root is a zero of the function. We can have "repeated" zeros. If a polynomial function *f* has a factor of $\left(x-c\right)^m$, and not $\left(x-c\right)^{m+1}$, then c is a zero of **multiplicity** *m* of *f*.

- Odd Multiplicity: *f* crosses the *x*-axis at *c*; $f\left(x\right)$ changes signs
- Even Multiplicity: *f* "kisses" or is tangent to the *x*-axis at *c*; $f\left(x\right)$ doesn't change signs

$y = (x+6)(x-7)$	$y = (x+6)(x-7)^2$
	
$x = -6$ once $x = 7$ once	$x = -6$ once $x = 7$ twice

All four graphs have the same zeros, at *x* = 6 and *x* = 7, but the multiplicity of the zero determines whether the graph crosses the *x*-axis at that zero or if it instead turns back the way it came.

$y = (x+6)^2(x-7)$	$y = (x+6)^2(x-7)^2$
	
$x = -6$ twice $x = 7$ once	$x = -6$ twice $x = 7$ twice

<u>Guidelines for Sketching Graphs Polynomial Functions:</u>

✓ <u>Zeros</u>: Factor the polynomial to find all its real zeros; these are the x-intercepts of the graphs.

✓ <u>Test points:</u> Make a table of values for the polynomial. Include test points to determine whether the graph of the polynomial lies above or below the x-axis on the intervals determined by the zeros. Include the y-intercept in the table.

✓ <u>End Behavior:</u> Determine the end behavior of the polynomial.

✓ <u>Graph:</u> Plot the intercepts and the other points you found in the table. Sketch a smooth curve that passes through these points and exhibits the required end behavior.

Ex 7: Graph the function $f\left(x\right)=\left(x-3\right)\left(x+1\right)^2$

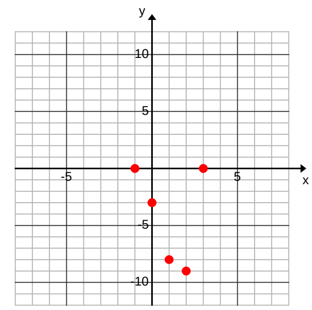
Step 1: Plot the *x*-intercepts. Since $x + 1$ and $x - 3$ are factors, -1 and 3 are zeros (*x*-intercepts)

Note: $x - 3$ is raised to an odd power so the graph crosses the *x*-axis at $x = 3$. $x + 1$ is raised to an even power so the graph is tangent to the *x*-axis at $x = -1$.

When a factor $x-k$ is raised to an odd power, the graph crosses through the x-axis.
When a factor $x-k$ is raised to an even power, the graph is tangent to the x-axis.

Step 2: Plot a few points between the x-intercepts.

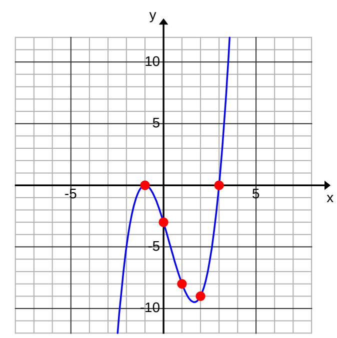
$$f(0)=-3;\ f(1)=-8;\ \ f(2)=-9$$



Step 3: Determine the end behavior of the graph.
Cubic function (odd degree) with positive leading coefficient

$$\text{as } x\to-\infty,\ f\left(x\right)\to-\infty$$
$$\text{as } x\to+\infty,\ f\left(x\right)\to+\infty$$

Step 4: Sketch the graph



Ex 8: Sketch the graph of $f\left(x\right)=-\dfrac{1}{2}\left(x-2\right)^2\left(x+3\right)^2$. Describe the multiplicity of the zeros.

Plot the intercepts. Because -3 and 2 are zeros of *f*, plot (-3, 0) and (2, 0).
$x=-3$; multiplicity 2 $\qquad x=2$; multiplicity 2

This means that it will "kiss" or is tangent to the x-axis (even multiplicity) at both intercepts.

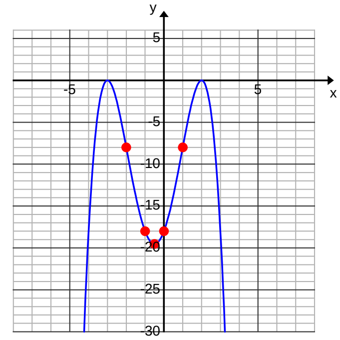
Plot points between and beyond the *x*-intercepts.

X	-2	-1	0	1	-.5
Y	-8	-18	-18	-8	-19.53

Determine the end behavior. Because *f* has four factors of the form $x - k$ it is a quartic function. It has a negative leading coefficient.

That means that $f(x) \to -\infty$ as $x \to -\infty \to$ as *x* and $f(x) \to -\infty$ as $x \to +\infty$.

Draw the graph so that it passes through the plotted points and has the appropriate end behavior.



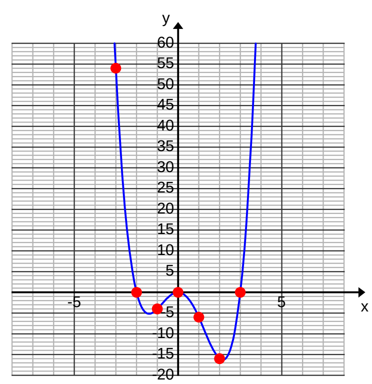
Ex 9: Sketch the graph of $g(x) = x^4 - x^3 - 6x^2$. Describe the multiplicity of the zeros.

First we will need to factor the polynomial as much as possible so we can identify the zeroes and get their multiplicities. $x^4 - x^3 - 6x^2 = x^2(x^2 - x - 6) = x^2(x-3)(x+2)$

$x = -2$: multiplicity 1 $x = 0$: multiplicity 2 $x = 3$: multiplicity 1 *y*-intercept is zero

End behavior (even degree of 4, positive leading coefficient): $\lim_{x\to-\infty} = \infty$; $\lim_{x\to\infty} = \infty$

X	-3	-1	1	2
Y	54	-4	-6	-16

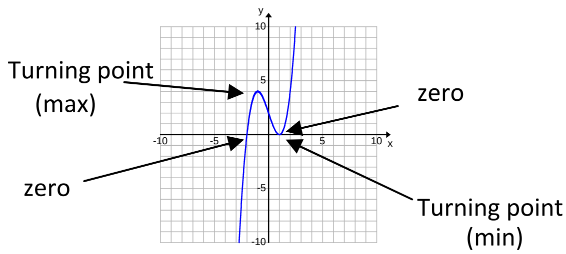


Finding Turning Points

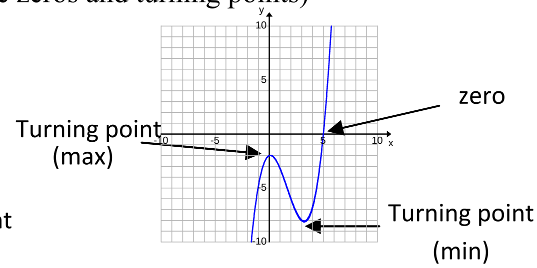
Turning points of polynomial functions: Another important characteristic of graphs of polynomial functions is that they have turning points corresponding to local maximum and minimum values. The *y* – coordinate of a turning point is a ***local or relative maximum*** if the point is higher than all nearby points. The *y* – coordinate of a turning point is a ***local or relative minimum*** if the point is lower than all nearby points. Global or absolute minimums and maximums are the greatest or least values of the entire function.

The graph of every polynomial function of degree *n* has **at most** *n* – 1 turning points. Moreover, if a polynomial has *n* distinct real zeros, then its graph has exactly *n* – 1 turning points.

Ex 10: Identify the zeros and turning points (estimate the zeros and turning points)

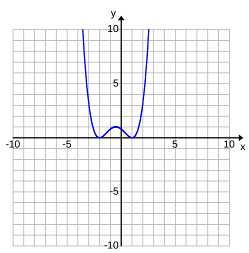


Leading coefficient positive
3 real zeros (including the double zero)
{-2, 1, 1}
2 turning points
(-1,4); (1,0)
1 local max; 1 local min



Leading coefficient positive
1 real zero, 2 imaginary zeros
{5}
2 turning points
(0,-2); (3,-8)
1 local max; 1 local min

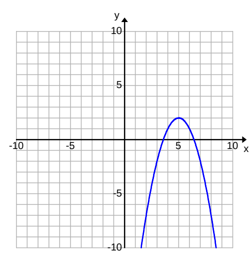
You Try:



Leading coefficient __________
____real zeros

____turning points

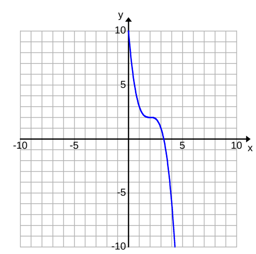
____local max; ____local min



Leading coefficient __________
____real zeros

____turning points

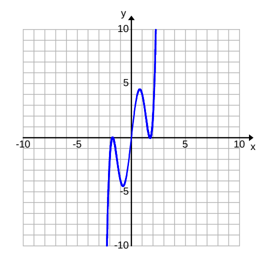
____local max; ____local min



Leading coefficient __________
____real zeros

____turning points

____local max; ____local min



Leading coefficient __________
____real zeros

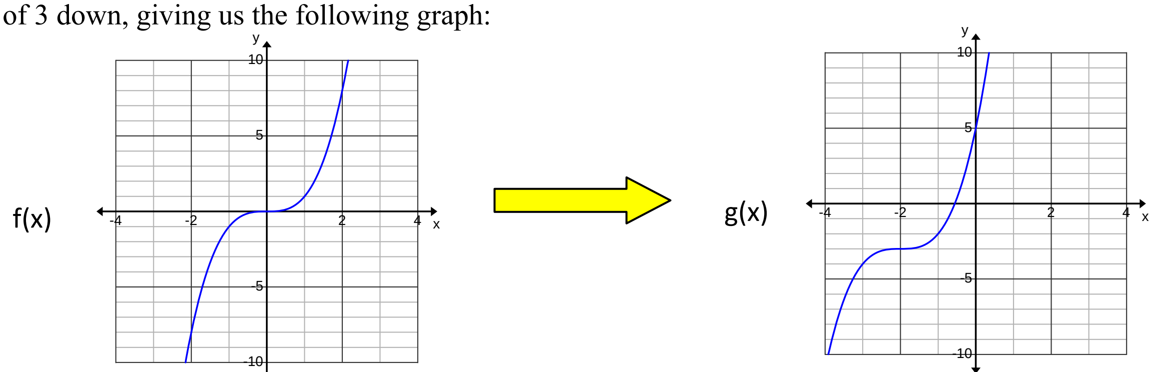
____turning points

____local max; ____local min

More on Graphing Polynomial Functions:

Continue to reinforce transformations while graphing polynomial functions. Relate this to the functions previously translated in Unit 1 Quadratics. These students also spent much time transforming functions in Algebra I and Algebra II. This is review.

Ex 11: Consider the cubic function: $f(x) = x^3$. How can we graph $g(x)$ if $g(x) = (x+2)^3 - 3$?
The graph of $f(x)$ would start in the third quadrant, increase, cross at the origin and continue increasing in the first quadrant. $g(x)$ is transformed with a horizontal translation of 2 to the left and a vertical translation of 3 down, giving us the following graph:



Ex 12: Consider $p(x)=2x^4-x^3-11x^2+5x+5$. $h(x)$ is $p(x)$ translated 4 units right and 2 units up and a vertical stretch of 3. What is the equation of $h(x)$?

Recall from transformations the following general format: $f(x)=a(x-h)^p+k$. The variable *a* relates to vertical stretches or shrinks, *k* relates to the vertical translation and the variable *h* relates to the horizontal translation. Inputting the values 3, 2 and 4, we get:

$$h(x)=3p(x-4)+2=3\left[2(x-4)^4-(x-4)^3-11(x-4)^2+5(x-4)+5\right]+2$$

Ex 13: Consider the function $f(x)=3x^3-9x^2-3x+9$.

a) Use the leading coefficient and degree of $f(x)$ to describe the end behavior.

The leading term is $3x^3$ so as x increases without bound, $f(x)$ increases without bound and as x decreases without bound $f(x)$ decreases without bound. $\lim_{x\to\infty} f(x)\to\infty$ and $\lim_{x\to-\infty} f(x)\to-\infty$

b) Write the rule for the function $g(x)=f(-x)$, and describe the transformation.

$$g(x)=f(-x)=3(-x)^3-9(-x)^2-3(-x)+9=-3x^3-9x^2+3x+9$$

The transformation here is a reflection with respect to the *y*-axis since the input values were replaced with their opposites.

c) Describe the end behavior of $g(x)$. How does the end behavior of $g(x)$ relate to the transformation of $f(x)$?

Since $g(x)$ is a reflection of $f(x)$, as x increases without bound, $g(x)$ decreases without bound and as x decreases without bound, $g(x)$ increases without bound. $\lim_{x\to\infty} g(x)\to-\infty$ and $\lim_{x\to-\infty} g(x)\to\infty$

Ex 14: Describe how to transform the graph. Name the *y*-intercept. $f(x)=-2(x+4)^3+7$

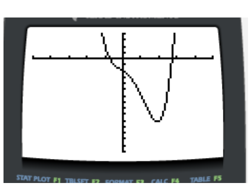
"Parent Function" = cubic function $y=x^3$

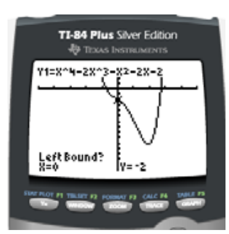
- Vertical stretch by a factor of 2
- Reflect over the *x*-axis
- Shift left 4 units
- Shift up 7 units
- *y*-intercept: $y=-2(0+4)^3+7=-121$

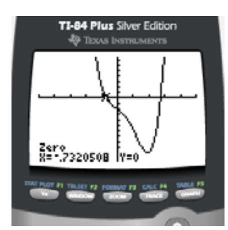
Using Technology to Approximate Zeros

Ex 15: Approximate the real zeros of $f(x)=x^4-2x^3-x^2-2x-2$

Use a graphing calculator to graph and calculate the zeros. Use the Calculate: zero

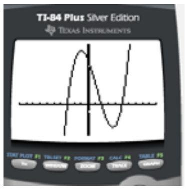


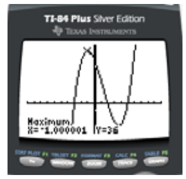
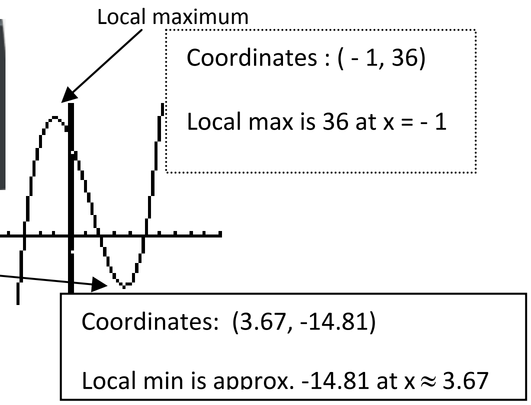




Ans: $x\approx-0.732, 2.732$

Ex 16: Use a graphing calculator to graph and calculate the approximate local maximum(s) and local minimum(s) of $f(x)=(x-2)(x+3)(x-5)$. Use the Calculate: minimum or Calculate: maximum

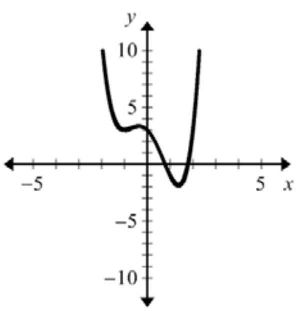


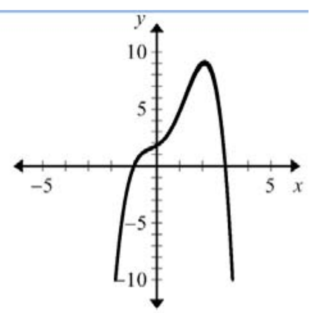




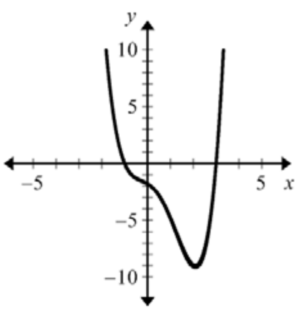
QOD: What is the difference between local and absolute maxima and minima?

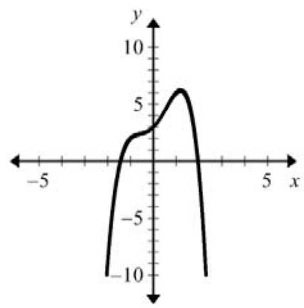
SAMPLE EXAM QUESTIONS

1. Which best represents the graph of the polynomial function $y=-x^4+2x^2+2x+3$?

A.


C.


B.


D.


Ans: D

2. Which describes the end behavior of the graph of $f(x)=x^4-5x+2$ as $x\to-\infty$?

A. $f(x)\to-\infty$

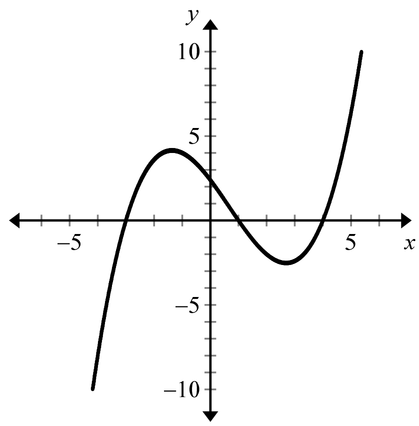
B. $f(x)\to 0$

C. $f(x)\to+\infty$

D. $f(x)\to 2$

Ans: B

3. Use the graph of the polynomial function.
What are the zeros of the polynomial?



A. $\{2\}$

B. $\{-2\}$

C. $\{-3,\ 1,\ 4\}$

D. $\{3,\ -1,\ -4\}$

Ans: C

4. The table lists all the real roots of a 5th degree polynomial $p(x)$ and the multiplicity of each root.

x	Multiplicity
-3	1
-1	1
1	2
2	1

Which general factorization correctly represents $p(x)$?

A. $a(x-3)(x-1)^2(x-2)$

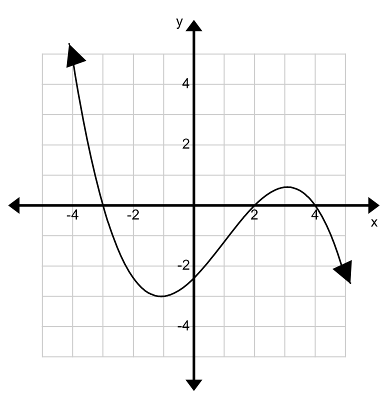
B. $a(x+3)(x-1)^3(x-2)$

C. $a(x+3)(x+1)(x-1)^2(x-2)$

D. $a(x+3)(x+1)^3(x-2)$

Ans: C

5. Consider the graph of $p(x)$ below. Which general factorization correctly represents $p(x)$.



Which general factorization correctly represents $p(x)$?

A. $a(x+3)(x-2)(x-4)$

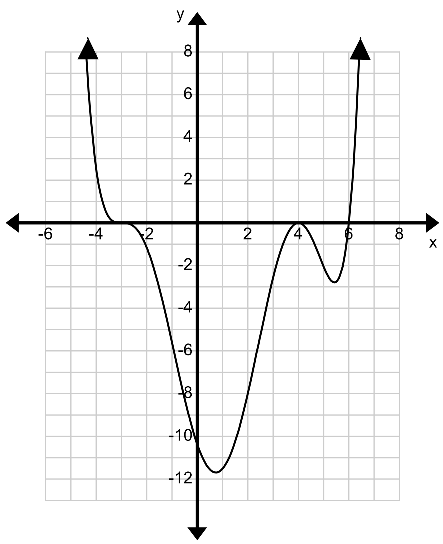
B. $a(x+3)(x+2)(x+4)$

C. $a(x-3)(x+2)(x-4)$

D. $a(x-3)(x-2)(x-4)$

Ans: A

6. Use the graph of $p(x)$ to answer questions



a) True or False: The leading term of $p(x)$, when written in standard form, is positive.

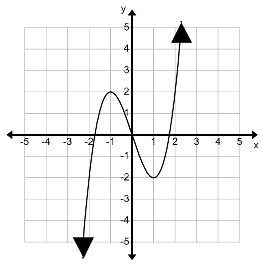
True

b) True or False: From the graph, $p(-3)=0$. The multiplicity of the factor $(x+3)$ is even. Explain your answer.

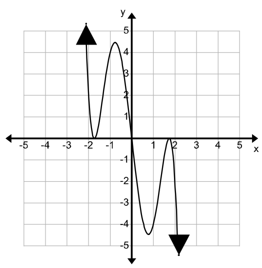
False. The multiplicity of the factor (x + 3) is odd because the graph crosses the x-axis at -3. It the multiplicity was even, it would not cross.

7. Which graph represents $f(x) = x^5 - 6x^3 + 9x$ **?**

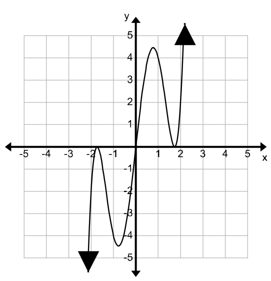
(A)



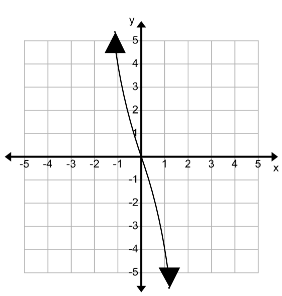
(B)



(C)



(D)



Ans: C

Unit 2.5 To use the Fundamental theorem of Algebra to determine the number of zeroes of polynomial functions.

Unit 2.6 To find all zeroes of a polynomial function using the Remainder Theorem and the Rational roots Theorem.

The Fundamental Theorem of Algebra

- If $f(x)$ is a polynomial of degree *n* where *n>0*,then the equation $f(x)=0$ has at least one root in the set of complex numbers
- A polynomial function of degree *n* has *n* complex zeros (some zeros may repeat)
- Every polynomial function of odd degree with real coefficients has at least one real zero.
- Complex zeros come in conjugate pairs

  Review: $a+bi \Rightarrow$ conjugate $\Rightarrow a-bi$

Finding the number of solutions or zeros

Review: Find the solutions of the following examples. State how many solutions each has and classify each zero as rational, irrational, or complex (imaginary).

Ex: $3x-1=0$ One rational zero: $x=\frac{1}{3}$

Ex: $x^2-9=0$ Two rational zeros: $x=-3,3$

Ex: $x^3-1=0$ (Hint: Use the factorization for the difference of cubes, then use the quadratic formula for the quadratic factor.)

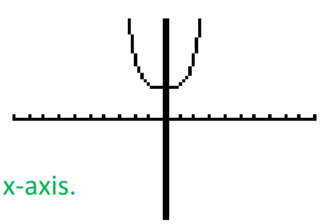
Three roots, one rational zero and two complex: $x=1,\frac{-1+i\sqrt{3}}{2},\frac{-1-i\sqrt{3}}{2}$

Do you notice a pattern with the degree of the polynomial and the number of solutions each has?

Ex: How many different solutions are there to $x^4-16=0$? Four, the degree is four.

How do you explain this number? There are two rational zeros, -2 and 2. There are two imaginary roots, -2i and 2i.

Ex: How many different solutions are there to $x^2+16=0$?



Solution: $\boxed{x=4i,-4i}$

Note: On the graph, the imaginary roots do not cross the x-axis.

Note: $4i,-4i$ are complex conjugate pairs. $1+i\sqrt{2}, 1-i\sqrt{2}$ are complex conjugate pairs. The complex roots of polynomial functions with real coefficients always occur in complex conjugate pairs. Is this also the case for irrational zeros?

Using the Rational Zero Theorem

Review: A rational zero is a rational number that produces a function value of 0. It can be visualized as $f(x)=0$ where x is a rational number (can be written as a ratio). On the graph it is an x-intercept. The Rational Zero Theorem will help to identify possible rational zeros.

The Rational Zero Theorem If $f(x)=a_nx^n+...a_1x+a_0$ has integer coefficients, then every rational zero of f has the following form: $\frac{\text{p}}{\text{q}}=\frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$

The first important step is to list the possible rational zeros. After they are listed we can test them using synthetic division to determine if they are rational zeros. If the value of the possible rational zeros = 0, they are called zeros.

List the possible rational zeros:

Ex 17: Find the possible rational zeros of $f(x)=x^2+9x+20$

Step 1: The leading coefficient is 1. 1 is the only factor of 1.

Step 2: The constant is 20. All of the factors of 20 are $\pm1, \pm2, \pm4, \pm5, \pm10, \pm20$.

Step 3: List the possible factors - $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{4}{1}, \pm\frac{5}{1}, \pm\frac{10}{1}$, and $\pm\frac{20}{1}$

*If we factored the quadratic, we would find that -4 and -5 are zeros.

Ex 18: Find the possible rational zeros of $f(x)=5x^3+7x^2+12x-6$

Step 1: The leading coefficient is 5. The factors of 5 are $\pm5, \pm1$.

Step 2: The constant is -6. All of the factors of -6 are $\pm1, \pm2, \pm3, \pm6$.

Step 3: List the possible factors - $\pm\frac{1}{1}, \pm\frac{2}{1}, \pm\frac{3}{1}, \pm\frac{6}{1}, \pm\frac{1}{5}, \pm\frac{2}{5} \pm\frac{3}{5}, \pm\frac{6}{5}$

*We will not test for actual zeros for this example.

When the leading coefficient is not 1, the list of possible zeros can increase dramatically. There are many tools that are used to find the rational zeros. Examples that follow will demonstrate some of them.

The following theorems will help us to find all zeroes of polynomial functions:

Remainder Theorem: If a polynomial $f(x)$ is divided by $x-k$, then the remainder is $r=f(k)$

Factor Theorem: A polynomial $f(x)$ has a factor $x-k$ if and only if $f(k)=0$.

Ex 19: Factor $f(x) = 3x^3 + 14x^2 - 28x - 24$ given that $f(-6) = 0$.

Because $f(-6) = 0$, we know that $x + 6$ is a factor of $f(x)$ by the Factor Theorem. We will use synthetic division to find the other factors.

−6	3	14	−28	−24
		−18	24	24
	3	−4	−4	0

$f(x) = (x+6)(3x^2 - 4x - 4)$

$f(x) = (x+6)(3x+2)(x-2)$

Note for graphing: This means that (-6,0) is an ordered pair that is on the graph of the function. -6 is called a zero. It is also an x-intercept.

Ex 20: One zero of $f(x) = 2x^3 - 9x^2 - 32x - 21$ is $x = 7$. Completely factor the function.

Because $f(7) = 0$, we know that $x - 7$ is a factor of $f(x)$ by the Factor Theorem. We will use synthetic division to find the other factors.

7	2	−9	−32	−21
		14	35	21
	2	5	3	0

$f(x) = (x-7)(2x^2 + 5x + 3)$

$f(x) = (x-7)(2x+3)(x+1)$

QOD: If $f(x)$ is a polynomial that has $x + a$ as a factor, what do you know about the value of $f(-a)$?

If a zero of the polynomial is not given, use the rational root theorem to find a zero.

Ex 21: Find all the real zeros of $f(x) = x^3 + 72 - 5x^2 - 18x$

Step 1: Put the function in standard order. $f(x) = x^3 - 5x^2 - 18x + 72$

Step 2: List possible rational zeros $\pm(1,2,3,4,6,8,9,12,18,24,36,72)$

Step 3: Try the possible zeros until you find one.

1	1	−5	−18	72
		1	−4	−22
	1	−4	−22	50

−1	1	−5	−18	72
		1	4	14
	−1	−4	−14	86

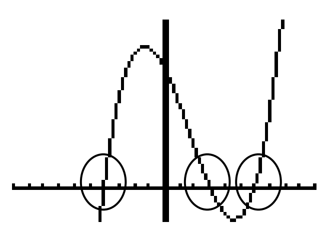
3	1	−5	−18	72
		3	−6	−72
	1	−2	−24	0

Since the polynomial degree is reduced, the function can be written as:

$f(x) = (x-3)(x^2 - 2x - 24)$

Then factored:

$f(x) = (x-3)(x-6)(x+4)$



$(x-3) = 0 \qquad (x-6) = 0 \qquad (x+4) = 0$

Zeros: $x = 3 \qquad x = 6 \qquad x = -4$

Note: Finding rational zeros is also referred to as finding real zeros. Rational numbers are also real numbers. There is a distinction between ***listing** possible rational zeros* and ***finding** rational (real) zeros*.

Ex 22: Find all the zeros of the function and write the polynomial as a product of linear factors.

$f(x) = x^4 - 6x^3 + 10x^2 - 6x + 9$

Step One: Check for any rational zeros.

Possible rational zeros: $\frac{\text{factors of } 9}{\text{factors of } 1} = \pm\frac{1}{1}, \pm\frac{3}{1}, \pm\frac{9}{1}$ = -1, 1, -3, 3, -9, 9

Use synthetic substitution:

$$\begin{array}{r|rrrrr} 3 & 1 & -6 & 10 & -6 & 9 \\ & & 3 & -9 & 3 & -9 \\ \hline & 1 & -3 & 1 & -3 & 0 \end{array}$$

So 3 is a zero, and $x-3$ is a factor.

Step Two: Rewrite the last line as a polynomial and find the zeros of the polynomial.

$f(x) = (x-3)(x^3 - 3x^2 + x - 3)$

$x^3 - 3x^2 + x - 3 = 0$ ⟶ Factor by grouping: $x^2(x-3) + 1(x-3) = 0 \Rightarrow (x^2+1)(x-3) = 0$

$x^2 + 1 = 0$

$\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

$x - 3 = 0$

$x = 3$

Step Three: List the zeros. Check that you have the correct number according to the Fundamental Theorem of Algebra (FTA).

$x = 3, -i, i$ The polynomial was degree 4, and there are 4 zeros (3 is repeated).

Note that *i* and –*i* are complex conjugates.

Step Four: Write the polynomial as a product of linear factors.

$$f(x) = (x-3)(x-3)(x+i)(x-i)$$

Ex 23: Find all of the real zeros of $f(x) = 3x^3 + 12x^2 + 3x - 18$.

Step 1: Notice that each term contains a common factor of 3. The problem can be factored to $f(x) = 3(x^3 + 4x^2 + x - 6)$ and since $3 \neq 0$ only $(x^3 + 4x^2 + x - 6)$ can be $= 0$.

Step 2: List possible rational zeros $\pm(1,2,3,6)$ (Since the leading coefficient is now 1)

Step 3: Try the possible zeros until you find one.

1 |	1	4	1	-6
		1	1	2
	1	1	2	-4

-1 |	1	4	1	-6
		-1	-3	2
	1	3	-2	-4

-2 |	1	4	1	-6
		-2	-4	6
	1	2	-3	**0**

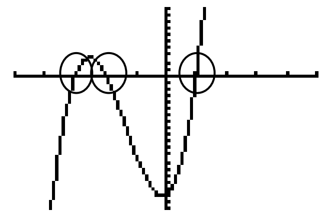
The function can be written as:

$$f(x) = (x+2)\underline{(x^2+2x-3)}$$

Then factored: $f(x) = (x+2)(x+3)(x-1)$

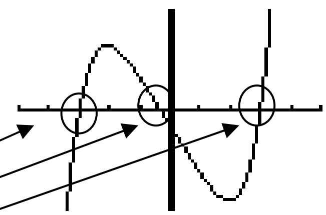
$x+2=0 \qquad x+3=0 \qquad x-1=0$

Zeros: $x=-2 \qquad x=-3 \qquad x=1$



Ex 24: Find all the zeros of $f(x) = 3x^5 + x^4 - 243x - 81$

Step 1: Maybe this could be graphed first.



Step 2: Look at the graph for reasonable choices

It appears they might be -3, 3, and $-\frac{1}{3}$

Step 3: Check the chosen values using synthetic division. Start with -3.

-3 |	3	1	0	0	-243	-81
		-9	24	-72	216	81
	3	-8	24	-72	-27	**0**

It is a root (zero).

The factored form so far is $f(x) = (x+3)(3x^4 - 8x^3 + 24x^2 - 72x - 27)$

Step 4: Repeat the steps above using a different reasonable choice.
Try 3.

3 |	3	-8	24	-72	-27
		9	3	81	27
	3	1	27	9	**0**

It is a root (zero).

Step 5: Repeat the steps above using a different reasonable choice. Try $-\frac{1}{3}$

$-\frac{1}{3}$ |	3	1	27	9
		-1	0	-9
	3	0	27	**0**

It is a root (zero).

Yes, all three work! And each time, the function (polynomial) is reduced by one degree.

Step 6: The function $3x^2 + 27$ is left to be factored.

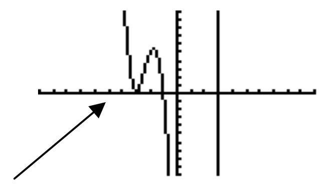
$3(x^2 + 9) \to x = \pm 3i$

Solution: There are 5 zeros: $-3, 3, -\frac{1}{3}, 3i, -3i$. Three are rational, two complex.

Finding the zeros of a polynomial function

This activity involves finding the rational zeros as learned in the previous section, then using other tools, such as the quadratic formula or technology, to find the irrational or complex roots.

Ex 25: Find all zeros of $f(x) = x^4 + 4x^3 - 6x^2 - 36x - 27$



Using the rational root theorem and synthetic division, it can be shown that
-3 is a repeated root and 3 and -1 are roots.

The factored form looks like this: $(x+3)^2(x-3)(x+1)$. The graph is shown.

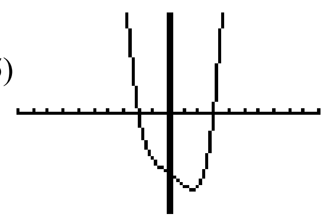
When a factor $x - k$ is raised to an odd power, the graph crosses through the *x*-axis. When a factor $x - k$ is raised to an even power, the graph is tangent to the *x*-axis.

Solution: There are four real zeros -3 is a repeated root and 3 and -1 are roots.

Ex 26: Find all zeros of $f(x) = x^4 - x^3 - x^2 - 5x - 30$

Using the rational root theorem and synthetic division, it can be shown that 3 and -2 are roots.

Rewrite the polynomial in factored form: $f(x) = x^4 - x^3 - x^2 - 5x - 30$
factors to $(x+2)(x-3)(x^2+5)$



Factor or quadratic formula: $x^2 + 5 = 0$

yields zeros of x= $\pm i\sqrt{5}$

Solution:

There are four zeros, 3 and -2 and $\pm i\sqrt{5}$. Two are real and two are complex conjugates.

SAMPLE EXAM QUESTIONS

1. Use the remainder theorem to find the remainder when f(x) is divided by x - c?

$$f(x) = x^4 + 8x^3 + 12x^2;\ x+1$$

A. 21 **B.** -21

C. 5 **D.** -5

Ans: C

2. Use the factor theorem to determine whether x – c is a factor of f(x).

$$f(x) = 5x^4 + 19x^3 - 4x^2 + x - 4;\ x+4$$

A. Yes **B.** No

Ans: A

3. Use the remainder theorem to find the remainder when f(x) is divided by x-k.

$$f(x) = 2x^2 - 3x + 1;\ k+2$$

A. 15 **B.** 9

C. 18 **D.** 3

Ans: D

4. Use the factor theorem to determine whether the first polynomial is a factor of the second polynomial.

$$x-1;\ f(x) = x^3 - x^2 + x - 1$$

A. No solution **B.** no

C. yes **D.** 0

Ans: C

5. Which of the following represents the solution set of the polynomial equation $x^4 - 7x^2 + 12 = 0$?

A. $\left\{2, -2, i\sqrt{3}, -i\sqrt{3}\right\}$

B. $\left\{2, -2, \sqrt{3}, -\sqrt{3}\right\}$

C. $\left\{2i, -2i, i\sqrt{3}, -i\sqrt{3}\right\}$

D. $\left\{2i, -2i, \sqrt{3}, -\sqrt{3}\right\}$

Ans: A

6. Which lists the set of all <u>real</u> zeros of the following polynomial function?

$$f(x) = x^3 + 3x^2 + 4x + 12$$

A. $\{-3\}$

B. $\{-3, -2, 2\}$

C. $\{-3, 2\}$

D. $\{-3, -2, 1, 2\}$

Ans: A

7. According to the Fundamental Theorem of Algebra, how many complex zeros does the polynomial $f(x) = -5x^4 + 2x^3 + x - 1$ have?

A. 2

B. 4

C. 3

D. 5

Ans: B

8. According to the Fundamental Theorem of Algebra, how many roots does the following equation have? $6x^2 + 4 = 11x$

(A) 2

(B) 4

(C) 6

(D) 11

Ans: A

9. A 4th degree polynomial with real coefficients is found to have exactly two distinct real roots. What must be true about the other two roots?

A. One root is real and the other is imaginary.

B. Both roots must be real.

C. Both roots are imaginary roots that are complex conjugates.

D. All the roots have been found.

Ans: C

10. If $f(x) = 2x^4 + 7x^3 + 3x^2 - 8x - 4$, find the possible rational roots of $f(x)$.

(A) $x = 1,\ 4$

(B) $x = 1,\ \pm 4$

(C) $x = \pm\frac{1}{2},\ \pm 1,\ \pm 2$

(D) $x = \pm\frac{1}{2},\ \pm 1,\ \pm 2,\ \pm 4$

Ans: D

11. Given polynomial $q(x)$, $q(4)=6$. Which statement is correct?

(A) $x=4$ is not a root

(B) $x=4$ is a root

(C) $(x-4)$ is a factor

(D) $(x+4)$ is not a factor

Ans: A

12. How many possible rational zeros exist for the polynomial function $y=3x^6+9x^2+4x-12$?

(A) 9

(B) 12

(C) 18

(D) 24

Ans: C

Unit 2.7 To solve and graph solutions of single variable and two variable polynomial inequalities.

Build on the foundation from Unit 1, Target 1.6 solving and graphing quadratic inequalities. The same steps are involved and this should be a quick review.

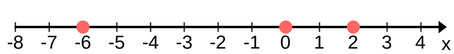
Ex 27: Solve $x^4+4x^3-12x^2 \le 0$

Step One: Solve the quartic equation $x^4+4x^3-12x^2=0$ using any method.

We will use factoring.

$$x^2(x-2)(x+6)=0$$
$$x=0 \quad x-2=0 \quad x+6=0$$
$$x=0 \quad x=2 \quad x=-6$$

Step Two: Draw a sign chart on a number line to test which values for *x* satisfy the inequality.



Choose an *x*-value to the left of -6, between -6 and 0, between 0 and 2 and right of 2 and substitute into the inequality.

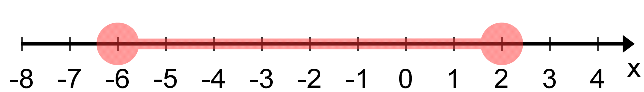
$x=-7:(-7)^2(-1)(-9)\le 0$ FALSE $\quad x=-1:(-1)^2(5)(-3)\le 0$ TRUE

$x=1:(1)^2(7)(-1)\le 0$ TRUE $\quad x=3:(3)^2(9)(1)\le 0$ FALSE

Step Three: Write the solution as a compound inequality or in set notation and graph.

$-6 \le x \le 2$

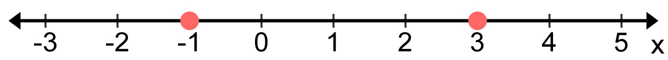
$[-6,2]$



Ex 28: Solve $(x+1)(x-3)^2 > 0$

Step One: Find the zeros. It is already in factored form so the zeros are 3 and -1.

Step Two: Draw a sign chart on a number line to test which values for *x* satisfy the inequality.



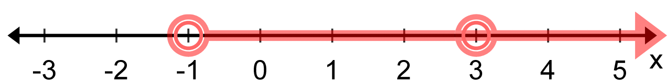
Choose an *x*-value to the left of -1, between -1 and 3, and right of 3 and substitute into the inequality.

$x=-2:(-1)(-5)^2 > 0$ FALSE $\quad x=0:(1)(-3)^2 > 0$ TRUE

$x=4:(5)(1)^2 > 0$ TRUE

Step Three: Write the solution as a compound inequality or in set notation and graph. For our solution to this inequality, we are looking for regions where the polynomial is positive, however, we don't want values where the polynomial is zero, so x = 3 is not included.

$-1 < x < 3 \cup 3 < x < \infty$



$(-1,3)\cup(3,\infty)$

Ex 29: Solve the polynomial inequality: $y \le x^3 + 2x^2$.

Step One: Graph the cubic function. Find the zeros. Look at end behavior.

$x^2(x+2)=0$

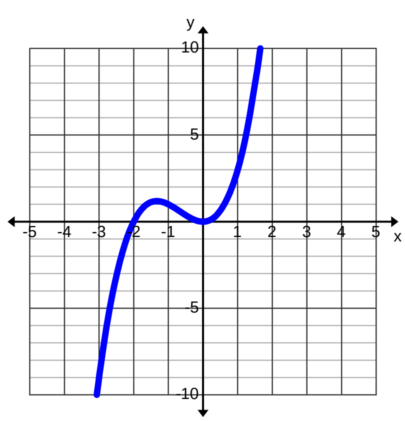
$x=0$ multiplicity 2, $x=-2$ multpilicity 1

y-intercept = 0

Cubic function (odd degree) with positive leading coefficient

$$\text{as } x \to -\infty,\ f(x) \to -\infty$$
$$\text{as } x \to +\infty,\ f(x) \to +\infty$$

Step Two: Graph the cubic function. Since the inequality has an equality symbol, it will be a solid line.



Step Three: Solve for the interval. A given *x* will solve the inequality if $f(x) \le 0$ or if *f(x)* is below the x-axis or on the *x*-axis. For our graph, the solution will be: $\boxed{(-\infty,-2]\cup[0]}$

Ex 30: Solve the polynomial inequality, $y \ge x^4 + x^3 - 2x^2 - 2x$.

Step One: Graph the quartic function. Find the zeros. Look at end behavior.

$$x(x^3+x^2-2x-2)=0 \rightarrow x[(x^3+x^2)+(-2x-2)]=0$$

$$x[x^2(x+1)-2(x+1)]=0 \rightarrow x[(x+1)(x^2-2)]=0$$

y-intercept = 0

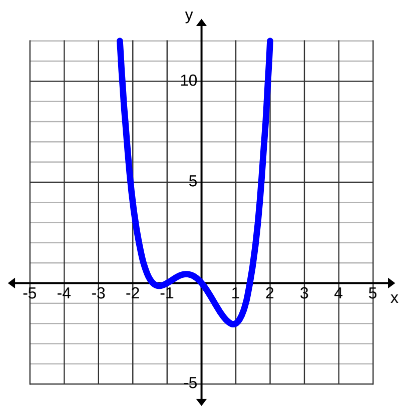
$x = 0$ multiplicity 1, $x = -1$ multpilicity 1, $x = \pm\sqrt{2}$

Quartic function (even degree) with positive leading coefficient

$$\text{as } x \to -\infty,\ f(x) \to +\infty$$

$$\text{as } x \to +\infty,\ f(x) \to +\infty$$

Step Two: Graph the cubic function. Since the inequality has an equality symbol, it will be a solid line.



Step Three: Solve for the interval. A given *x* will solve the inequality if $f(x) \ge 0$ or in other words, if *f(x)* is above the x-axis. For our graph, the solution will be:

$\boxed{\left(-\infty,-\sqrt{2}\right]\cup[-1,0]\cup\left[\sqrt{2},\infty\right)}$