Volume of Cylinders, Cones, and Spheres

Goal: To provide opportunities for students to practice calculating the volume of cylinders, cones, and spheres

Common Core Standards

Geometry

Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

8.G.9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

Student Activities Overview and Answer Key

Station 1

Students construct a cylinder out of a sheet of paper. They explore whether the way they choose to roll the paper affects the volume. They experiment and use calculations to answer this question.

Answers: Answers will vary; the cylinder with the 11-inch circumference; 76.9 cubic inches; 63.0 cubic inches; the cylinder with the 11-inch circumference

Station 2

At this station, students use the formula for the volume of a cone ($V = \frac{1}{3}\pi r^2 h$) to determine how many ice cream cones could fit into a cooler. They explain how they do this, and are instructed to be careful with their units.

Answers: πr^2; 4.19 cubic inches; 12 cubic feet; 4,948 cones; you need to change feet to inches or inches to feet so you are working in the same unit, then you can divide the volume of a cone into the volume of the cooler

Station 3

Students determine the volume of several spherical objects by using the formula for the volume of a sphere $V = \frac{4\pi r^3}{3}$. Students measure the circumference of each object using measuring tape, then derive the value of the radius r using the formula for the circumference of a circle ($C = 2\pi r$). They then substitute the value for r into the formula for volume.

Answers: Answers will vary depending on the objects chosen.

Station 4

Students use the formula for the volume of a sphere $V = \frac{4\pi r^3}{3}$ to solve problems as a group.

Answers: Answers may vary depending on whether students use a calculator with a π button or if they use the approximate value 3.14 for π. Answers derived using π button:

1. 448π cm^3 or 1407.43 cm^3
2. 576π in^3 or 1809.56in^3
3. $16{,}848\pi$ yd^3 or 52,929.56 yd^3
4. $14{,}580\pi$ mm^3 or 45804.42 mm^3
5. 103.67 cm^3
6. 804.25 mm^3
7. 1357.17 in^3
8. 3015.93 yd^3
9. 3053.63 ft^3
10. 2144.66 mm^3
11. 268.08 cm^3
12. 113.1 yd^3

Materials List/Setup

Station 1 two sheets of paper; tape; mini marshmallows

Station 2 calculator

Station 3 four spherical objects of varying sizes (Ping-Pong balls, orange, basketball, globe, etc.); measuring tape; calculator

Station 4 calculator

Discussion Guide

To support students in reflecting on the activities and to gather some formative information about student learning, use the following prompts to facilitate a class discussion to "debrief" the station activities.

Prompts/Questions

1. Why does the way you roll a rectangle affect the volume of the cylinder you create?
2. Using a real-life example, when is it important to know the volume of an object?
3. What is the formula for the volume of a cylinder?
4. What is the formula for the volume of a cone?
5. What is the formula for the volume of a sphere?

Think, Pair, Share

Have students jot down their own responses to questions, then discuss with a partner (who was not in their station group), and then discuss as a whole class.

Suggested Appropriate Responses

1. If you roll it one way, the length of the rectangle is the height of the cylinder; if you roll it the other way, the length of the rectangle is the circumference of the cylinder.
2. Examples include a can of soup, a storage container, etc.
3. $\pi r^2 h$
4. $(V = \frac{1}{3}\pi r^2 h)$
5. $V = \frac{4\pi r^3}{3}$

Possible Misunderstandings/Mistakes

- Incorrectly measuring objects
- Incorrectly substituting measurements in the formulas
- Mistaking circumference for radius
- Making arithmetical errors
- Not understanding that the radius is half the diameter of an object

Station 1

At this station, you will find two sheets of paper, tape, and mini marshmallows. Each sheet of paper is 8 ½" by 11". You will be using these to investigate volume.

Your task is to answer this question: Which way should you roll the paper into a cylinder to get the greatest volume? With 11 inches as the circumference or the height? Does it matter?

Tape one sheet of paper into a cylinder so that 11 inches is the circumference. Tape one sheet of paper into a cylinder so that 11 inches is the height.

Which do you think holds more? _____

Fill both cylinders with mini marshmallows.

Which cylinder holds more? _____

Now calculate the volume of the two cylinders. Write your solution below.

Volume of cylinder with 11" circumference: _____

Volume of cylinder with 11" height: _____

Which cylinder has a greater volume? _____

Station 2

At this station, you will pretend you are an ice cream vendor. You have a cooler that is 3 feet by 2 feet by 2 feet. You want to fit in as many ice cream cones as possible. Each ice cream cone is pre-wrapped and has ice cream in it, so you cannot fit them inside one another.

The way to find the volume of a cone is the same as finding the volume of a pyramid: $V = \frac{1}{3}Bh$

In the case of the cone, the base is not found by multiplying the length times the width.

How do we find the area of the base of a cone? _____

_____

_____

Each ice cream cone is 4 inches tall, and the radius is 1 inch.

What is the volume of one ice cream cone? _____

What is the volume of the cooler? _____

How many ice cream cones can you fit in the cooler? (Be careful of your units!)

_____

Explain how you arrived at your solution. _____

_____

_____

_____

Station 3

At this station, you will find a calculator, measuring tape, and several round objects. Work as a group to determine each object's approximate volume by following the steps below.

1. First, use the measuring tape to measure around the middle of each object. This is the circumference (*C*).

   _____

2. Use this variation of the formula for circumference of a circle to find the radius (*r*) of the object.

   Radius of a circle $r = \frac{C}{2\pi}$

   _____

3. Finally, substitute the radius into the formula for the volume of a sphere:

   Volume of a sphere: $V = \frac{4}{3}\pi r^3$

   _____

4. Use the information to fill in the table.

Object circumference	Radius	Volume

Station 4

Work as a group to solve the following problems. You may use a calculator. Round to the nearest hundredth if necessary.

$\pi \approx 3.14$

For problems 1–4, find the volume of a cylinder using the given information.

Volume of a cylinder: $\pi r^2 h$

1. Radius = 8 cm; height = 7 cm

   Volume = _____

2. Radius = 6 in.; height = 16 in.

   Volume = _____

3. Radius = 36 yd; height = 13 yd

   Volume = _____

4. Radius = 18 mm; height = 45 mm

   Volume = _____

continued

For problems 5–8, find the volume of a cone using the given information.

Volume of a cone: $V = \frac{1}{3}\pi r^2 h$

5. Radius = 3 cm; height = 11 cm

   Volume = _____

6. Radius = 8 mm; height = 12 mm

   Volume = _____

7. Radius = 9 in.; height = 16 in.

   Volume = _____

8. Radius = 12 yd; height = 20 yd

   Volume = _____

continued

For problems 9–12, find the volume of a sphere using the given information.

Volume of a sphere: $V = \frac{4\pi r^3}{3}$

9. Radius = 9 ft

   Volume = _____

10. Radius = 8 mm

    Volume = _____

11. Radius = 4 cm

    Volume = _____

12. Radius = 3 yd

    Volume = _____