



Quadratics

Math Background

Previously, you

- Identified and graphed quadratic functions in Algebra II
- Applied transformations to parent functions
- Solved quadratic functions in Algebra II
- Worked with complex number operations in Algebra II

In this unit you will

- Review graphing quadratic functions, with and without technology
- Review solving quadratic functions using graphing, factoring, completing the square and the quadratic formula
- Review operations with complex numbers

You can use the skills in this unit to

- Predict the number and nature of the roots using the discriminant.
- Manipulate complex numbers including finding the quotients of complex numbers.
- Solve quadratic equations that have complex solutions.
- Graph solutions of single variable and two variable quadratic inequalities.

Vocabulary

- **Axis of symmetry** – A line that passes through a figure in such a way that the part of the figure on one side of the line is a mirror reflection of the part on the other side of the line.
- **Completing the square** – A way of simplifying or solving a quadratic equation by adding an expression to both sides to make one part of the equation a perfect square.
- **Complex number** – Any number that can be written as $a+bi$, where a and b are real numbers and $i = \sqrt{-1}$.
- **Complex solutions** – The x -values that make the function equal to zero are complex numbers (have an imaginary part).
- **Discriminant** – The expression $b^2 - 4ac$ in a quadratic equation $ax^2 + bx + c = 0$. It can be used to determine the characteristics of the solution.
- **Domain** – The input values of the function. For a quadratic function, the domain is all real numbers.
- **Maxima** – The largest value (highest point) of the function.
- **Minima** – The smallest value (lowest point) of the function.
- **Parabola** – The U-shaped graph of a quadratic function.
- **Parent Quadratic function** – The simplest quadratic function, $f(x) = x^2$.
- **Quadratic Equation** – A polynomial equation in which the highest power of the variable is two. The general form of such equations in the variable x is $ax^2 + bx + c = 0$.
- **Quadratic Formula** – The formula for determining the roots of a quadratic equation from its coefficients.
- **Quadratic function** – A function where the highest exponent of the variable is a square.
- **Range** – The output values of a function.
- **Reflection** – A transformation in which every point of a figure is mapped to a corresponding image across a line of symmetry.



- **Transformation** – A change in the position, size, or shape of a figure or graph.
- **Translation** – A figure is moved from one location to another on the coordinate plane without changing its size, shape or orientation.
- **Vertex** – The point of intersection of a parabola and its line of symmetry. The minima or maxima of the quadratic function.
- **Zeros of a Function** – The x -value or x -values that make the function equal to zero. A zero may be a real number or a complex number.

Essential Questions

- What are the key features of the graphs of quadratic functions?
- What are the different ways to solve quadratic equations and which ways are more efficient? Do all quadratics have real solutions?
- What is a complex number? What is the purpose for a complex number?
- When do you use an equation versus an inequality?

Overall Big Ideas

A quadratic function is represented by a U-shaped curve, called a parabola. It intercepts one or both axes and has one maximum or minimum value.

Solving quadratic functions can be done by different techniques like square rooting, factoring, completing the square, graphing and using the quadratic formula. Some techniques are more efficient than others based on the characteristics of the quadratic function. Some quadratics do not have real solutions. Changing the form of the expression by factoring reveals important attributes about the function and its graph.

Complex numbers expand the number system to include square roots of negative numbers and allows applications of complex numbers to electronics. We use the properties of operations as it applies to complex numbers to simplify expressions and to build foundations to solve quadratic equations having complex solutions.

Variable equations or inequalities model real-life situations and generalize applications, building a foundation for solving equations and inequalities with more than one variable.

**Skill**

To graph quadratic functions in both standard and vertex form, with and without technology.

To solve quadratic equations by graphing, factoring, completing the square, and the quadratic formula.

To predict and analyze the nature of the roots of a quadratic using the discriminant.

To identify, simplify, and perform operations with complex numbers.

To solve quadratic equations with complex numbers.

To solve and graph solutions of single variable and two variable quadratic inequalities.

Related Standards**A.APR.B.2**

Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a , the remainder on division by $x - a$ is $p(a)$, so $p(a) = 0$ if and only if $(x - a)$ is a factor of $p(x)$.

A.APR.B.3

Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

F.IF.A.1

Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.

F.IF.A.2

Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

F.IF.C.7a

Graph linear and quadratic functions and show intercepts, maxima, and minima. *(Modeling Standard)

A.SSE.B.3a

Factor a quadratic expression to reveal the zeros of the function it defines. *(Modeling Standard)

A.SSE.B.3b

Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**A.REI.B.4a**

Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.

A.REI.B.4b

Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .

F.IF.C.8a

Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

N.CN.A.1

Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a+bi$ with a and b real.

N.CN.A.2

Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

N.CN.A.3

Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N.CN.C.7

Solve quadratic equations with real coefficients that have complex solutions.

A.CED.A.1-1

Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear, quadratic, and exponential functions. Limit exponentials to have integer inputs only. *(Modeling Standard)

A.CED.A.2-1

Create linear, exponential, and quadratic equations in two or more variables to represent relationships between quantities and graph equations on coordinate axes with labels and scales. Limit exponentials to have integer inputs only. *(Modeling Standard)



Notes, Examples, and Exam Questions

Parent Function: The most basic quadratic function is $f(x) = x^2$. This function is often called the parent quadratic function. Once you understand the parent function you can shift, reflect, and stretch the parent graph to get graphs of other quadratic functions. The graph of the function $y = f(x)$ is the set of all points $(x, f(x))$, x is in the domain of f . We match domain values along the x -axis with their range values along the y -axis to get the ordered pairs that yield the graph of $y = f(x)$.

Unit 1.1 To graph quadratic functions in both standard and vertex form, with and without technology.

NOTE: This unit is a review unit. Unit 3, Quadratic Functions, is a five week unit covered in the first quarter of Algebra II. Most of these topics are not new and should be treated as review. Division with complex numbers and solving and graphing two variable quadratic inequalities are the only new topics in this unit.

Graphing a Quadratic Function in Standard Form:

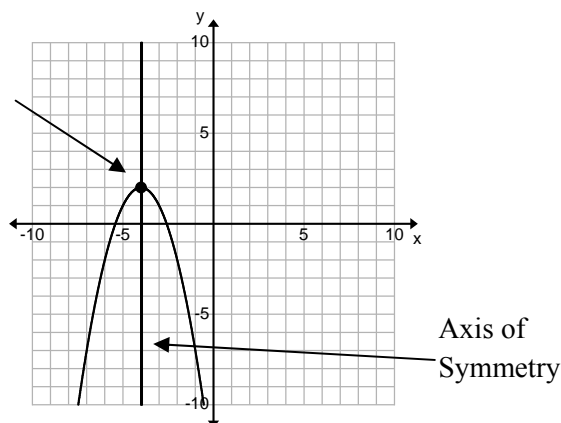
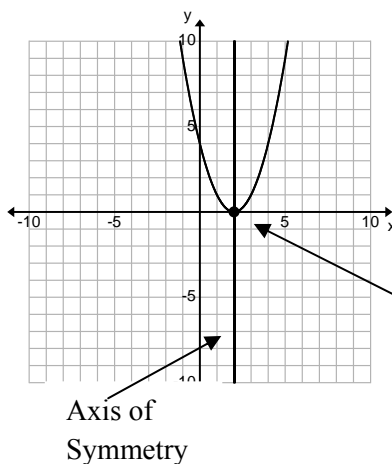
Standard Form of a Quadratic Function $y = ax^2 + bx + c$, when $a \neq 0$; a , b , and c are real numbers

Vertex: the vertex is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$ Axis of Symmetry: $x = -\frac{b}{2a}$ y-intercept: c

Minimum Value: When a parabola opens upward, the y -value of the vertex is the minimum value.

Maximum Value: When a parabola opens downward, the y -value of the vertex is the maximum value.

Axis of Symmetry: It is the vertical line that passes through the vertex of a quadratic function.



$D: \{x \mid x \in \mathbb{R}\}$ The domain is all real numbers
 $R: \{y \mid y \geq k\}$ The range is all values greater than or equal to the minimum.

$D: \{x \mid x \in \mathbb{R}\}$ The domain is all real numbers
 $R: \{y \mid y \leq k\}$ The range is all values less than or equal to the maximum.

**First, find the vertex:**

- list $a = \underline{\quad}$, $b = \underline{\quad}$, $c = \underline{\quad}$

If a is positive, it opens up, if negative, it opens down.

$$\text{- find } x = \frac{-b}{2a}$$

- plug this x -value into the function - this point $(\underline{\quad}, \underline{\quad})$ is the vertex of the parabola

Second, find the axis of symmetry:

$$\text{It is the line } x = \frac{-b}{2a}.$$

Third, graph the points:

- put the vertex you found on the graph
- graph the axis of symmetry
- plot two points on one side of the axis of symmetry.
- Use symmetry to plot two more points on the opposite side.
- graph all 5 points

Fourth:

- Draw a parabola through the points

Ex 1: Graph the quadratic function $y = x^2 - 6x - 1$. State the vertex and axis of symmetry.

Step One: $a = 1$, $b = -6$, $c = -1$. The parabola opens up. Find $x = -\frac{b}{2a}$, the x -coordinate of the vertex.

$$x = -\frac{-6}{2(1)} = 3$$

$$y = (3)^2 - 6(3) - 1 = -10$$

$$\text{vertex: } \boxed{(3, -10)}$$

Step Two: Find the axis of symmetry.

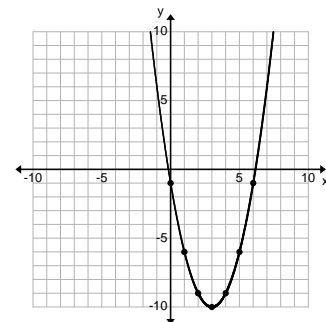
The axis of symmetry is the line $x = 3$

Step Three: Pick two points on one side of the vertex. Let $x = 1$. Substituting into the function, we get $y = -6$, or $(1, -6)$. Let $x = 0$. Since $c = -1$, the y -intercept is -1 , so we have the point $(0, -1)$. Using symmetry, notice that $(1, -6)$ is 2 units left of the axis of symmetry. The point on the parabola symmetrical to $(1, -6)$ is 2 units to the right of the axis at $(5, -6)$. Notice that $(0, -1)$ is 3 units left of the axis of symmetry. The point on the parabola symmetrical to $(0, -1)$ is 3 units to the right of the axis at $(6, -1)$.

Step Four: Connect points with a smooth curve to draw the parabola.

$$\text{Vertex: } \boxed{(3, -10)}$$

$$\text{Axis of Symmetry: } \boxed{x = 3}$$





Ex 2: Graph the quadratic function $y = -3x^2 + 12x - 6$. State the vertex and axis of symmetry.

Step One: $a = -3$, $b = 12$, $c = -6$. The parabola opens down. Find $x = -\frac{b}{2a}$, the x -coordinate of the vertex.

$$x = -\frac{12}{2(-3)} = 2 \qquad y = -3(2)^2 + 12(2) - 6 = 6$$

vertex: $\boxed{(2,6)}$

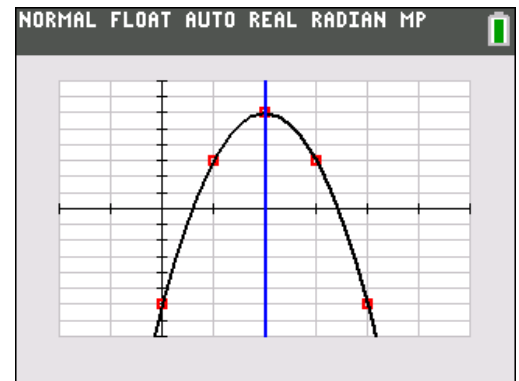
Step Two: Find the axis of symmetry. The axis of symmetry is the line $x = 2$

Step Three: Pick two points on one side of the vertex. Let $x = 1$. Substituting into the function, we get $y = 3$, or $(1, 3)$. Let $x = 0$. Since $c = -6$, the y -intercept is -6 , so we have the point $(0, -6)$. Using symmetry, notice that $(1, -6)$ is 1 unit left of the axis of symmetry. The point on the parabola symmetrical to $(1, -6)$ is 1 unit to the right of the axis at $(3, -6)$. Notice that $(0, -6)$ is 2 units left of the axis of symmetry. The point on the parabola symmetrical to $(0, -6)$ is 2 units to the right of the axis at $(4, -6)$.

Step Four: Connect points with a smooth curve to draw the parabola.

Vertex: $\boxed{(2,6)}$

Axis of Symmetry: $\boxed{x = 2}$



Graphing a Quadratic Function in Vertex Form:

$$y = a(x - h)^2 + k$$

a indicates a reflection across the x -axis and/or a vertical stretch or compression.

h indicates a horizontal translation

k indicates a vertical translation

Axis of Symmetry $x = h$

Vertex is (h, k)

If a is positive the parabola opens up.

If a is negative the parabola opens down.

Vertical Stretch: $|a| > 1$

Vertical Shrink: $0 < |a| < 1$

Horizontal Translation: $h < 0$ moves left

$h > 0$ moves right

Vertical Translation: $k < 0$ moves down

$k > 0$ moves up

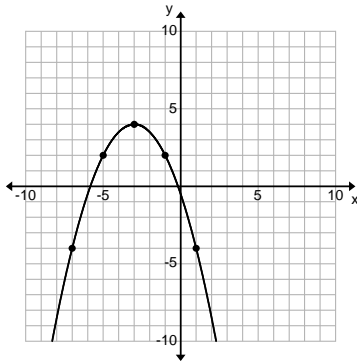


Ex 3: Graph the quadratic function $y = -\frac{1}{2}(x+3)^2 + 4$. State the vertex and axis of symmetry.

Step One: Determine if the graph opens up or opens down. Because a is $-\frac{1}{2}$ the graph opens down.

Step Two: Identify the vertex and axis of symmetry. Note: Another way of writing the function is $y = -\frac{1}{2}(x - (-3))^2 + 4$. So the vertex is $(-3, 4)$ and the axis of symmetry is $x = -3$.

Step Three: Plot the vertex and sketch the graph. Notice the parent graph is reflected over the x axis, is wider (vertical shrink), shifted to the left 3 (opposite of what you think), and is translated up four units.



Optional: Make a table of values. When choosing x -values for the T-table, use the vertex, a few values to the left of the vertex, and a few values to the right of the vertex. (Note: Because of the fraction, you may want to choose values that will guarantee whole numbers for the y -coordinates.) Five to seven points will give a nice graph of the parabola.

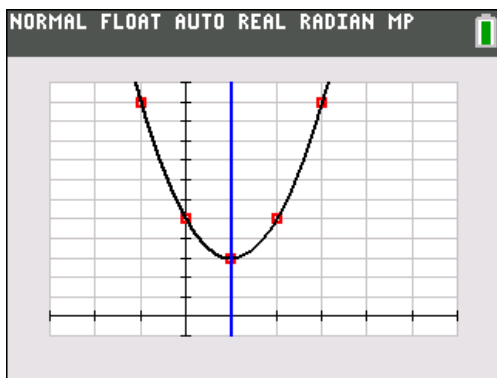
x	-9	-7	-5	-3	-1	1	3
y	-14	-4	2	4	2	-4	-14

Ex 4: Graph the quadratic function $y = 2(x-1)^2 + 3$. State the vertex and axis of symmetry.

Step One: Determine if the graph opens up or opens down. Because a is 2 the graph opens up.

Step Two: Identify the vertex and axis of symmetry. The vertex is $(1, 3)$ and the axis of symmetry is $x = 1$.

Step Three: Plot the vertex and sketch the graph. Notice the parent graph is narrower (vertical stretch), shifted to the right 1 and is translated up three units.



Optional:

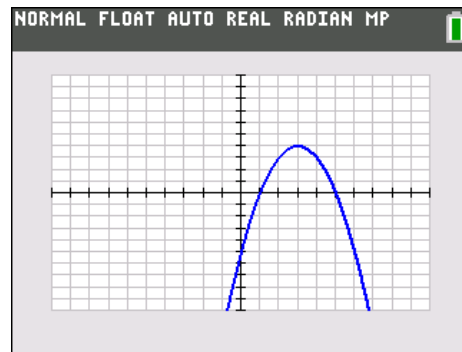
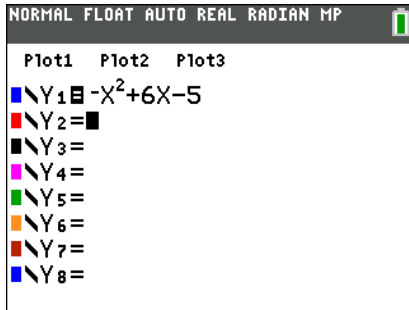
x	-1	0	2	3
y	11	5	5	11



Ex 5: Use technology to graph quadratic functions.

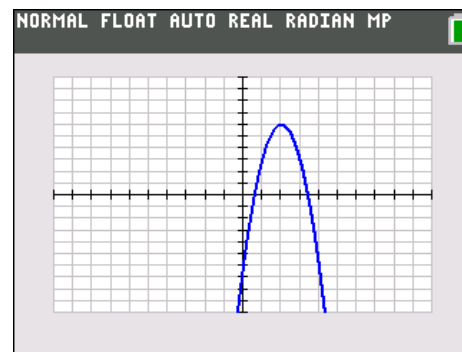
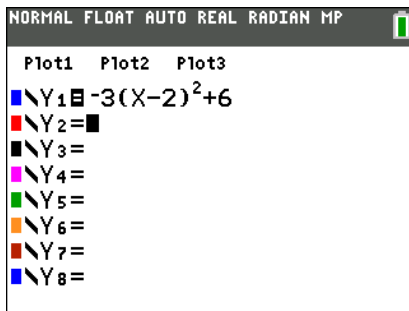
a) $y = -x^2 + 6x - 5$

Keystrokes:



b) $y = -3(x - 2)^2 + 6$

Keystrokes:



SAMPLE EXAM QUESTIONS

1. Find the vertex of the parabola and determine whether the parabola opens up or down.

$$y = -(x - 2)^2 - 2$$

- (A) (0, -2), up (B) (2, -2), down
 (C) (2, 2), down (D) (-2, 0), up

Ans: B

2. Find the coordinates of the vertex of $y = 4x^2 - 32x + 69$.

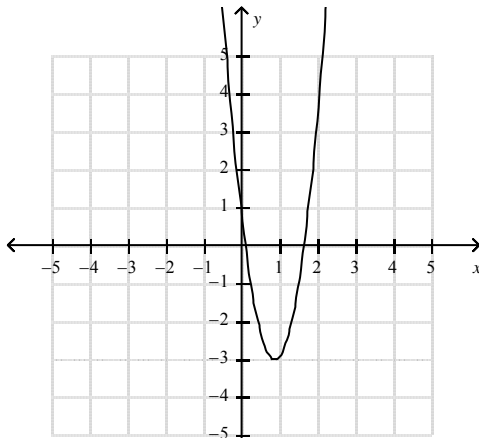
- (A) (-5, -4) (B) (-4, -5)
 (C) (5, 4) (D) (4, 5)

Ans: D

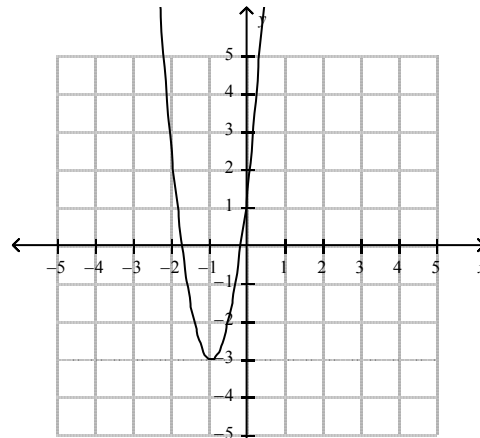


3. Graph the quadratic function $f(x) = 5x^2 + 9x + 1$.

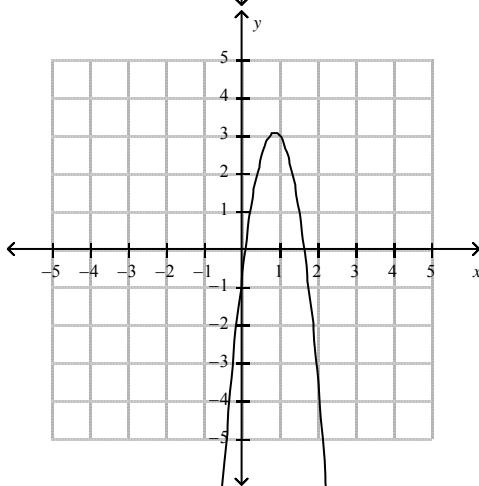
a.



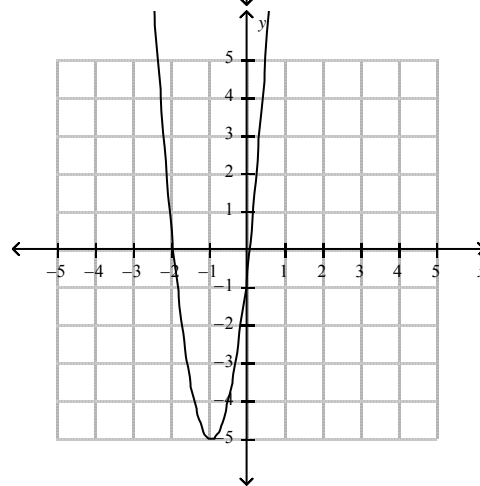
c.



b.



d.



Ans: C

4. Which of the following parabolas will have a maximum rather than a minimum?

- (A) $y = (2 - x)^2$
- (B) $y = 3x - x^2$
- (C) $y = x^2 - 5x - 10$
- (D) $y = (2 - x)(3 - x)$

Ans: B

5. What is the equation of the axis of symmetry of the parabola with equation $y = 2(x + 3)^2 - 4$?

- (A) $x = 3$
- (B) $x = 2$
- (C) $x = 4$
- (D) $x = -3$

Ans: D



6. Find the domain and range of the function: $f(x) = (x+2)^2 + 5$

(A) Domain: $(-\infty, \infty)$; range: $[-5, \infty)$

(B) Domain: $(-5, \infty)$; range: $(-\infty, \infty)$

(C) Domain: $(5, \infty)$; range: $(-\infty, \infty)$

(D) Domain: $(-\infty, \infty)$; range: $[5, \infty)$

Ans: D

7. Identify the vertex of $g(x) = (x+14)^2 - 8$.

a. $(-14, -8)$

b. $(-14, 8)$

c. $(14, -8)$

d. $(14, 8)$

Ans: A

8. Use this description to write the quadratic function in vertex form:

The parent function $f(x) = x^2$ is vertically stretched by a factor of 3 and translated 8 units right and 1 unit down.

a. $g(x) = \frac{1}{3}(x-8)^2 - 1$

b. $g(x) = \frac{1}{3}(x+8)^2 - 1$

c. $g(x) = 3(x-8)^2 - 1$

d. $g(x) = 3(x+8)^2 - 1$

Ans: C

9. What is the maximum of the quadratic function $f(x) = -2x^2 + 4x + 6$?

A. $f(x) = 1$

B. $f(x) = 6$

C. $f(x) = 3$

D. $f(x) = 8$

Ans: D

10. Consider $h(x) = 3x^2 + 6x + 2$. What are its vertex and y-intercept?

a. vertex: $(-1, -1)$, y-intercept: $(0, 2)$

b. vertex: $(-2, 2)$, y-intercept: $(0, -2)$

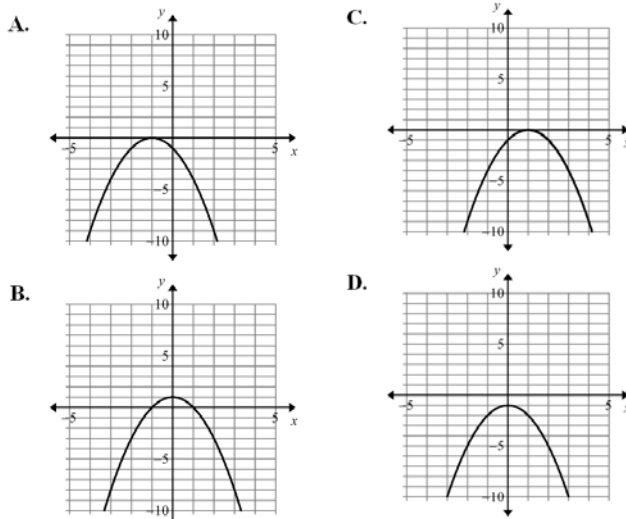
c. vertex: $(1, 1)$, y-intercept: $(0, 2)$

d. vertex: $(-2, 1)$, y-intercept: $(0, -2)$

Ans: A



11. Which graph represents $f(x) = -x^2 + 2x - 1$?



Ans: C

Unit 1.2 To solve quadratic equations by graphing, factoring, completing the square, and the quadratic formula.

A quadratic equation in the form $ax^2 + bx + c = 0$ has a related function $f(x) = ax^2 + bx + c$. The zeros of the function are the x -intercepts of its graph. These x -values are the solutions or roots of the related quadratic equation. A quadratic equation can have one real solution, two real solutions, or no real solutions. Note that the three words - zeros, roots and x -intercepts - all represent the same thing – the x -values that make the function equal to zero. Graph the function and find where the graph crosses the x -axis to solve the quadratic equation.

Using a Graphing Calculator to Solve Quadratic Equations



Ex 6: Approximate the solution(s) of $x^2 = 1 - 4x$ using a graphing calculator.

Step One: Write the equation in the form $ax^2 + bx + c = 0$. $x^2 + 4x - 1 = 0$

Step Two: Graph the function $y = ax^2 + bx + c$. $y = x^2 + 4x - 1$

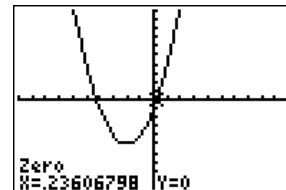
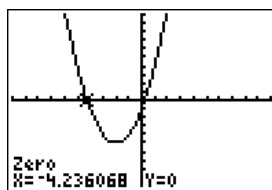
Press $\boxed{Y=}$, plug in the equation and then hit $\boxed{\text{ZOOM}}\boxed{6}$ for a nice window for the graph.

Step Three: Find the zero(s) of the function.

Press $\boxed{2\text{nd}}\boxed{\text{TRACE}}\boxed{2}$ to find the zeros.

To find each zero, make sure the cursors are to the left and right of the zero.

$$x \approx -4.236, 0.236$$





Factoring

Factoring Method: Write the equation in standard form: $ax^2 + bx + c = 0$

1. Factor the quadratic expression.
2. Use the Zero Product Property. This property states that if the product of two factors is zero, then one or both of the factors must equal zero. Set each factor equal to zero.
3. Solve each corresponding linear equation for the variable.
4. The solutions to a quadratic equation are roots. The roots of an equation are the value(s) of the variable that make the equation true.

Ex 7: Solve the equation $4x^2 - 10x - 7 = -2x^2 + x$.

Step One: Write the equation in standard form.

$$6x^2 - 11x - 7 = 0$$

Step Two: Factor the quadratic using the “ac method”.

$$a \cdot c = -42$$

$$6x^2 - 14x + 3x - 7 = 0$$

$$b = -11$$

$$2x(3x - 7) + 1(3x - 7) = 0$$

$$-14 \text{ and } 3$$

$$(3x - 7)(2x + 1) = 0$$

Step Three: Set each factor equal to zero and solve.

$$3x - 7 = 0 \quad 2x + 1 = 0$$

$$x = \frac{7}{3} \quad x = -\frac{1}{2}$$

The solutions can be written in set notation:

$$\left\{ \frac{7}{3}, -\frac{1}{2} \right\}$$

Ex 8: Solve the equation $49 = -28w - 4w^2$.

Step One: Write the equation in standard form.

$$4w^2 + 28w + 49 = 0$$

Step Two: Factor the quadratic. It is a perfect square trinomial.

$$(2w)^2 + 28w + (7)^2$$

$$(2w + 7)^2 = 0$$

Note: $2(7)(2w) = 28w$

Step Three: Set each factor equal to zero and solve.

$$2w + 7 = 0$$

$$w = -\frac{7}{2}$$

$$\left\{ -\frac{7}{2} \right\}$$



Completing the Square

If the equation is not factorable, a method called completing the square can be used to rewrite the equation so that the trinomial is a perfect square and can be factored.

1. Make sure the leading coefficient is ONE. If it is not, DIVIDE the entire equation by the leading coefficient, a.
2. Isolate the variable terms ax^2+bx . Place the constant on a side by itself.
3. Find $b/2$ and ADD its **square** to both sides.
4. Factor the perfect square.
5. Take the square root of both sides.
6. Solve the resulting equation.

Ex 9: Solve $2x^2 + 12x - 4 = 0$ by completing the square.

Step One: Rewrite to make the lead coefficient 1.

$$\frac{2x^2}{2} + \frac{12x}{2} - \frac{4}{2} = \frac{0}{2}$$

$$x^2 + 6x - 2 = 0$$

$$x^2 + 6x = 2$$

Step Two: Take the constant term to the other side.

$$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 2 + \left(\frac{6}{2}\right)^2$$

Step Three: Complete the square (add $\left(\frac{b}{2}\right)^2$ to **both** sides).

$$x^2 + 6x + 9 = 11$$

Step Four: Factor the perfect square trinomial.

$$(x+3)^2 = 11$$

Step Five: Take the square roots of both sides.

$$\sqrt{(x+3)^2} = \sqrt{11}$$

$$|x+3| = \sqrt{11}$$

Step Six: Solve for the variable.

$$x+3 = \sqrt{11} \quad x+3 = -\sqrt{11}$$

$$x = -3 + \sqrt{11} \quad x = -3 - \sqrt{11}$$

The solution set is $\{-3 - \sqrt{11}, -3 + \sqrt{11}\}$

Ex 10: Solve $x^2 - 4x + 1 = 0$ by completing the square.

Step One: The leading coefficient is 1, go to step two.

$$x^2 - 4x + 1 = 0$$

Step Two: Take the constant term to the other side.

$$x^2 - 4x = -1$$

Step Three: Complete the square (add $\left(\frac{b}{2}\right)^2$ to **both** sides).

$$x^2 - 4x + \left(\frac{-4}{2}\right)^2 = -1 + \left(\frac{-4}{2}\right)^2$$

$$x^2 - 4x + 4 = 3$$

Step Four: Factor the perfect square trinomial.

$$(x-2)^2 = 3$$

Step Five: Take the square roots of both sides.

$$\sqrt{(x-2)^2} = \sqrt{3}$$

Step Six: Solve for the variable.

$$x-2 = \sqrt{3} \quad x-2 = -\sqrt{3}$$

$$x = 2 + \sqrt{3} \quad x = 2 - \sqrt{3}$$

$$2 \pm \sqrt{3}$$



Quadratic Formula

The Quadratic Formula can be used to solve any equation in the form $ax^2 + bx + c = 0$, where $a \neq 0$. To determine the roots of the equation, substitute the coefficients a and b and the constant c into the quadratic formula and then simplify the resulting expression.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Ex 11: Solve the quadratic equation $x^2 - 8x = -1$ using the quadratic formula.

Step One: Rewrite in standard form (if necessary). $x^2 - 8x + 1 = 0$

Step Two: Identify a , b , and c . $a = 1, b = -8, c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step Three: Substitute the values into the quadratic formula.

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(1)(1)}}{2(1)}$$

Step Four: Simplify.

$$x = \frac{8 \pm \sqrt{64 - 4}}{2} = \frac{8 \pm \sqrt{60}}{2} = \frac{8 \pm 2\sqrt{15}}{2} = 4 \pm \sqrt{15}$$

The solution set is $\{4 - \sqrt{15}, 4 + \sqrt{15}\}$

Ex 12: Solve the quadratic equation $4 - 5x = 3x^2$ using the quadratic formula.

Step One: Rewrite in standard form (if necessary). $0 = 3x^2 + 5x - 4$

Step Two: Identify a , b , and c . $a = 3, b = 5, c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step Three: Substitute the values into the quadratic formula.

$$x = \frac{-(5) \pm \sqrt{(5)^2 - 4(3)(-4)}}{2(3)}$$

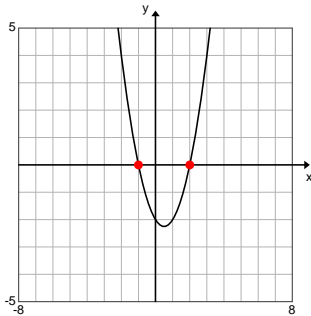
Step Four: Simplify.

$$x = \frac{-5 \pm \sqrt{24 + 48}}{6} = \frac{-5 \pm \sqrt{73}}{6} \quad \boxed{\frac{-5 \pm \sqrt{73}}{6}}$$

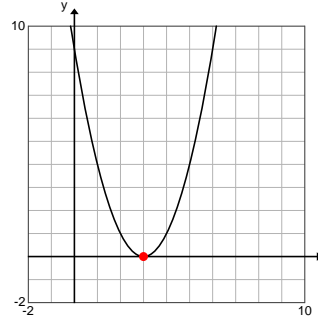


Unit 1.3 To predict and analyze the nature of the roots of a quadratic using the discriminant.

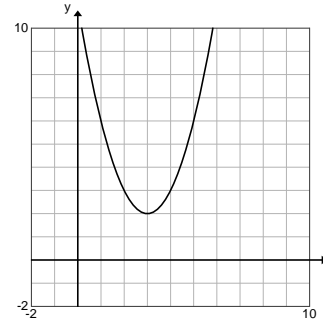
Models: A quadratic equation can have one real solution, two real solutions, or no real solutions depending on the number of times the graph crosses the x-axis.



Two real solutions



One Real Solution



No real solution

Discriminant: The number under the square root in the quadratic formula.

$$\boxed{b^2 - 4ac}$$

The sign of the discriminant determines the number and type of solutions of a quadratic equation.

- ❖ If $b^2 - 4ac > 0$, then the equation has **two real solutions** (two x -intercepts) and no imaginary solutions.
- ❖ If $b^2 - 4ac = 0$, then the equation has **one real solution** (one x -intercept) and one imaginary solution.
- ❖ If $b^2 - 4ac < 0$, then the equation has **two imaginary solutions** (no x -intercept).

Ex 13: Determine the number and type of roots for the quadratic equation $4x^2 - 4x + 1 = 0$.

$$\text{Discriminant: } b^2 - 4ac = (-4)^2 - 4(4)(1) = \boxed{0}$$

Since the discriminant is 0, there is one real solution and one imaginary.

Ex 14: Determine the number and type of roots that the quadratic equation $18n^2 + 4n = 10n^2 - 6n + 5$ has.

Put the equation in standard form: $8n^2 + 10n - 5 = 0$. Therefore, $a = 8, b = 10$ and $c = -5$.

$$\text{Discriminant: } b^2 - 4ac = (10)^2 - 4(8)(-5) = \boxed{\sqrt{260} \approx 16.12}$$

Since the discriminant is a positive value, there are two real roots (solutions).



To summarize: Five methods for solving quadratics have been discussed. When to use each one? Here's a plan of attack:

- If there is no linear term (bx), factor (if possible).
- If you have a trinomial, try to factor it first.
- If it is not factorable, then use the quadratic formula or the completing the square method.
- The graphing method can only estimate roots if they are not integers unless technology (TI-84) is handy.

SAMPLE EXAM QUESTIONS

1. What are the solutions of the quadratic equation $9x^2 - 6x - 8 = 0$?

- A. $x = -\frac{2}{9}, x = 4$
- B. $x = -\frac{2}{3}, x = \frac{4}{3}$
- C. $x = -\frac{4}{3}, x = \frac{2}{3}$
- D. $x = -4, x = \frac{2}{9}$

Ans: B

2. What is the solution set for the quadratic equation $x^2 + 6x - 3 = 0$?

- A. $\{-3 - 2\sqrt{3}, -3 + 2\sqrt{3}\}$
- B. $\{3 - 2\sqrt{3}, 3 + 2\sqrt{3}\}$
- C. $\{-3 - \sqrt{6}, -3 + \sqrt{6}\}$
- D. $\{3 - \sqrt{6}, 3 + \sqrt{6}\}$

Ans: A

3. Which is one of the appropriate steps in finding solutions for $x^2 + 4x - 3 = 0$ when completing the square?

- A. $(x+4)^2 = 3$
- B. $(x+2)^2 = 3$
- C. $(x+4)^2 = 7$
- D. $(x+2)^2 = 7$

Ans: D



4. How many real and imaginary solutions are there for the equation $7x^2 - x + 10 = 0$?

- A. no real solutions, 2 imaginary solutions
- B. 1 real solution, no imaginary solutions
- C. 1 real solution, 1 imaginary solution
- D. 2 real solutions, no imaginary solutions

Ans: A

5. Solve the equation by factoring: $2x^2 + x = 15$

- A. $\left\{-3, \frac{5}{2}\right\}$
- B. $\left\{3, \frac{5}{2}\right\}$
- C. $\left\{-3, -\frac{5}{2}\right\}$
- D. $\left\{3, -\frac{5}{2}\right\}$

Ans: A

6. Use the quadratic formula to solve the equation: $2n^2 = -10n - 7$

- A. $\left\{\frac{-10 + \sqrt{11}}{2}, \frac{-10 - \sqrt{11}}{2}\right\}$
- B. $\left\{\frac{-5 + \sqrt{11}}{2}, \frac{-15 - \sqrt{11}}{2}\right\}$
- C. $\left\{\frac{-5 + \sqrt{39}}{2}, \frac{-5 - \sqrt{39}}{2}\right\}$
- D. $\left\{\frac{-5 + \sqrt{11}}{4}, \frac{-5 - \sqrt{11}}{4}\right\}$

Ans: B

7. Which equation below has only one real root?

- A. $x^2 + 2x + 1 = -1$
- B. $x^2 + 2x + 1 = 1$
- C. $x^2 + 2x + 1 = 0$
- D. $x^2 + 2x + 1 = 2$

Ans: C

Unit 1.4 To identify, simplify, and perform operations with complex numbers.

A Complex Number is a combination of a real number and an imaginary number. Imaginary numbers are special because when squared, they give a **negative** result. Normally this doesn't happen, because when we square a positive number we get a positive result, and when we square a negative number we also get a positive result. But just imagine there is such a number, because we need it! The "unit" imaginary number (like 1 is for Real numbers) is i , which is the square root of -1 . So, a Complex Number has a real part and an imaginary part, but either part can be 0, so all Real numbers and Imaginary numbers are also Complex Numbers. Complex Number (in Standard Form): $a + bi$, where a is the real part of the complex number and bi is the imaginary part of the complex number.



Imaginary Unit: $i = \sqrt{-1}$

Complex Number	Real Part	Imaginary Part
$3 + 2i$	3	2
5	5	0
$-6i$	0	-6

$$i^1 = \sqrt{-1}$$

$$i^2 = \sqrt{-1} \cdot \sqrt{-1} = -1$$

Powers of i :

$$i^3 = i \cdot i^2 = -i$$

$$i^4 = i^2 \cdot i^2 = 1$$

$$i^5 = i \cdot i^4 = i$$

...

▲Note: The pattern continues every 4th power of i .

Ex 15: Simplifying Square Roots of Negative Numbers

$$\sqrt{-5} = \sqrt{-1}\sqrt{5} = i\sqrt{5}$$

$$\sqrt{-64} = \sqrt{-1}\sqrt{64} = 8i$$

$$(\sqrt{-1})^4 = i^4 = i^2 \cdot i^2 = (-1)(-1) = 1$$

$$2\sqrt{-25} = 2\sqrt{25}\sqrt{-1} = 2 \cdot 5\sqrt{-1} = 10i$$

$$-\sqrt{-45} = -\sqrt{(45)(-1)} = -\sqrt{9}\sqrt{5}\sqrt{-1} = -3i\sqrt{5}$$

Sum and Difference of Complex Numbers: Add or subtract the real parts and the imaginary parts separately.

Ex 16: Find the sum: $(-2 + 7i) + (-3 - 4i)$

$$\begin{aligned} &(-2 + -3) + (7i - 4i) \\ &= \boxed{-5 + 3i} \end{aligned}$$

Ex 17: Find the difference: $(9 - 5i) - (-4 - 3i)$

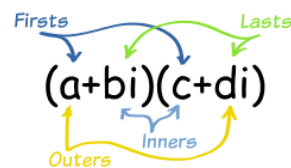
$$\begin{aligned} &(9 - (-4)) + (-5i - (-3i)) \\ &= \boxed{13 - 2i} \end{aligned}$$

Product of Complex Numbers: Use the distributive property or FOIL method to multiply two complex numbers.

Ex 18: Find the product $(-5 - i)(4 + 3i)$.

Use FOIL:

$$\begin{aligned} &(-5)(4) + (-5)(3i) + (-i)(4) + (-i)(3i) \\ &= -20 - 15i - 4i - 3i^2 \\ &= -20 - 19i - 3(-1) \\ &= \boxed{-17 - 19i} \end{aligned}$$



- Firsts: $a \times c$
- Outers: $a \times di$
- Inners: $bi \times c$
- Lasts: $bi \times di$

$$(a+bi)(c+di) = ac + adi + bci + bdi^2$$



Ex 19: Multiply $7i(3 - 5i)$.

Use the distributive property.

$$\begin{aligned} &7i(3 - 5i) \\ &21i - 35i^2 \rightarrow 21i - 35(-1) \\ &= 21i + 35 \\ &= \boxed{35 + 21i} \end{aligned}$$

Complex Conjugate: The complex conjugate of $a + bi$ is $a - bi$.

Quotient of Complex Numbers: To divide two complex numbers, multiply the numerator and denominator by the complex conjugate of the divisor (denominator). Why do we do this? When we multiply a complex number by its conjugate, we

get a real number: $\boxed{(a + bi)(a - bi) = a^2 - abi + abi - b^2i^2 = a^2 + b^2}$

Ex 20: Find the quotient $\frac{3 + 5i}{1 - 2i}$.

Multiply by the conjugate: $(1 + 2i)$:

$$\begin{aligned} &\frac{3 + 5i}{1 - 2i} \cdot \frac{1 + 2i}{1 + 2i} &&= \frac{-7 + 11i}{5} \\ &= \frac{3 + 6i + 5i + 10i^2}{1 - 4i^2} &&= \boxed{-\frac{7}{5} + \frac{11}{5}i} \\ &= \frac{3 + 11i - 10}{1 + 4} \end{aligned}$$

Note: The final answer is written in standard form.

Ex 21: Find the quotient $\frac{3 - i}{2 + 7i}$.

Multiply by the conjugate: $(2 - 7i)$:

$$\begin{aligned} &\frac{3 - i}{2 + 7i} \cdot \frac{2 - 7i}{2 - 7i} &&= \frac{-1 - 23i}{53} \\ &= \frac{6 - 2i - 21i + 7i^2}{2^2 + 7^2} &&= \boxed{-\frac{1}{53} - \frac{23}{53}i} \\ &= \frac{6 - 23i - 7}{4 + 49} \end{aligned}$$



Unit 1.5 To solve quadratic equations with complex numbers.

Ex 22: Solve $3x^2 + 8 = 0$.

Solve using square roots:

$$3x^2 = -8 \rightarrow x^2 = -\frac{8}{3}$$

$$\sqrt{x^2} = \sqrt{-\frac{8}{3}}$$

$$x = \pm \sqrt{\frac{8}{3}} \sqrt{-1}$$

Write the answer(s) in complex form:

$$x = \pm i \sqrt{\frac{8}{3}}$$

Ex 23: Solve $3q^2 + 11 = 5q$.

Use the quadratic formula: $3q^2 - 5q + 11 = 0$ $a = 3, b = -5, c = 11$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(11)}}{2(3)} \quad \text{Simplify: } x = \frac{5 \pm \sqrt{25 - 132}}{6} = \frac{5 \pm \sqrt{-107}}{6}$$

Write the answer(s) in complex form:

$$\frac{5}{6} \pm \frac{i\sqrt{107}}{6}$$

SAMPLE EXAM QUESTIONS

1. What are the solutions of the quadratic equation $3x^2 + 5x = -4$?

A. $x = \frac{-5 + i\sqrt{23}}{6}, x = \frac{-5 - i\sqrt{23}}{6}$

B. $x = \frac{-5 + i\sqrt{73}}{6}, x = \frac{-5 - i\sqrt{73}}{6}$

C. $x = \frac{5 + i\sqrt{23}}{6}, x = \frac{5 - i\sqrt{23}}{6}$

D. $x = \frac{5 + i\sqrt{73}}{6}, x = \frac{5 - i\sqrt{73}}{6}$

Ans: A



2. Write the expression $\frac{7+3i}{3+9i}$ as a complex number in standard form.

A. $\frac{1}{12} - \frac{3}{4}i$

B. $\frac{8}{15} - \frac{3}{5}i$

C. $\frac{8}{15} + \frac{4}{5}i$

D. $\frac{1}{12} + i$

Ans: B

3. What are the solutions of $x^2 - 8x + 25 = 0$?

A. $x = 8 - 6i$ or $x = 8 + 6i$

B. $x = 4 - 3i$ or $x = 4 + 3i$

C. $x = 4 - 2i$ or $x = 4 + 2i$

D. $x = -4 - 3i$ or $x = -4 + 3i$

Ans: B

4. What is the product $(3+2i)(3-2i)$?

A. $5+12i$

C. $13+12i$

B. 5

D. 13

Ans: D

5. What is the product $(8-3i)(3+2i)$?

A. $30+7i$

C. $18-25i$

B. $-6i^2 + 7i + 24$

D. $30-7i$

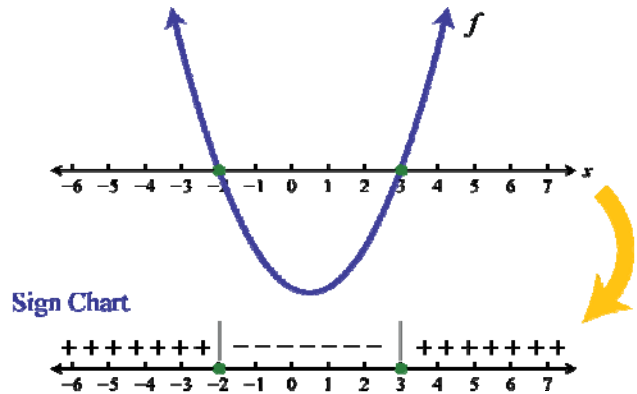
Ans: A



Unit 1.6 To solve and graph solutions of single variable and two variable quadratic inequalities.

Solving a Quadratic Inequality in One Variable: Solving inequalities is very similar to solving equations. We do most of the same things. When solving equations, we try to find POINTS, such as the ones marked “= 0”. But when we solve inequalities, we try to find INTERVAL(S), such as one marked “< 0”. So, this is what we do:

- Find the roots – the “= 0” points using the methods already learned. Start with factoring and use quadratic formula if it is not factorable.
- In between the roots are intervals that are either
 - Greater than zero (> 0), or
 - Less than zero (< 0)
- Then, pick a test value to find out which it is



****Note:** In this diagram, the roots were -2 and 3. Using a number line, pick values in each interval to determine whether the function is positive or negative. The corresponding graph is shown above.

Ex 24: Solve $x^2 - 5x + 6 \geq 0$.

Step One: Solve the quadratic equation $x^2 - x - 6 = 0$ using any method.

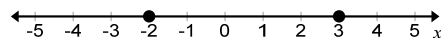
$$(x - 3)(x + 2) = 0$$

We will use factoring.

$$x - 3 = 0 \quad x + 2 = 0$$

$$x = 3 \quad x = -2$$

Step Two: Draw a sign chart on a number line to test which values for x satisfy the inequality.



Choose an x -value to the left of -2 and substitute into the inequality. We will try -4 .

$$(-4 - 3)(-4 + 2) \geq 0$$

$$(-7)(-2) \geq 0 \quad \text{true}$$

Choose an x -value between -2 and 3 and substitute into the inequality. We will try 0 .

$$(0 - 3)(0 + 2) \geq 0$$

$$(-3)(2) \geq 0 \quad \text{false}$$

Choose an x -value to the right of 3 and substitute into the inequality. We will try 4 .

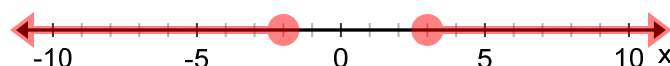
$$(4 - 3)(4 + 2) \geq 0$$

$$(1)(6) \geq 0 \quad \text{true}$$

Step Three: Write the solution as a compound inequality or in set notation and graph.

$$x \leq -2 \text{ or } x \geq 3$$

$$(-\infty, -2] \cup [3, \infty)$$





Ex 25: Solve $2x^2 + x - 4 < 0$.

Step One: Solve the quadratic equation $2x^2 + x - 4 = 0$ using any method.

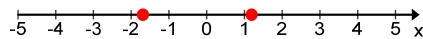
$$a = 2, b = 1, c = -4$$

We will use the quadratic formula.

$$x = \frac{-1 \pm \sqrt{1^2 - 4(2)(-4)}}{2(2)} \rightarrow x = \frac{-1 \pm \sqrt{33}}{4}$$

$$x \approx -1.69 \text{ or } x \approx 1.19$$

Step Two: Draw a sign chart on a number line to test which values for x satisfy the inequality.



Choose an x -value to the left of -1.69 and substitute into the inequality. We will try -2 .

$$2(-2)^2 + (-2) - 4 < 0$$

$$2 < 0 \text{ false}$$

Choose an x -value between -1.69 and 1.19 , substitute into the inequality. We will try 0 .

$$2(0)^2 + 0 - 4 < 0$$

$$-4 < 0 \text{ true}$$

Choose an x -value to the right of 1.19 and substitute into the inequality. We will try 2 .

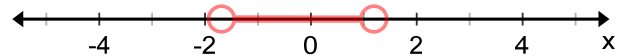
$$2(2)^2 + 2 - 4 < 0$$

$$6 < 0 \text{ false}$$

Step Three: Write the solution as a compound inequality or in set notation and graph.

$$-1.69 < x < 1.19$$

$$(-1.69, 1.19)$$



Ex 26: Solve $x^2 - x + 1 < 0$.

Step One: Solve the quadratic equation $x^2 - x + 1 < 0$ using any method.

A quick check of the discriminant tells us that there are no real solutions or no x -intercepts.

$$b^2 - 4ac = (-1)^2 - 4(1)(1) = -3$$

There are no roots to put on the sign chart. But this makes things easier! Because the line does not cross the x -axis, the function must be either:

- Always > 0 , or
- Always < 0

So, all we have to do is test one value (say $x = 0$) to see if the parabola is above or below the x -axis.

$(0)^2 - 0 + 1 = 1$ This means the function is always greater than zero. Since we are trying to find when it is less than zero, this never occurs and our answer is:

No Solution

If we were trying to solve: $x^2 - x + 1 > 0$, then our solution would be: **All Real Numbers**



Solving a Quadratic Inequality in Two Variables: First, graph the corresponding quadratic equation. If the inequality symbols are $<$ or $>$, graph a dashed parabola. If the symbols are \leq or \geq , graph a solid parabola. If the inequality symbol is greater than or greater than or equal to, shade up and if the symbol is less than or less than or equal to, shade down. A test point can also be used. Pick a point (choose $(0, 0)$ if possible – it's easy!) either inside or outside the parabola. If the test point makes the inequality true, shade that area (either inside or outside). If the test point makes the inequality false, shade the other area not tested.

Ex 27: Graph the quadratic inequality, $y \geq -x^2 + 2x + 1$.

Step One: Graph the parabola.

Find the vertex and axis of symmetry. Since a is negative, it opens down.

$$x = \frac{-2}{2(-1)} = 1 \quad \text{vertex: } (1, 2); \text{ axis of symmetry: } x = 1$$

Note: Since the symbol includes equality, we draw a solid parabola.

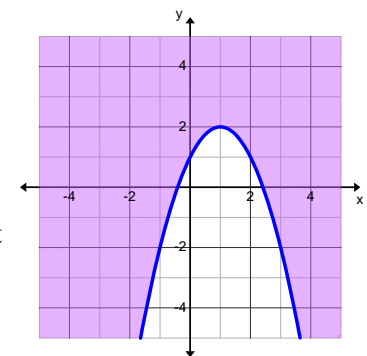
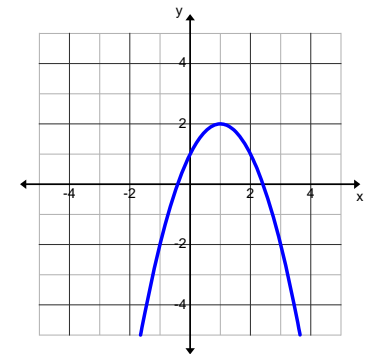
Step Two: Choose a test point inside the parabola and substitute it into the inequality.

$$y \geq -x^2 + 2x + 1$$

We will choose $(0, 0)$. $0 \geq -(0)^2 + 2(0) + 1$

$$0 \geq 3 \text{ false}$$

Step Three: If the test point makes the inequality true, shade inside the parabola. If it does not, shade outside the parabola. Also, note from above, that since we had a greater than symbol, it states to shade up.



Ex 28: Graph the quadratic inequality, $y > 2(x - 2)^2 - 3$.

Step One: Graph the parabola.

Find the vertex and axis of symmetry. Since a is positive, it opens up.

$$\text{vertex: } (2, -3); \text{ axis of symmetry: } x = 2$$

Note: Since the symbol does not include equality, we draw a dashed parabola.

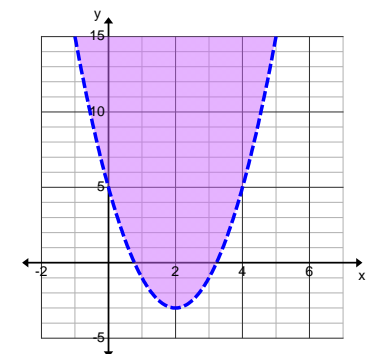
Step Two: Choose an easy test point and substitute it into the inequality.

$$y > 2(x - 2)^2 - 3$$

We will choose $(0, 0)$. $0 > 2(0 - 2)^2 - 3$

$$0 > 5 \text{ false}$$

Step Three: If the test point makes the inequality true, shade outside the parabola. If it does not, shade inside the parabola. Also, note from above, that since we had a greater than symbol, it states to shade up.





Graphing Calculator Activity

Ex 29: Graph the quadratic inequality $y > -x^2 + 9$ by hand and then check your graph on the graphing calculator using the “Inequalz” Application.

Step One: Graph the parabola. Make the parabola **dashed** if $<$ or $>$ and **solid** if \leq or \geq .

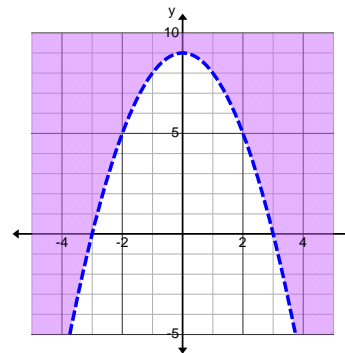
The vertex of the parabola is the point $(0, 9)$. Note: We draw a dashed parabola that opens down.

Step Two: Choose a test point inside (not on) the parabola and substitute it into the inequality.

$$\begin{array}{l} \text{We will choose } (0, 0). \\ y > -x^2 + 9 \\ 0 > (0)^2 + 9 \end{array} \quad \begin{array}{l} \\ \\ 0 > 9 \text{ false} \end{array}$$

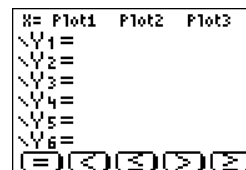
Step Three: If the test point makes the inequality true, shade inside the parabola. If it does not, shade outside the parabola.

We will shade outside the parabola.

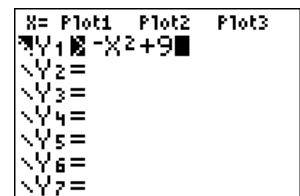


To check your graph, turn on the application by choosing “INEQUALZ” after pressing the APPS key.

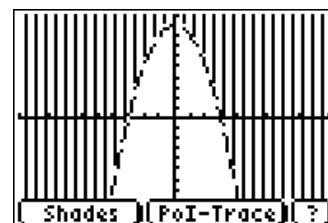
Press any key, and now your Y= screen should look like this:



Enter the function $y = -x^2 + 9$ into Y1. Then use the command (function) buttons along the bottom of your calculator screen to choose $>$. Note – In order to use the command buttons, you must first type the ALPHA key. So to choose $>$, we will press ALPHA TRACE.



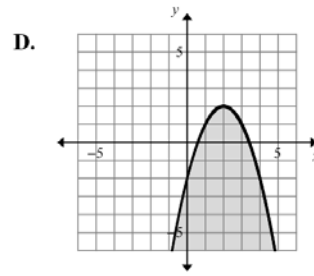
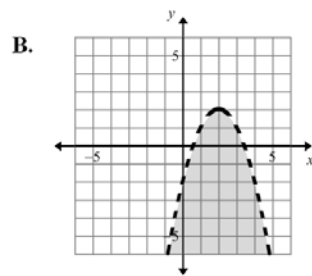
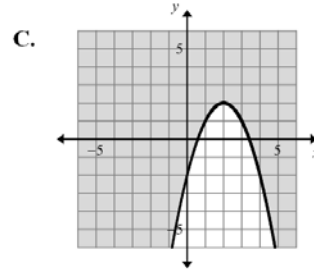
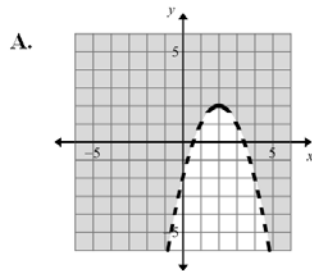
Graph the inequality. (For the graph shown, we used ZOOM STANDARD).





SAMPLE EXAM QUESTIONS

1. Which of the following graphs represents the quadratic inequality $y \leq -x^2 + 4x - 2$?



Ans: D

2. What is the solution of $x^2 + 7x - 8 > 0$?

A. $x = -8$ or $x = 1$

B. $x < -8$ or $x > 1$

C. $-8 < x < 1$

D. $x < -1$ or $x > 8$

Ans: B

3. What is the solution set of $y^2 - 2y \leq 3y + 14$?

A. $y \leq 7$

B. $y \leq -2$ or $y \geq 7$

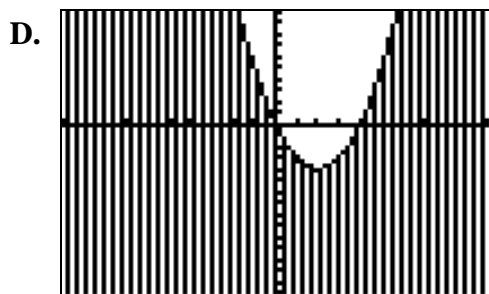
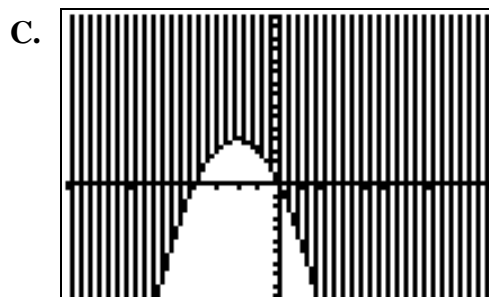
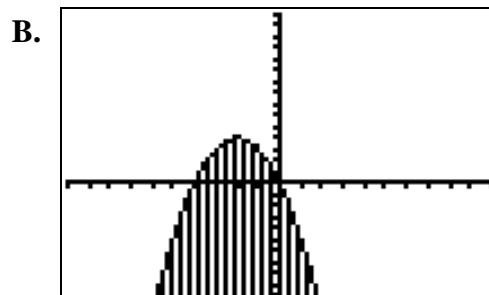
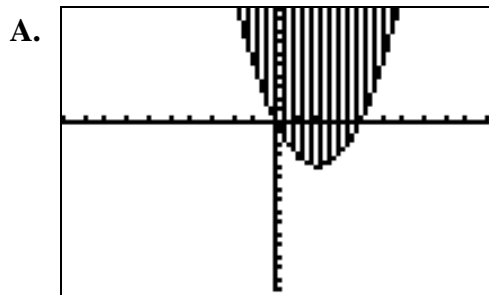
C. $-7 \leq y \leq 2$

D. $-2 \leq y \leq 7$

Ans: D



4. Which of the following screens from a graphing calculator represents $y \leq x^2 - 4x$? (Assume the scale on each graph is one unit per tick mark.)



Ans: D