

#	Question Type	Unit	Common Core State Standard(s)	CE obj	DOK Level	Key
1	МС		G.SRT.10	6.15H	1	С
2	МС		G.SRT.10	6.15H	1	С
3	MC		G.SRT.11	6.16H	1	А
4	MC		G.SRT.11	6.16H	2	А
5	MTF		G.SRT.10	6.16H	1	В
6	MTF		G.SRT.10	6.16H	1	А
7	MTF		G.SRT.10	6.16H	1	В
8	МС		G.SRT.11	6.16H	1	А
9	MTF		G.SRT.11	6.16H	1	А
10	MTF		G.SRT.11	6.16H	1	В
11	MTF		G.SRT.11	6.16H	1	В
12	CR		G.SRT.11	6.16H	2	-
13	CR		G.SRT.11	6.16H	2	-
14	МС		G.SRT.9	6.18H	2	А
15	МС		G.SRT.9	6.18H	1	С
16	МС	3	G.GMD.3	7.11	1	С
17	МС	5	G.C.5	7.2	1	С
18	МС	5	G.C.5	7.2	1	С
19	МС	5	G.C.5	7.2	1	В
20	МС	3	G.GMD.1	7.4	2	С
21	CR	3	G.GMD.1	7.4	3	
22	МС	5	G.C.5	7.5	2	С
23	МС	3	G.GMD.4	7.6	2	D
24	МС	3	G.GMD.3	7.8	1	D
25	МС	3	G.GMD.3	7.8	2	D
26	ER	3	G.GMD.3, G.MG.1	7.8	2	
27	МС	3	G.GMD.3	7.11	1	С
28	МС	3	G.GMD.3	7.11	1	В
29	МС	3	G.GMD.3	7.11	2	В
30	МС	3	G.GMD.3	7.11	2	D
31	МС	3	G.MG.1	7.11	2	А



32	MC	3	G.GMD.4	7.13	2	А
33	MC	3	G.GMD.3	7.14	2	С
34	MC	3	G.GMD.1	7.16	1	А
35	MC	3	G.GMD.4	7.17	2	А
36	CR	3	G.GMD.3, G.GMD.4	7.17	2	—
37	MC	4	G.GPE.7	7.18	2	С
38	CR	4	G.GPE.5, G.GPE.7	7.18	2	
39	CR	5	G.C.5, G.MG.1	7.18	2	
40	CR	5	G.C.5, G.MG.1	7.18	2	
41	CR	5	G.GPE.4	7.18	2	
42	MC	3	G.MG.2	7.19	2	С
43	MC	3	G.MG.2	7.19	2	С
44	CR	3	G.GMD.3, G.MG.1	7.19	3	
45	CR	3	G.GMD.3, G.MG.1, G.MG.2	7.19	2	
46	CR	5	G.GPE.7	7.19	2	
47	CR	5	G.C.1	8.1	3	
48	MC	5	G.C.1	8.1	2	С
49	MTF	5	G.C.2	8.2	1	А
50	MTF	5	G.C.2	8.3	1	В
51	MC	5	G.C.3	8.5	1	А
52	MC	5	G.C.3	8.6	1	В
53	MC	5	G.C.3	8.6	1	А
54	MC	5	G.C.3	8.6	1	В
55	CR	5	G.C.3	8.6	2	
56	MC	5	G.C.3	8.7	1	D
57	MC	5	G.C.3	8.7	1	D
58	CR	5	G.C.3	8.7	2	
59	MC	5	G.C.2	8.9	1	В
60	MC	5	G.C.2	8.9	1	В
61	MC	5	G.C.2	8.9	2	В
62	MC	5	G.C.2	8.9	1	В
63	MC	5	G.C.2	8.9	1	В
64	MC	5	G.C.2	8.9	1	D



65	MC	5	G.C.2	8.9	1	С
66	MTF	5	G.C.2	8.9	1	А
67	MTF	5	G.C.2	8.10	1	В
68	MTF	5	G.C.2	8.10	1	А
69	MTF	5	G.C.2	8.10	1	А
70	MTF	5	G.C.2	8.10	1	В
71	MTF	5	G.C.2	8,10	1	В
72	CR	5	G.C.2	8.10	2	
73	CR	5	G.GPE.5	8.10	2	
74	MC	5	G.C.2	8.11	1	А
75	MC	5	G.C.2	8.11	2	С
76	MTF	5	G.C.2	8.11	1	А
77	MTF	5	G.C.2	8.11	1	В
78	MC	5	G.C.2	8.11	1	D
79	MC	5	G.C.2	8.11	1	С
80	MC	5	G.C.2	8.11	1	А
81	MC	5	G.C.2	8.11	1	В
82	MC	5	G.C.2	8.11	1	А
83	MC	5	G.C.2	8.11	2	В
84	MTF	5	G.C.2	8.11	1	В
85	MTF	5	G.C.2	8.11	2	А
86	MTF	5	G.C.2	8.11	2	В
87	MTF	5	G.C.2	8.11	2	А
88	MC	5	G.C.2	8.11	2	D
89	MC	5	G.C.2	8.11	1	D
90	MC	5	G.C.2	8.11	2	В
91	MC	5	G.C.2	8.11	1	С
92	MC	5	G.C.2	8.11	1	D
93	MC	5	G.C.2	8.11	1	В
94	CR	5	G.C.4	8.12H	2	—
95	CR	5	G.GPE.1	9.1	3	
96	MC	5	G.GPE.1	9.2	2	D
97	MC	5	G.GPE.1	9.2	2	D



98	MC	5	G.GPE.1	9.2	1	D
99	MC	5	G.GPE.1	9.2	1	В
100	MC	5	G.GPE.1	9.3	2	А
101	MC	5	G.GPE.1	9.3	1	В
102	CR	4	G.GPE.1, G.GPE.4	9.4	3	_
103	MC	5	G.GPE.1	9.4	1	D
104	CR	5	G.C.5	9.5	2	
105	MC	4	G.GPE.2	9.7	1	В
106	CR	4	G.GPE.2, G.GPE.4	9.8	3	
107	CR	4	G.GPE.2	9.9	2	
108	MC	6	S.CP.1	10.4	1	А
109	CR	6	S.CP.1, S.CP.2, S.CP.6, S.CP.7	10.6	1	
110	MTF	6	S.CP.8	10.7H	2	А
111	MC	6	S.CP.8	10.7H	2	В
112	MC	6	S.CP.9	10.7H	2	А
113	MTF	6	S.MD.6	10.7H	2	В
114	MTF	6	S.MD.6	10.7H	2	А
115	MTF	6	S.MD.6	10.7H	2	В
116	MTF	6	S.MD.6	10.7H	2	А
117	MTF	6	S.CP.2	10.8	1	А
118	MTF	6	S.CP.1	10.9	1	А
119	MTF	6	S.CP.5	10.10	2	В
120	MTF	6	S.CP.3	10.11	2	В
121	MC	6	S.CP.3	10.11	2	D
122	MC	6	S.CP.5	10.12	1	А
123	MC	6	S.CP.7	10.13	1	С
124	CR	6	S.CP.3, S.CP.4, S.CP.5, S.CP.6	10.14	2	
125	MC	6	S.CP.5	10.15	2	А
126	MC	6	S.CP.7	10.15	1	С
127	MC	6	S.CP.4	10.17	2	А
128	CR	6	S.CP.5, S.CP.6, S.CP.8, S.MD.7	10.18	2	
129	MC	6	S.CP.9	10.19H	1	В
130	MC	6	S.CP.9	10.19H	1	D



12. (6.16) (HONORS) In triangle $\triangle ABC$, $m \angle B = 25^{\circ}$, a = 6.2, and b = 4. Find all possible measures of the remaining two angles and the third side.

This question assesses the student's ability to use the law of sines. Using the law of sines, $\frac{\sin A}{6.2} = \frac{\sin 25^{\circ}}{4}, \text{ so } A \approx 41^{\circ}. \quad C \approx 114^{\circ}. \quad \frac{\sin 114^{\circ}}{c} = \frac{\sin 25^{\circ}}{4} \text{ so, } c \approx 8.6.$

There is a second possible solution, as this is an ambiguous case: $A \approx 139^{\circ}$. $C \approx 16^{\circ}$. $\frac{\sin 16^{\circ}}{c} = \frac{\sin 25^{\circ}}{4}$, so $c \approx 2.6$.

13. (6.16) (HONORS) In triangle $\triangle ABC$, $m \angle B = 25^{\circ}$, a = 6.2, and c = 4.

- (a) Find all possible measures of the remaining two angles and the third side.
- (b) Find all possible areas of the triangle

This question assesses the student's ability to use the law of cosines, law of sines, and the general formula for the area of the triangle.

(a) Using the law of cosines, $b^2 = 4^2 + 6.2^2 - 2(4)(6.2)\cos 25^\circ$, so $b \approx 3.1$. Using the law of sines, $\frac{\sin C}{4} = \frac{\sin 25^\circ}{3.1}$, so $C \approx 33^\circ$, $A \approx 122^\circ$.

(b) The area of the triangle is $A = \frac{1}{2} (4) (6.2) \sin 25^\circ \approx 5.2 \text{ units}^2$

21. This question assesses the student's understanding of the use of limiting principles to derive formulas.

As the circle is cut into increasingly larger numbers of sectors and rearranged as shown, the shape of the resulting figure becomes more like a parallelogram, where the base has a length close to half the circle's circumference and the height is its radius. The area of the parallelogram is $(r)(\pi r) = \pi r^2$.





26. This question assesses the student's ability to apply volume formulas in modeling situations.

(a) The aquarium has a rectangular cross section 50 cm by 25 cm. The marbles have a diameter of 1 cm, so an array of 50×25 marbles will cover the bottom of the tank. To fill the marbles about 5 cm deep will take $50 \times 25 \times 5 = 6250$ marbles. That is 6250/500 = 13 (12.5) bags of marbles. (Note: if a student realizes that the marbles will probably not form a 3-D array $50 \times 25 \times 5$, but have "layers" of different sizes, say 50×25 for the first, 49×24 for the second, 50×25 for the third, etc. this is acceptable. Precisely how the marbles are "packed" needs only be reasonable.)

(b) The total volume of water and marbles in the tank is 50 cm \times 25 cm \times 27 cm = 33,750 cm³.

The volume of marbles in the tank is $6250 \times \frac{4}{3} \pi \left(\frac{1}{2} \text{ cm}\right)^3 \approx 3272.5 \text{ cm}^3$. So the volume of water is

 $30,477.5 \text{ cm}^3$ or about 30.5 liters. (Treating the marbles as a solid 50 cm \times 25 cm \times 5 cm block of glass is incorrect; there is space between the marbles. If a student makes the case that 6 layers of marbles is closer to 5 cm tall than 5 layers, this is an acceptable solution.)

36. This question assesses the student's ability to visualize threedimensional objects generated by rotations of two-dimensional objects and the application of volume formulas for solids.



B

(a) The resulting shape is a frustum of a cone with a cylinder cut out of the middle.

(b) The resulting volume would be the volume of a cone generated by the large triangle (ΔABC) minus the sum of the volume of the small cone (generated by ΔAED) and the inner cylinder.

$$\begin{cases} V_{\text{large cone}} = \frac{1}{3}\pi r^{2}h \\ V_{\text{large cone}} = \frac{1}{3}\pi (5)^{2} 5 \\ V_{\text{large cone}} = \frac{125\pi}{3} \end{cases} - \begin{cases} V_{\text{small cone}} = \frac{1}{3}\pi r^{2}h \\ V_{\text{small cone}} = \frac{1}{3}\pi (3)^{2} 3 + V_{\text{inner cylinder}} = \pi (3)^{2} (2) \\ V_{\text{small cone}} = \frac{27\pi}{3} \\ V_{\text{inner cylinder}} = 18 \end{cases} \end{cases}$$

$$Volume of solid = \frac{125\pi}{3} - \left(\frac{27\pi}{3} + 18\pi\right) = \frac{44\pi}{3}$$



38. This question assesses the student's to graph linear functions, construct parallel and/or perpendicular lines to given lines on the coordinate plane, and compute the perimeter of a figure using coordinate geometry.

- (a) See graph.
- (b) The line *m* has equation y = -2x + 20. See graph.
- (c) See graph.

(d) AO = 10. OB = 5. $BK = \sqrt{45} = 3\sqrt{5}$. $KA = \sqrt{80} = 4\sqrt{5}$. Perimeter = $15 + 7\sqrt{5}$.



39. This questions assesses the student's ability apply the measurement of arc lengths.

 $s = 2\pi r \frac{\theta}{360^{\circ}}$ = $2\pi (250000 \text{ miles}) \frac{0.50^{\circ}}{360^{\circ}}$ = 2181.661... miles

The approximate diameter of the moon is 2200 miles.



40. This questions assesses the student's ability apply the measurement of arc lengths.

The arc length from *A* to *B* is given as 500 miles. It is $\frac{7.2}{360}$ of the circumference of the earth. So, the approximate circumference of the earth is 500 miles $\times \frac{360^{\circ}}{72^{\circ}} = 25000$ miles.





41. This questions assesses the student's ability to apply the area of sectors of circles.

See diagram.

The dog's area is three parts: $\frac{3}{4}$ of a circle of radius 9 feet, $\frac{1}{4}$ of a circle of radius 5 feet, and $\frac{1}{4}$ of a circle of radius 3 feet.

$$A = \frac{3}{4}\pi (9 \text{ ft})^2 + \frac{1}{4}\pi (5 \text{ ft})^2 + \frac{1}{4}\pi (3 \text{ ft})^2$$
$$A = 69.25\pi \text{ ft}^2$$
$$A \approx 218 \text{ ft}^2$$



44. This question assesses the student's ability to apply volume formulas in modeling situations.

(a) The beaker is well approximated by a cylinder:

 $400 \text{ cm}^3 = \pi (4 \text{ cm})^2 h$

 $h \approx 8 \text{ cm}$

(b) The Erlenmeyer Flask well approximated by a cone:

$$400 \text{ cm}^3 = \frac{1}{3}\pi (4 \text{ cm})^2 h$$

 $h \approx 24 \text{ cm}$

(c) The Florence Flask is well approximated by a sphere:

$$400 \text{ cm}^3 = \frac{4}{3}\pi r^3$$
$$r \approx 4.6 \text{ cm}$$

This is larger than the beaker's radius, so it will NOT fit inside the beaker.

(d) As one moves upward from the bottom of the flask, the cross-sectional area decreases. Therefore, greater height (of the cone) is needed to hold an equivalent volume.

(e) The 200-ml mark must be roughly half way between the bottom of the flask and the 400-ml mark The 100-ml and 300-ml must be closer to the 200-ml mark than the bottom and 400-ml mark, respectively.





45. This question assesses the student's ability to apply volume formulas and the concept of density in modeling situations.

density =
$$\frac{\text{mass}}{\text{volume}}$$

(a)
$$1.2 \frac{\text{mg}}{\text{mm}^3} = \frac{0.06 \text{ mg}}{V}$$
$$V = \frac{0.06 \text{ mg}}{1.2 \frac{\text{mg}}{\text{mm}^3}}$$
$$V = 0.05 \text{ mm}^3$$



(b) The photoresist covers the circular wafer with a uniform thickness, essentially making it a cylinder with a height equal to the thickness:

$$V = \pi r^{2}h$$

0.05 mm³ = π (40 mm)² h
 $h = \frac{0.05 \text{ mm}^{3}}{\pi (40 \text{ mm})^{2}}$
 $h = 0.000009947... \text{ mm}$
 $h \approx 10^{-5} \text{ mm} \approx 10 \text{ nm}$

46. This questions assesses the student's ability to apply the area of triangles and quadrilaterals.

(a) Esmeralda county can be divided into a triangle (south of dotted line) and a quadrilateral.

The quadrilateral north of the dotted line has an area equal to the large triangle north of the dotted line minus the small shaded triangle added to the figure.

South triangle: base is about 70 miles and height is about 60 miles. Area equals

$$\frac{1}{2}$$
(60 mi)(70 mi) = 2100 mi².

North quadrilateral is the difference between the large triangle with base 70 miles and height about 36 miles, and the shaded triangle with base and height of 6 miles. Area equals

$$\frac{1}{2}(36 \text{ mi})(70 \text{ mi}) - \frac{1}{2}(6 \text{ mi})(6 \text{ mi}) = 1242 \text{ mi}^2$$
.

The area of Esmeralda County is approximately 3342 square miles.





(b) The population density of Esmeralda County is $\frac{775 \text{ people}}{3342 \text{ mi}^2} \approx 0.23$ people per square mile.

47. This question assesses the student's ability prove all circles are similar.

To prove circles A and B are similar, there must be a sequence of similarity

transformations that map circle A to circle B.

Translate circle A by vector \overrightarrow{AB} . Under the translation, A' = B.

Dilate circle A' by a factor $\frac{b}{a}$. Since circle A was the set of all points distance a from

point A, circle A' is the set of all points distance $a \cdot \frac{b}{a} = b$ from point A'. That is the same set of points as

circle B. The composition of the transformation and dilation, both similarity transformations, maps circle A to circle B. Thus, circles A and B are similar.

55. This question assesses the student's ability to construct a circle inside a triangle.

The student must (1) construct at least two angle bisectors, (2) locate the point of intersection of the bisectors (point O), (3) construct a perpendicular to one side through O, (4) locate the point of intersection of the side and the perpendicular line (point P), and (5) construct a circle centered at Owhose radius is OP.

A

58. This question assesses the student's ability to construct a circle circumscribed about a triangle.

The student must (1) construct at least two perpendicular bisectors, (2) locate the point of intersection of the bisectors (point O), and (3) construct a circle centered at O whose radius is OA (or OB or OC).





72. This question assesses the student's ability apply relationships between lines and angles in circles.

Triangle AHO is a right triangle.

 $AO = \sqrt{210^2 + 2000^2}$ $AO \approx 2011 \text{ km}$

The mountain's height, $AM = AO - 2000 \text{ km} \approx 11 \text{ km}$.

73. This question assesses the student's ability to graph a circle, apply properties of parts of circles, and use coordinate geometry to find equations of lines.

The radius shown has a slope of 1. Thus, the slope of the tangent line is -1. The equation of the tangent line is y-4=-1(x-4) or y=-x+8.

94. This question assesses the student's ability to construct a tangent to a circle from a point.

The student must (1) construct \overline{AO} , (2) construct the midpoint of \overline{AO} at M, (3) construct a circle of radius OM centered at M, (4) locate one intersection of circle M and circle O (point P), and construct \overrightarrow{AP} which is tangent to circle O.







210 km

4000 km



95. This questions assesses the student's understanding of the relationship between the distance formula, Pythagorean Theorem, and the equation of a circle.

Triangle *ABC* is a right triangle, so by the Pythagorean Theorem $AB^2 = AC^2 + BC^2$.

The distance *d* from *A* to *B* can be rewritten at $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

The circle centered at *A* is the set of all points a distance *d* from point *A*. Using *B* as an arbitrary point, the equation $d^{2} = (x_{2} - x_{1})^{2} + (y_{2} - y_{1})^{2}$ holds. If points *A* and *B* are described as having coordinates (*h*, *k*) and (*x*, *y*), respectively, and the circle is described as having a radius of *r*, then $r^{2} = (x - h)^{2} + (y - k)^{2}$.



102. This question assesses the student's to graph linear functions, construct parallel and/or perpendicular lines to given lines on the coordinate plane, and y

compute the perimeter of a figure using coordinate geometry.

- (a) See graph.
- (b) A is (-6, 0) and B(6, 0). See graph.

$$m_{\overline{AP}} = \frac{3.6 - 0}{4.8 - (-6)} = \frac{3.6}{10.8} = \frac{1}{3}$$

(c) $m_{\overline{BP}} = \frac{3.6 - 0}{4.8 - 6} = \frac{3.6}{-1.2} = -3$
 $(m_{\overline{AP}})(m_{\overline{BP}}) = (\frac{1}{3})(-3) = -1$

Since the product of the slopes of \overline{AP} and \overline{BP} equal -1, the segments are perpendicular and, therefore, $\angle APB$ is a right angle. Thus, $\triangle ABP$ is a right triangle.

104. This question assesses the student's ability to apply circles and their equations in the coordinate plane.

(a) The most obvious choice is to center the circle at the midpoint of (1, 3) and (-5, 9), or (-2, 6). The radius of the circle will be $\sqrt{(1-(-2))^2+(3-6)^2} = \sqrt{18}$. The equation is then $(x+2)^2+(y-6)^2 = 18$. Any circle that goes through the points (1, 3) and (-5, 9) is correct.

(b) Answers will vary depending on the equation, but obvious choices are y-intercepts

 $(0, 6+\sqrt{14})$ or $(0, 6-\sqrt{14})$, or the lattice points 90° from the given points i.e. (1, 9) or (-5, 3).

106. This question assesses the student's ability to apply the geometric definition of a parabolas on the coordinate plane.







The focus is *p* units from the vertex, therefore, points *A* and *B* have a *y*-coordinate equal to *p*.

 $p = \frac{1}{4p}x^{2}$ Thus, $4p^{2} = x^{2}$. The distance from A(-2p, 0) to B(2p, 0) is 4p. $\pm 2p = x$

107. This question assesses the student's ability to apply the geometric definition of a parabola on the coordinate plane.

- (a) See graph.
- (b) See graph. The focus is at (0, 1)
- (c) See graph. Point P is at (3, 2.25).

 $PF = \sqrt{(3-0)^2 + (2.25-1)^2}$ (d) = 3.25 PD = 2.25 - (-1). The distances are the same

(which is how the parabola is defined.)

109. This questions assesses the student's understanding of probability rules.

(a)
$$P(A \text{ and } B) = P(A) \cdot P(B) = (0.3)(0.4) = 0.12$$

- (b) P(A or B) = P(A) + (B) P(A and B) = 0.3 + 0.4 0.12 = 0.58
- (c) P(not A) = 1 P(A) = 1 0.3 = 0.7
- (d) Since A and B are independent, P(A|B) = P(A) = 0.3

124. This questions assesses the student's ability to compile data into a two way table and apply rules of probability, and understanding of the concept of independence.

(a) See table.

		Have siblings?			
		Yes	No	Total	
	Yes	15	6	21	
Own pets?	No	6	8	14	
F		i	i		



(b, i) $P(\text{siblings yes}) = \frac{21}{35} = \frac{3}{5}$ (b, ii) $P(\text{siblings yes or pets yes}) = \frac{21}{35} + \frac{21}{35} - \frac{15}{35} = \frac{27}{35}$ (b, iii) $P(\text{siblings yes } | \text{ pets yes}) = \frac{15}{21} = \frac{5}{7}$

(e) No, because

 $P(\text{siblings yes}) \neq P(\text{siblings yes} | \text{ pets yes})$

$$\frac{3}{5} \neq \frac{5}{7}$$

128. This questions assesses the student's ability to apply rules for probability to compound events and real-world situations.

This question can be solved using multiple methods,

including formulas, a tree diagram, or a table

(as shown). Values based on given information is **bold**.

(a) P(Classified as spam?) = 16.25%

(b) $P(\text{Spam} | \text{Classified spam}) = \frac{12\%}{16.25\%} \approx 74\%$

(c) P(Legitimate and classified Spam) + P(Spam and classified Legitimate) = 4.25% + 3% = 7.25%

(d) P(Classified Spam | Legitimate) = 5% (given)

 $P(\text{Not Classified Spam} | \text{Spam}) = \frac{3\%}{15\%} = 20\%$. It is 4 times more likely that a spam message will get through the filter than a legitimate message will be classified as spam.

		Message Classified as			
		Spam	Legitimate	Total	
	Legitimate	5% of 85% = 4.25%	80.75%	85%	
Message Type	Spam	80% of 15% = 12%	3%	15%	
	Total	16.25%	83.75%	100%	



teresponse mey