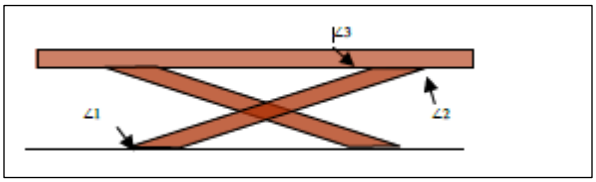
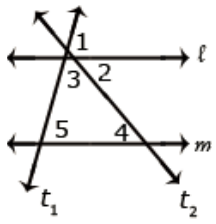
Unit 13: Angle Relationships in Triangles and Parallel Lines

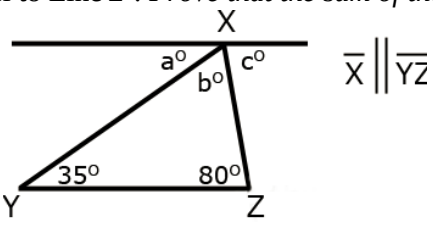
Clusters: Understand congruence and similarity using physical models, transparencies, or geometry software. Understand the connections between proportional relationships, lines, and linear equations.

Nevada Academic Content Standard	What does this standard mean that a student will know and be able to do? (adapted from North Carolina *8th Grade Standards, Unpacked Content*)
8.G.A.5 Use informal arguments to establish facts about the angle sum and exterior angles of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*	**8.G.5** Students use exploration and deductive reasoning to determine relationships that exist between the following: a) angle sums and exterior angle sums of triangles, b) angles created when parallel lines are cut by a transversal, and c) the angle-angle criterion for similarity of triangle. Students construct various triangles and find the measures of the interior and exterior angles. Students make conjectures about the relationship between the measure of an exterior angle and the other two angles of a triangle (the measure of an exterior angle of a triangle is equal to the sum of the measures of the other two interior angles), and the sum of the exterior angles (360º). Using these relationships, students use deductive reasoning to find the measure of missing angles. Students construct parallel lines and a transversal to examine the relationships between the created angles. Students recognize vertical angles, adjacent angles and supplementary angles from 7th grade and build on these relationships to identify other pairs of congruent angles. Using these relationships, students use deductive reasoning to find the measure of missing angles. *Example 1:* You are building a bench for a picnic table. The top of the bench will be parallel to the ground. If $m\angle 1 = 148°$, find $m\angle 2$ and $m\angle 3$. Explain your answer.  *Solution*: Angle 1 and angle 2 are alternate interior angles, giving angle 2 a measure of 148º. Angle 2 and angle 3 are supplementary. Angle 3 will have a measure of 32º so the $m\angle 2 + m\angle 3 = 180º$. *Example 2:* Show that $m\angle 3 + m\angle 4 + m\angle 5 = 180º$ if line l and m are parallel lines and t_1 and t_2 are transversals.  *Solution*: $m\angle 1 + m\angle 2 + m\angle 3 = 180°$ $\angle 5 \cong \angle 1$ (corresponding angles are congruent) therefore $\angle 1$ can be substituted for $\angle 5$.

$\angle 4 \cong \angle 2$ (alternate interior angles are congruent)
therefore $\angle 4$ can be substituted for $\angle 2$
Therefore $m\angle 3 + m\angle 4 + m\angle 5 = 180^o$.

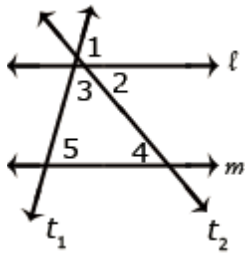
Students can informally conclude that the sum of the angles in a triangle is 180^o (the angle-sum theorem) by applying their understanding of lines and alternate interior angles.

Example 3:
In the figure below Line *X* is parallel to Line $\overline{Z}$. *Prove that the sum of the angles of a triangle is 180º.*



Solution: Measure of angle *a* is 35º because it alternates with the angle inside the triangle that measures 35º. Measure of angle *c* is 80º because it alternates with the angle inside the triangle that measures 80º. Because lines have a measure of 180º, and angles *a* + *b* + *c* form a straight line, then angle *b* must be 65 º → 180 – (35 + 80) = 65. Therefore, the sum of the angles of the triangle is 35º + 65 º + 80 º.

Example 4:
What is the measure of angle 5 if the measure of angle 2 is 45ºand the measure of angle 3 is 60º?



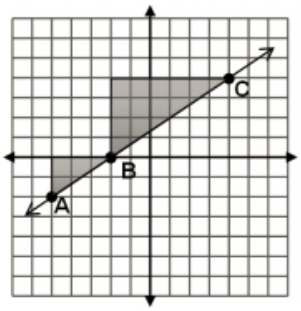
Solution: Angles 2 and 4 are alternate interior angles, therefore the measure of angle 4 is also 45º. The measure of angles 3, 4 and 5 must add to 180º. If the measures of angles 3 and 4 add to 105º, then the measure of angle 5 must be equal to 75º.

Students construct various triangles having line segments of different lengths but with two corresponding congruent angles. Comparing ratios of sides will produce a constant scale factor, meaning the triangles are similar. Students solve problems with similar triangles

8.EE.6 Triangles are similar when there is a constant rate of proportionality between them. Using a graph, students construct triangles between two points on a line and compare the sides to understand that the slope (ratio of rise to run) is the same between any two points on a line.

8.EE.B.6
Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; ~~*derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.*~~

Example:
The triangle between *A* and *B* has a vertical height of 2 and a horizontal length of 3. The triangle between *B* and *C* has a vertical height of 4 and a horizontal length of 6.
The simplified ratio of the vertical height to the horizontal length of both triangles is 2 to 3, which also represents a slope of $\frac{2}{3}$ for the line, indicating that the triangles are similar.



Approximate Time Frame: *1-2 weeks*

Terms:

- ✓ angle
- ✓ similar
- ✓ parallel
- ✓ transversal
- ✓ exterior angle
- ✓ interior angle
- ✓ angle-angle criterion
- ✓ vertical angles
- ✓ adjacent angles
- ✓ supplementary angles
- ✓ complementary angles

Resources

MGH – McGraw Hill, Glencoe Math (2015)
ML – McDougal Littell, Pre-Algebra Book; Larson, 2005
EX – Explorations in Core Math (Holt McDougal)
NY – *Engage* New York
LZ – Learn Zillion Website

	Suggested Topics for Lessons	***Suggested Resources***
8.G.A.5	**Angle Relationships in Triangles** **Angle Relationships in Parallel Lines** **Angle-Angle Similarity**	➢ MGH 5-1 Inquiry Lab*: Parallel Lines* (page 369) ➢ MGH 5-1 *Lines* (page 371) ➢ MGH 5-3 Inquiry Lab*: Triangles* (page 387) ➢ MGH 5-3 *Angles of Triangles* (page 389) ➢ MGH 5-4 *Polygons and Angles* (page 397) ➢ ML10.1 *Triangles* (page 511) ➢ ML13.1 *Angle Relationships* (page 709) ➢ ML13.2 *Angles and Parallel Lines* (page 716) ➢ ML13.3 *Angles and Polygons* (page 722) ➢ EX 4-3 *Similar Figures* (page 151) ➢ EX 5-1 *Angle Relationships* (page 175) ➢ EX 5-2 *Parallel and Perpendicular Lines* (page 179) ➢ EX 5-3 *Triangles* (page 185) ➢ NY Module 2, Topic C, Lessons 12: *Angles Associated with Parallel Lines* ➢ NY Module 2, Topic C, Lessons 13: *Angle Sum of a Triangle* ➢ NY Module 2, Topic C, Lessons 14: *More on the Angles of a Triangle* ➢ PBS Learning Media: *Corresponding Angles—School Yourself Geometry* ➢ PBS Learning Media: *Vertical Angles—School Yourself Geometry* ➢ PBS Learning Media: *Finding Unknown Angles—Math Shorts* ➢ Khan Academy Videos: *Angles between intersecting lines* ➢ Khan Academy Skills Practice: *Questions—8.G.A.5* ➢ LZ Lesson Set: *Understand angle sum, exterior angles, transversal, angle-angle* ➢ LZ Lesson Plan: *Identify angle relationships of parallel lines cut by transversals*
8.EE.B.6	**Slope and Similar Triangles**	➢ MGH 7-6 *Slope and Similar Triangles* (page 561) ➢ LZ Lesson: *Derive y=mx using similar triangles* ➢ LZ Lesson*: Derive y=mx+b using similar triangles* ➢ PBS Learning Media*: Math Shorts Video: Understanding Slope with Similar Triangles* ➢ Khan Academy: *Similar triangles to prove that the slope is constant for a line*