Got Math?

A Newsletter from the Secondary Mathematics Team

Southern Nevada Regional Professional Development Program

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Sequences and Polynomial Expressions Part II

As an extension to the sequences generated by linear expressions, some of the patterns we study in secondary math are sequences generated by higher degree polynomials. In linear expressions we notice that the first difference is constant, and in quadratic expressions we find that the second difference is constant, and so on. ***First difference*** refers to the differences we find the first time we subtract previous terms of the sequence, ***second difference*** refers to the differences we find the second time we subtract previous first differences of the terms of the sequence. See the example below.

Consider the sequence 3, 6, 13, 24, 39

Specific case

Original sequence	3		6		13		24		39
First difference		3		7		11		15	
Second difference			4		4		4		4

If the pattern works, the sequence can be generated by a quadratic polynomial, so let's again substitute 1, 2, 3, 4, … for x in the general form of a quadratic expression, Ax^2+Bx+C.

This gives us the general case listed below.

Ax^2+Bx+C

General case

$x=1$, $x=2$, $x=3$, $x=4$

Original sequence	$A+B+C$		$4A+2B+C$		$9A+3B+C$		$16A+4B+C$
First difference		$3A+B$		$5A+B$		$7A+B$	
Second difference			$2A$		$2A$		

The second difference is constant and is equal to $2A$ in the general case and 4 in the specific case. Should they (4 and $2A$) be equal? Yes, so $A = 2$.

Now we see a system of equations that is fairly easy to solve.

The equations

$A+B+C = 3$

$3A+B = 3$

$2A = 4$

The solution: $2x^2 - 3x + 4$

Start with $2A = 4$ ⟹ $A = 2$

$3A+B = 3$ ⟹ $B = -3$

$A+B+C = 3$ ⟹ $C = 4$

The expression $2x^2 - 3x + 4$ generates the sequence **3, 6, 13, 24, 39, …**

Note: Any term can be found by simple substitution into this expression. What is the 10th term?

Math Resources

www.rpdp.net

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Sequences and Polynomial Expressions Part II — continued

Practice: Let's try it with the sequence 0, 0, 2, 6, 12…. First, check for first and second differences. Can you find the quadratic expression that generates this sequence?

1	0		
		0	
2	0		2
		2	
3	2		2
		4	
4	6		2
		6	
5	12		

$Ax^2 + Bx + C$

$2A = 2,\ \text{so } A = 1$

$3A + B = 0,\ \text{so } B = -3$

$A + B + C = 0,\ \text{so } C = 2$

$\therefore \underline{x^2 - 3x + 2}$

Solution: The expression that generates this sequence is $x^2 - 3x + 2$.

Check:

On Your Own: Find the expression that generates the sequence −3, 9, 25, 45, . . .

Summary: If we know the terms in a sequence with constant second differences, the sequence could be generated by the general quadratic expression Ax^2+Bx+C, where $2A$ is the common second difference. $3A + B$ is equal to the first value of the first differences of the sequence, and can be used to find B. $A + B + C$ is the first term of the sequence and can then be used to determine C in the expression. With these three equations it is easy to back-solve them to evaluate the coefficients of the expression.

Challenge 1: Try putting the lists for the On Your Own problem above into L_1 and L_2 in the graphing calculator. Then have the calculator calculate the quadratic regression. Do you find that the graph of the equation goes through all of the points in the list? Does the table of values duplicate the terms of the sequence.

Challenge 2: Find an expression generator for 3, 7, 19, 45, 91, 163, …..

Hint: This sequence is not arithmetic. It does not have common *first differences*. It is not quadratic; it does not have common *second differences*. But it may have common *third differences*. Try to find the patterns.

Sequences Generated by Quadratic Expressions

Problems 1-4: Write the expression that generates the given sequence.

1. 0, 4, 12, 24, ...

2. $-10, -7, -10 -19, -34, \ldots$

3. 6, 11, 18, 27, 38, ...

4. 3, 5, 11, 21, . . .

This form may be helpful in finding the function given a quadratic sequence. For example, in problem #3, the table might look like this:

1	6	5	2
2	11	7	2
3	18	9	2
4	27	11	
5	38		

$Ax^2 + Bx + C$

$2A = 2 \rightarrow\ A = 1$

$3A + B = 5 \rightarrow B = 2$

$A + B + C = 6 \rightarrow C = 3$

$\therefore \underline{x^2 + 2x + 3}$

Problems 5-6: Write the first four terms of a sequence that is generated by the quadratic expression.

5. $4x^2 + 7x - 5$

6. $-3x^2 + x + 2$

Summarize:

Solutions:
1. $2x^2 - 2x$
2. $-3x^2 + 12x - 19$
3. $x^2 + 2x + 3$
4. $2x^2 - 4x + 5$
5. 6, 25, 52, 87, . . .
6. 0, −8, −22, −42, . . .