Got Math?

A Newsletter from the Secondary Mathematics Team

Southern Nevada Regional Professional Development Program

VOLUME II, ISSUE I

Sequences and Polynomial Expressions

Some of the patterns we study in secondary math are arithmetic sequences. Each number in the sequence is called a ***term***. Each successive term of an arithmetic sequence is separated by a ***common difference***. For example, if you started with the first term of 5 then added 3, you get 8 for the second term. Then add 8 + 3 to get the next term, and so on. The arithmetic sequence 5, 8, 11, 14… is generated, and the common difference is 3.

5		8		11		14
	3		3		3	

In these problems we notice that the first difference is constant, and in this case is equal to 3. (***First difference*** refers to the differences we find the first time we subtract previous terms of the sequence, $(a_n - a_{n-1})$.

Note: 3 is a rate of change.

Look at it another way. We could also generate the same sequence by using the expression, $3x + 2$, and substituting 1, 2, 3, … for x.

$3(1)+2=5 \qquad 3(2)+2=8 \qquad 3(3)+2=11$

5		8		11		14
	3		3		3	

Specific case

Now we get to the interesting part. If we did not know the linear expression that generated this sequence, could we find it? Since the first differences between the terms remain constant, this is a linear or arithmetic sequence that could generally be written as $Ax+B$. Let's substitute 1, 2, 3, … for x to generate the general-case sequence.

$$\begin{cases} A(1)+B=A+B \\ A(2)+B=2A+B \\ A(3)+B=3A+B \\ A(4)+B=4A+B \end{cases}$$

$A+B$		$2A+B$		$3A+B$		$4A+B$
	A		A		A	

General case

Note: The first difference is constant and is equal to A.

Aha! If the first difference is 3 in the specific case and the first difference is A in the general case, should they (3 and A) be equal? Yes, so $A = 3$. Does it make sense that the rate of change is the coefficient of x in the linear equation $Ax + B$? What does the A represent in a linear equation?

If $A = 3$, that means that the first term, $A + B = 5$. Solving for B, $3 + B = 5$, so $B = 2$.

Solution: The sequence was generated by the expression $3x + 2$.

Do you see the pattern? Can you describe it and write it?

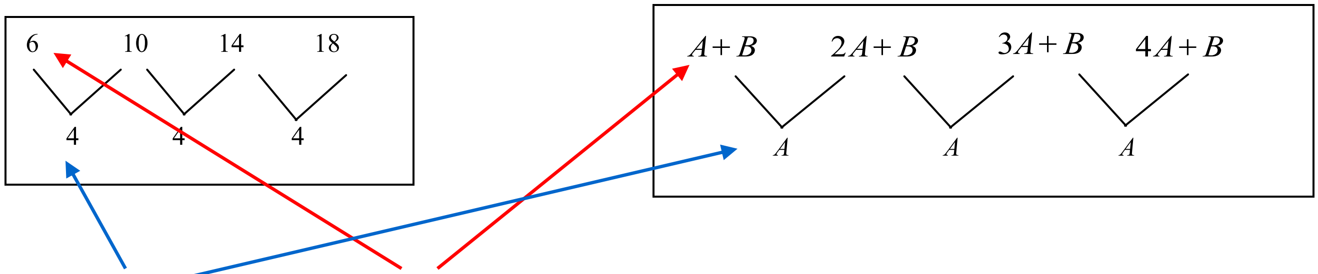
Math Resources

www.rpdp.net

This issue:

Sequences and Polynomial Expressions— continued

Practice: Let's try it with a sequence 6, 10, 14, 18…. First, is it a linear (arithmetic) sequence? Yes. Can you find the linear expression that generates this sequence?



$A = 4$ $\quad A + B = 6$ $\quad B = 2$ $\quad$ (Why is $B = 2$?)

Solution: The linear expression that generates this arithmetic sequence is $4x + 2$.

Check: $\begin{cases} 4x+2 \\ 4(1)+2=6 \\ 4(2)+2=10 \\ 4(3)+2=14 \end{cases}$

Variations: What would happen if we were not given the first term of the sequence, but did know other terms such as $a_3, a_4, a_5, a_6, \ldots$?

In the previous problem, A would still be 4, but we would use the third term , $3A + B$, which is 14, to solve and find the linear expression. $\begin{cases} 3A+B=14 \\ 3(4)+B=14 \\ B=2 \end{cases}$.

We would still find the linear expression for this sequence to be $4x + 2$.

On your own: Find the linear expression that generates the sequence 9, 13, 17, 21, . . .

Summary: If we know the numbers in a sequence with constant first differences, the sequence could be generated by a linear expression, $Ax + B$, where A is the common difference, and $A + B$ is the first term of the sequence. $A + B$ can then be used to determine B in the linear expression.

Next issue: Find an expression generator for 6, 17, 34, 57, 86,…..

Hint: This sequence is not arithmetic. It does not have a common *first difference*, but does have a common ***second difference***.

Sequences and Polynomial Expressions - Practice Problems (Variations)

Sequences Generated by Linear Expressions

Problems 1-4: Write the linear expression that generates the given sequence.

1. $0, 11, 22, 33...$

2. $-10, -7, -4, -1, 2, ...$

3. $14, 7, 0, -7, ...$

4. 3, 4, 5, 6, . . .

Problems 5-6: Write the sequence that is generated by the linear expression.

5. $4x + 7$

6. $-3x - 10$

Summarize:

Solutions:
1. 11x - 11
2. 3x -13
3. -7x + 21
4. $x + 2$
5. 11, 15, 19, 23, . . .
6. -13, -16, -19, -22, . . .

This instruction may be helpful in finding the function given a linear sequence. For example, in problem #2, the table might look like this:

1	-10	
		3
2	-7	
		3
3	-4	
		3
4	-1	
		3
5	2	

$y = Ax + B$
$y = 3x + B$
Then substitute any ordered pair to find B.
$y = 3x - 13$