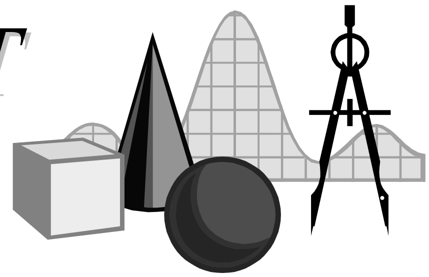
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



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This issue of *Take It to the MAT* addresses common methods to compare fractions.

The first and most common method of comparing fractions is to write each with a common denominator. For example, when comparing $\frac{5}{16}$ and $\frac{9}{28}$ we would often rewrite each fraction with the least common denominator of 112; $\frac{5}{16}\cdot\frac{7}{7}=\frac{35}{112}$ and $\frac{9}{28}\cdot\frac{4}{4}=\frac{36}{112}$. Since $\frac{35}{112}<\frac{36}{112}$, then $\frac{5}{16}<\frac{9}{28}$.

Finding the least common denominator (LCD) is often time consuming, so an alternative is to choose a common denominator that is not the LCD. The simplest is the product of the denominators, in our example $16\cdot 28=448$. Comparing the fractions above using this approach we get $\frac{5}{16}\cdot\frac{28}{28}=\frac{140}{448}$ and $\frac{9}{28}\cdot\frac{16}{16}=\frac{144}{448}$. Since $\frac{140}{448}<\frac{144}{448}$, then $\frac{5}{16}<\frac{9}{28}$.

A third recipe for comparing fractions is the "cross product" technique. This method calls for multiplying each fraction's denominator by the other fraction's numerator. When the products are compared, the numerator which was a factor of the larger product is the numerator of the larger fraction. Looking at our example again asking $\frac{5}{16}\boxed{?}\frac{9}{28}$, we multiply 5 by 28 and 9 by 16, often diagrammed as $\frac{5}{16}\times\frac{9}{28}$. Multiplying, $5\cdot 28=140$ and $9\cdot 16=144$. Since 144 is the larger product and the numerator 9 was one of the factors of 144, the fraction $\frac{9}{28}$ must be larger than $\frac{5}{16}$, or $\frac{5}{16}<\frac{9}{28}$. Unfortunately, we often give students this comparison algorithm without explaining why it works. The *why* can be illustrated in two ways.

First, look at the example. Multiplying the numerator 5 by the denominator 28 yields 140, the same numerator when we found the simple common denominator 448. Similarly, multiplying the numerator 9 by the denominator 16 yields 144, again the numerator we found using the second method. This is no coincidence.

A second way we can examine the procedure is algebraically. Consider that two fractions $\frac{a}{b}$ and $\frac{c}{d}$ exist (where a, b, c, d are non-negative real numbers and $b, d \neq 0$) and that $\frac{a}{b}<\frac{c}{d}$. Applying our technique of comparing with a simple common denominator, which is bd, we have $\frac{a}{b}\cdot\frac{d}{d}<\frac{c}{d}\cdot\frac{b}{b}$ or $\frac{ad}{bd}<\frac{bc}{bd}$. (Look at those numerators carefully; they are important.) The "cross product" method, multiplying a by d and b by c, would yield $ad<bc$. Hey, ad and bc were the numerators when we used the common denominator procedure!

That's why the "cross product" technique works—we are simply comparing the numerators once a common denominator is found!