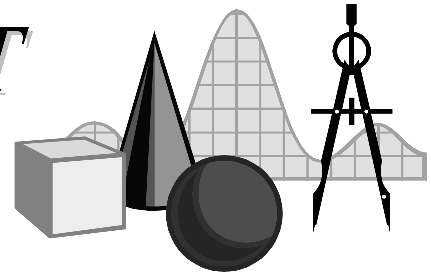
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Math Audit Team
Regional Professional Development Program
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In the last edition of *Take It to the MAT* we discussed the concept of division and how it relates to zero. Division by a given divisor was approached as breaking the dividend into groups *of* the divisor, the quotient being the resulting number of groups. For example, $6 \div 2$ was developed as taking six objects and diving them into groups of 2, the outcome being three groups. In this issue, we will look at division by a fraction in a similar light.

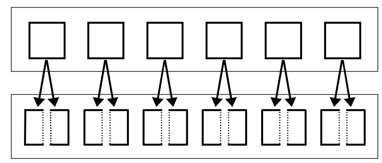

Figure 1

What would one think if asked to find $6 \div \frac{1}{2}$? Using the logic previously described, one would divide the six objects into *groups of* $\frac{1}{2}$. See *Figure 1*. Six objects when divided into groups of one-half yields twelve groups of $\frac{1}{2}$, so $6 \div \frac{1}{2} = 12$.

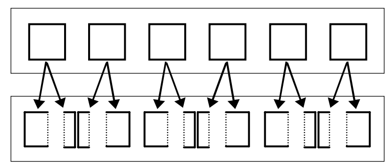

Figure 2

Now consider *Figure 2* that illustrates $6 \div \frac{2}{3}$. Each individual unit can have two-thirds of it "sliced off" leaving one-third of a unit left. A pair of those one-thirds can be then combined into another group of two-thirds. This leaves us with nine groups of $\frac{2}{3}$, $6 \div \frac{2}{3} = 9$.

When we teach students to divide by a fraction, we often spout out that tired, old phrase, "Yours is not to question why, just invert and multiply." In other words, "multiply by the *reciprocal* of the divisor." One byproduct of this is that students go through life either remembering the rule but not understanding why it works, or forgetting the rule because they never understood the concept of division in the first place.

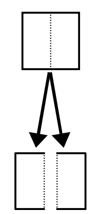

Figure 3

The algorithm for division does have its place. Let's look carefully at the rule about *inverting and multiplying*. Start with the simple division problem, $1 \div \frac{1}{2}$, one unit divided into groups of one-half unit. A whole divided into halves gives two groups of $\frac{1}{2}$. (See *Figure 3*.) As a matter of fact, every whole unit will yield two groups when divided into halves. Thus, dividing by one-half is equivalent to multiplying by two. We knew that though; $1 \div \frac{1}{2}$ by our algorithm equals $1 \times \frac{2}{1}$, or 1×2.

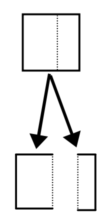

Figure 4

In the case of dividing by two-thirds, use the same logic. If you have $1 \div \frac{2}{3}$, a whole unit divided into groups of two-thirds of a unit, you end up with one group of two-thirds and one-third left over. Since one-third is half of two-thirds, the result is one and one-half groups. (See *Figure 4*.) The quotient tells how many groups of the divisor are made from the dividend. For every whole unit, we get $1\frac{1}{2}$ groups of $\frac{2}{3}$. Going back to the arithmetic, $1 \div \frac{2}{3}$ is the same as $1 \times \frac{3}{2}$, which equals $\frac{3}{2}$ or $1\frac{1}{2}$.

Once our students understand the concept of division, division by fractions becomes less complicated and easier to remember. If one forgets the algorithm, the concept can aid in its redefinition, if needed. Your challenge—consider it homework(?)—is to examine the topic of this issue further when the quotient of a number divided by a fraction is not a whole number.