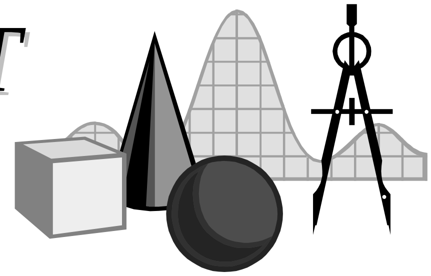
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Math Audit Team
Regional Professional Development Program
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In the February 14, 2000 issue of *Take It to the MAT*, multiplication of decimals was addressed, particularly the rules regarding the placement of the decimal point in the product. In this edition, we will study the rules for "movement" of decimal points when dividing decimal numbers using the traditional division algorithm, and *why* we do it.

First of all, we know that when we use the division algorithm, the decimal point in the quotient is placed directly above the decimal point in the dividend. For example, $5\overline{)1.5}$. **But, that's actually the *last* thing we do before the arithmetic.**

The first action taken is to look and see if there are any decimal places in the divisor. For example: Given $0.15 \div 0.5$, we typically rewrite this as $.5\overline{).15}$, then we notice that the divisor has one decimal place. What do we do? *Move the decimal point in the divisor as many places to the right as necessary to make it a whole number. Then, move the decimal point in the dividend an equal number of places to the right.* So, in $.5\overline{).15}$ we apply the rule—$.5\overline{).15}$—and end up with $5\overline{)1.5}$. The big question is, "Why can we do this?" Here's why.

Consider writing the division problem as a fraction, that is, $0.15 \div 0.5 = \frac{0.15}{0.5}$. You might be asking whether it is appropriate to write decimal numbers in the numerator and denominator of a fraction. It's perfectly fine to do so and as we will see, quite helpful. By now students should understand the connection between the ratio of two numbers—a fraction—and division. Anyway, we now have this fraction $\frac{0.15}{0.5}$; what do we do with it? When we did division by whole numbers, we just went ahead and worked the division algorithm. How can we change the divisor, or the denominator in the case of the fraction, into a whole number? Multiply by a power of 10; $\frac{0.15}{0.5} = \frac{0.15}{0.5} \times \frac{10}{10} = \frac{1.5}{5}$ which we can then rewrite as $5\overline{)1.5}$. Now, we're dividing by a whole number—which is what we like—and since $\frac{0.15}{0.5}$ is equivalent to $\frac{1.5}{5}$, the quotient for $.5\overline{).15}$ and $5\overline{)1.5}$ will be the same. Notice we multiplied both the numerator and denominator by 10; that is, we multiplied by $\frac{10}{10} = 1$, not 10. This is equivalent to moving the decimal point one place to the right in both the numerator and denominator, or alternately the dividend and divisor.

Again, division of decimals is a topic where we teach the algorithm, but do not impress the reasons behind it. Students in middle school routinely can evaluate $15695 \div 365$. When the question of dividing $15.695 \div 3.65$ comes up, we casually add the "decimal shift" step to the algorithm but neglect to say *why*. If students can relate the process to that of equivalent fractions, they will be less likely to forget what to move, when to move it, and why.