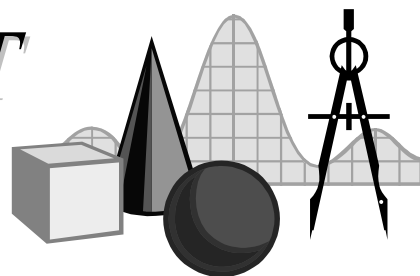# TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
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Several courses in high school include the study of the ways sets of objects can be arranged, specifically, combinations and permutations. We usually provide our students formulas to calculate the number of ways a set of things can be ordered or grouped, but don't often explore how the rules are obtained. In this issue of *Take It to the MAT*, we will examine permutations and see that their formula is not magical, but can be easily derived.

A *permutation* is *an ordered arrangement of a set (or subset) of things*. The two big ideas here are that we must begin with a set of things, from which we may choose all or part, and that order is important in the selection of those things. These sets of "things" need not be concrete objects; they may be destinations for airplanes or chores to do around the house.

For example, let's say we have three chores to do: take out the trash, vacuum the carpets, and wash the dishes. (For simplicity we shall refer to them as T, V, and W.) In what order shall we do our chores? In how many *different* orders can we do our chores? Since our chores can be thought of as the set {T, V, W}, each unique order is a permutation.

The list of permutations of our set of chores is TVW, TWV, VTW, VWT, WTV, and WVT. There are six different orders of our jobs. Listing them is problematic, however, as it is easy to miss one if we are not systematic about making the list. An alternate method to determine the number of permutations is by making a tree diagram. As we move down the sets of branches to their ends, we draw the same conclusion that there are six permutations.
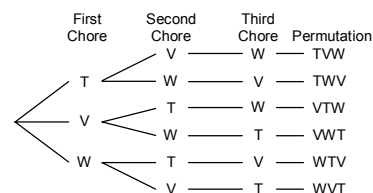
The fact that there are six permutations should not come as a surprise; counting principles require it. *If Event A can occur in a ways and Event B can occur in b ways, then the two events occur together in a · b ways*. In our example there are three ways to choose the first chore, two for the second chore once the first is chosen, and one remains for the last chore. That's $3 \cdot 2 \cdot 1 = 6$ different permutations.

Now imagine a baseball coach trying to figure out how many different batting orders could be made from the 9-player starting lineup. There are nine choices for the first spot, eight for the second, and so on until only one choice remains for slot nine. Applying the counting principle, there are $9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362,800$ permutations. We could shorten this to 9! = 362,800.

What if that same baseball coach had to decide in what order 5 of the team's 15 players take batting practice? There would be $15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 = 360,360$ permutations. We can't shorten the left side of the equation to 15! like we did before. But consider changing the expression to something where we could use factorial notation: $15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdot \dfrac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \dfrac{15!}{10!}$. That's much nicer.

Lastly, consider a coach having *n* players and selecting an order of *r* of them to take batting practice. The coach could create $n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-(r-1))$ permutations. Re-expressed as in the last example,

$$n \cdot (n-1) \cdot (n-2) \cdot \ldots \cdot (n-(r-1)) \cdot \dfrac{(n-r) \cdot \ldots \cdot 2 \cdot 1}{(n-r) \cdot \ldots \cdot 2 \cdot 1} = \dfrac{n!}{(n-r)!}.$$

We now have the general formula with which we are so familiar, that is, $_nP_r = P(n,r) = \dfrac{n!}{(n-r)!}$.