Take It to the MAT

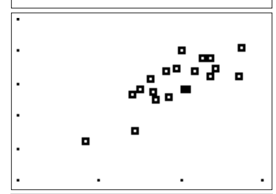
A Newsletter Addressing the Finer Points of Mathematics Instruction

Regional Professional Development Program
October 29, 2001 — High School Edition

When teaching linear functions, it is important to discuss the fitting of lines to real-world data. In courses such as Algebra 1, it is enough to let students make a scatter plot on graph paper then simply draw a line that seems to fit the data by "eyeballing" it. Finding the *median-median line* is also a simple technique that Algebra 1 students could use. Later, in Algebra 2, Precalculus, and Statistics, students use graphing calculators to view plots of the data and find the *least squares regression line*, or "line of best fit" as it is sometimes called. In any case, there are key concepts that are frequently glossed over when fitting lines to data. In this issue of *Take It to the MAT,* we'll look two of those big ideas: the *meaning* of slope and intercept.

Let's start with an example similar to one we may do with our students, graphing weight vs. height. Shown at right is the TI-83 scatter plot (and its window parameters) of 20 randomly selected adult males. The horizontal axis is height in inches and the vertical axis is weight in pounds. There *appears* to be a positive linear relationship between height and weight.

```
WINDOW
 Xmin=60
 Xmax=75
 Xscl=5
 Ymin=100
 Ymax=200
 Yscl=20
 Xres=1
```

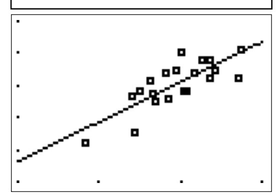


After students look at a plot of the data—they must do this *first* to have some idea what model they will use—they find the equation of line through some mechanism, in this case the LinReg function of the TI-83. Our linear model is

Weight ≈ –190 + 5.0(*Height*).

Note that the equation is written using words for the variables, not single letters. This is good practice and students should be as comfortable with this as writing $w \approx -190 + 5.0h$ or $y \approx -190 + 5.0x$. The line is shown with the scatter plot in the last frame.

```
LinReg
 y=a+bx
 a=-189.9372124
 b=4.973565276
```



The big idea here is understanding what those values of 190 and 5.0 mean. First, we recognize 5.0 as the *slope* of the line. But the slope isn't 5.0! It's really $5.0\frac{\text{pounds}}{\text{inch}}$. If we pick any two points and find the slope of the line through them, the vertical and horizontal changes are in the units of the vertical and horizontal axes, respectively. Students should understand that the slope of a line might have units, and that the slope has meaning. Essentially, a slope of $5.0\frac{\text{pounds}}{\text{inch}}$ says that if height is increased by one inch, then, on average, weight will increase by 5.0 pounds. *Slope shows the change in the dependent variable if the independent variable increases by one unit.*

What about the intercept of –190, which is actually 190 *pounds*? What does that intercept really mean? *The vertical-axis intercept is the value of the dependent variable when the independent variable is zero.* In our example, that means an adult male 0 inches tall would weigh –190 pounds. That doesn't make sense, though. While the slope always has meaning in the context of the situation, the intercept may not. No adult male, or any person for that matter, could be zero inches tall. Such a case is nonsensical and students should be able to say as much.

Situations exist where the intercept may have meaning. One case would be to plot natural gas usage versus average temperature (in °C) for North Dakota. There, an average temperature of 0°C certainly is plausible, unlike our heightless male. One must remember, regardless of the situation, that making predictions outside the range of the data—in our case predicting weight outside the range of 64–74 inches for height—is not recommended as the model derived may not hold beyond those values.

Thus, after all of this, our actual model is *Weight* ≈ –190 **pounds** + $5.0\frac{\textbf{pounds}}{\textbf{inch}}$(*Height*).