TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
Regional Professional Development Program
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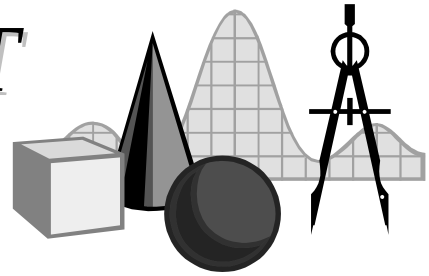


Take a good look at the graph in *Figure 1*. What do you see?

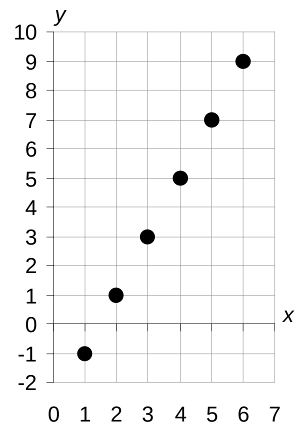


Figure 1

If you said, "A line," you're partially correct. If you said, "A linear function defined over the domain of the positive integers," you are right on. But did you think of is as an *arithmetic sequence*, where x represents the term number and y represents the terms themselves? That's OK, few people do. But the link between *arithmetic sequences* and *linear equations* is something that should not be overlooked. In this issue of *Take It to the MAT*, we will focus on this connection.

The function in Figure 1 can be defined as $y = 2x - 3$, $x \in$ {positive integers}. A simple x-y table that we might ask students to make would look like:

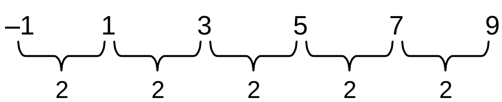
x	1	2	3	4	5	6
y	–1	1	3	5	7	9

If we were to change the notation to that used when defining sequences, $t_n = 2n - 3$, we would end up with the same table. We need not define n to be an element of the positive integers, since that is a constituent in the definition of *sequence*.

n	1	2	3	4	5	6
t_n	–1	1	3	5	7	9

Look at the similarity between the right-hand sides of the two equations: $2x - 3$ versus $2n - 3$. Plainly, an *arithmetic sequence* appears to be a *linear function*.

Think about some of the techniques that we teach students to use with sequences. First, we tell them to find the differences between consecutive terms. If it's constant, then we say the sequence is arithmetic—the terms have a *common difference*. In this case, the common difference is 2.



Next, we may ask them to look at the first term and apply the rule $t_n = t_1 + (n - 1)d$, where t_1 is the first term, d is the common difference, and n is the term number. In this case, $t_n = -1 + (n - 1)2$, which simplifies to $t_n = 2n - 3$. An additional and conceptually meaningful way of approaching this would be to ask students, "Looking at the sequence's pattern, what would term zero be, if one existed?" Working backward, using the common difference of 2, the zeroth term is –3.

But what does the common difference really mean? For every increase of one by n, t_n increases by two. What else behaves like that? That's right—slope of a line! Slope means that for every increase in x by one, the value of y changes by the slope. The connection between the "zeroth" term of a sequence and the y-intercept of the line is also clear. If x is zero, the value of y is the y-intercept. These connections are subtle and deserve full exploration by students.