Take It to the MAT

A Newsletter Addressing the Finer Points of Mathematics Instruction

Regional Professional Development Program
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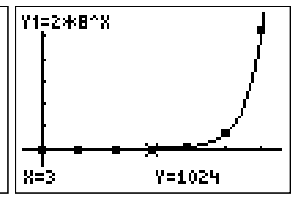
Exponential functions are important in mathematics because of their use in modeling growth. Logarithms are equally as significant due to their use in finding solutions of exponential equations. Logarithms have another purpose: to make our lives easier when dealing with extremely spread out data. This issue of *Take It to the MAT* will focus on using logarithms to transform data, making it easier to picture and possibly easier to manage.

X	Y_1
0	2
1	16
2	128
3	1024
4	8192
5	65536
6	524288

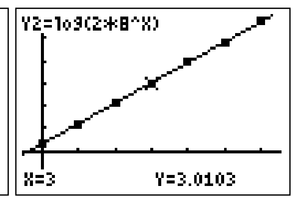
X=3



We'll begin by looking at the function $y = 2 \cdot 8^x$, a TI-83 table of it, and its graph. (The window range is X: [–0.5, 6.5], Y: [–150000, 600000].) From the graph, we see what might be called a "classic growth curve." The problem is the loss of detail when $x < 5$. It's very hard to see what's really going on at smaller values of *x*.

X	Y_1	Y_2
0	2	.30103
1	16	1.2041
2	128	2.1072
3	1024	3.0103
4	8192	3.9134
5	65536	4.8165
6	524288	5.7196

X=3



A solution to our problem is to take a logarithm of the *y* values. Any base logarithm will do; we'll use base ten. To the right is a table where $Y_2 = \log\left(2 \cdot 8^x\right)$ and the graph of $y = \log\left(2 \cdot 8^x\right)$. (The window range is X: [–0.5, 6.5], Y: [–1.5, 6].) The result of our transformation of the *y* values is that we now have a graph where the detail is not lost. As a matter of fact, it seems to be a line rather than a curve of rapidly increasing slope.

Let's explore the notion that this is a line. A quick inspection of the table seems to confirm that it is a line. For every increase of 1 by *x*, *y* increases by about 0.903. Thus, we appear to have a line with a slope of 0.903 and a *y*-intercept of 0.301 (Is it coincidence that our *y*-intercept is one-third of our slope?). The equation of the line is easy to derive: $\log y \approx 0.903x + 0.301$. Notice that the dependent variable is "log *y*", rather than just "*y*." This is because our graph is actually of log *y* vs. *x*, not *y* vs. *x*.

What if the *x*-*y* table above were real data? We might have a tough time discerning what type of model to use to fit the data. However, there is no doubt about the model to use for the transformed data—a line. But, what can we determine about the original function if we have an equation for the transformed data? Well, we transformed the *y*-values in our table by taking the base-ten logarithm of them. Let's "undo" that by creating an exponential function of base ten. Solving for *y* is shown to the right.

$$\log y \approx 0.903x + 0.301$$
$$10^{\log y} \approx 10^{0.903x + 0.301}$$
$$y \approx \left(10^{0.903}\right)^x \cdot 10^{0.301}$$
$$y \approx 7.998^x \cdot 2.000$$

Wow! The result is our original function, $y = 2 \cdot 8^x$. That means that our slope of 0.903 is actually log 8, and the intercept of 0.301 is log 2. Thus, an exponential function $y = a \cdot b^x$ can be transformed by taking the log of both sides into the linear function $\log y = \log a + \left(\log b\right)x$.

The moral of the story: if you are having trouble determining if a set of data can be modeled by an exponential function, do a log transformation on *y* and assess if the transformed data is linear.