TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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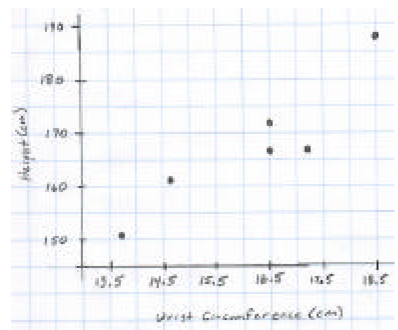
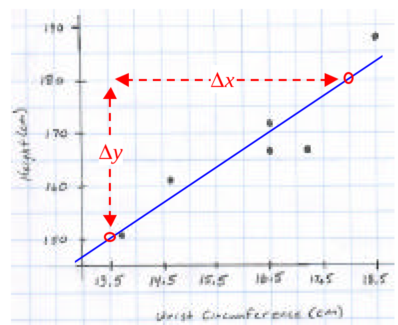
Graphing linear functions is a key topic in first-year algebra. However, it goes beyond sketching graphs of $y = mx + b$ and plotting points from x-y tables. Students should have experience with linear functions that have some context, some real meaning. In the November 26, 2001 edition of *Take It to the MAT*, the meaning of slope was discussed and its connections to ratio and rate. In this letter, we'll look at some real data, find a line that represents the data, and interpret what the line means.

Wrist Circumference (cm)	Height (cm)
13.7	151
14.6	161
16.5	167
16.5	172
17.2	167
18.5	188



Shown is a table of wrist circumference and height for six people and its scatterplot. The first question that students should ask is, "Does the data exhibit a linear pattern?" If the answer to that is, "Yes," and it appears to be in this case, then students should look for a line that represents the data. But how does one find the line? There are several statistical techniques that computers and calculators will do to find the equation of the line, including the median-median line and least squares regression. We will not address these here. The first method we want students to use is the "eyeball" method.



The graph at right shows a student's "eyeballed" line and it seems to fit the data fairly well. The next step is to find out what the equation of the line is, so that we may quantify the relationship between wrist circumference and height.

First, what is the slope of the line? Two points have been marked on the line: (13.5 cm, 150 cm) and (18.0 cm, 180 cm).

$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{180\text{cm} - 150\text{cm}}{18.0\text{cm} - 13.5\text{cm}} = \frac{30\text{cm}}{4.5\text{cm}} \approx 6.7$$

Thus, the slope is about 6.7. Notice that the units are the same in the numerator and denominator. They can, in a sense, be thought of to "divide out" or "cancel out." Regardless of the terminology we use, it means the slope here is not dependent on the unit of measure. (This is not always the case.) But what does 6.7 really mean? It says that for each one unit increase in wrist circumference, height increases *on average* by 6.7 units. This is the *relationship* between wrist circumference and height.

What if we want to predict height from wrist circumference? We need to write the equation of the line, but the slope is not enough. It would be nice to know the y-intercept. Looking at the graph, students will say it is about 147 cm, but that's not correct because the vertical axis is not $x = 0$. Students could redraw the scatterplot with the axes intersecting at (0,0), but a fair amount of accuracy would be lost. Let's just substitute one of our points and the slope into the general slope-intercept equation and solve for the intercept.

$$y = mx + b$$
$$180\text{cm} = (6.66...)(18.0\text{cm}) + b$$
$$b = 60\text{cm}$$

Now, we can see that *Height* ≈ 6.7 × *Wrist Circumference* + 60 cm. In this context, however, the intercept has little meaning. If a person has a wrist circumference of 0 cm, then that person is 60 cm tall? This does exhibit the danger of extrapolation, making predictions outside the range of the data, which is another story!