TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
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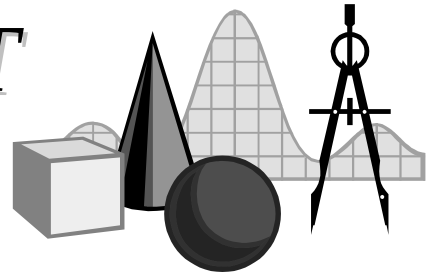

In the January 16, 2001 issue of *Take It to the MAT*, we looked at the derivation of the permutation formula $_nP_r = P(n,r) = \frac{n!}{(n-r)!}$. In this edition, the combination formula is derived and hopefully some of its mystery dismissed.

To review, a *permutation* is *an ordered arrangement of a set (or subset) of things.* The key with permutations is the characteristic of order. If we do the set of three chores take out the trash (T), vacuum the carpets (V), and wash the dishes (W) in the order TVW, that is a different permutation than VTW which is different than WVT.

In combinations, order is irrelevant. A *combination* is *a set (or subset) of things without regard to order*. Thus, TVW, VTW, and WVT are all the same combination; the order in which we do the chores is immaterial. If we were to do only two of our three chores today, say trash and vacuum, TV and VT would be different *permutations* because of their different orders. But TV and VT would be the same *combination* since they are the same subset of chores and the order in which they are done is unimportant.

How many combinations of all three chores can one find? Obviously one, the whole set. How many combinations can be created of two of the three chores? We may list them all: TV, TW, and VW. So, there are three. But what about VT, WT, and WV? VT is the same *combination* as TV, WT is the same as TW, and WV is the same as VW, because order doesn't matter. What is interesting is that there are 2 orders—2 permutations—for each combination. More about that later.

In the last issue we discussed the example of a baseball coach finding that there are 360,360 different *permutations* for 5 of the team's 15 players to take batting practice. How many *combinations* of 5 players are there? That is, coach wants to send five players to hit balls but doesn't care about the order in which they go.

We could list all of the combinations, but that would take a very long time. Or we can start with the fact that there are 360,360 permutations of five players out of the fifteen and think about them. Let's name the players A, B, C, D, and so on, up to O. One of the myriad permutations is CJGBM. Another is GCBJM, and yet another is MGCJB. Again, we won't record all 360 thousand arrangements, but we can see that the three just listed are the same *combination*.

So the question becomes how many permutations of B, C, G, J, and M can we make? From the last issue we know that the answer is 5! or 120. As a matter of fact, for any *combination* of 5 players, there are 120 different *permutations*. So, we can look at those 360,360 total permutations as coming in groups of 120. Consequently, there are 360,360 ÷ 120 = 3,003 groups of 5 players.

We know that the number of arrangements of n things taken r at a time is $\frac{n!}{(n-r)!}$. We now also know that there are r! ways to arrange any particular *group* of those r objects. Thus, there are $\frac{\frac{n!}{(n-r)!}}{r!} = \frac{n!}{(n-r)!r!}$ different combinations of n things taken r at a time. Viola! We have derived the general formula with which we are familiar, $_nC_r = C(n,r) = \frac{n!}{(n-r)!r!}$.