TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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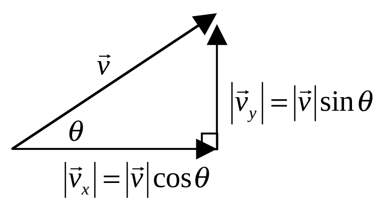
A very important, and sometimes underrepresented, topic in upper-level mathematics is *parametric equations*. The understanding of parametric equations and the ability to apply them is very important in higher mathematics and science, particularly physics. In this issue of *Take It to the MAT*, we will look at some applications.

In mathematics we use equations to model physical, financial, biological, and other processes. A single equation may be sufficient to describe a process. There are times, however, at which one formula is too complex, or does not give enough information to relate what is occurring. These are times where parametric equations are useful. That is, we will define multiple equations based on a single variable which is called the *parameter*.

A classic example of the use of parametric equations is the motion of projectiles. Take the case of a golfer hitting a ball off a tee. The ball leaves the tee position (x_0, y_0) with some velocity v, at some angle θ relative to the ground. Assuming that air resistance is not present—it is in reality, but let's not complicate matters—we can model the motion of the ball. We have all learned that the path of the ball through the air is a parabola, thus it's motion can be modeled with a quadratic equation.

Let x represent the ball's distance downrange from the tee and y represent the ball's height off the ground. So, we can model the motion of the ball with the equation $y = y_0 + (x - x_0)\tan\theta + \frac{a}{2v^2\cos^2\theta}(x - x_0)^2$, where a is the acceleration due to Earth's gravity. (a is usually given as –9.8 m/s^2 or –32 ft/s^2.) That's a pretty ugly equation and it only tells us how high the ball is for a given distance downrange. It doesn't tell us anything about where the ball is *at a given time*. Actually, the above equation was derived from the parametric equations of motion.

In general, the motion of a body in a given dimension under uniform acceleration is position now = initial position + (initial velocity)(time) + 0.5(acceleration)(time)^2. Thus, the equations of motion for the x and y dimensions are $x = x_0 + (v\cos\theta)t$ and $y = y_0 + (v\sin\theta)t + \frac{1}{2}at^2$. The initial velocities for the x and y dimensions were derived by breaking the velocity vector into components as shown by the diagram to the right. Note that the equation for x has no acceleration term, since we are assuming there is no air resistance and because gravity has no effect in the horizontal dimension.



The advantages that the parametric equations have over the function of y in terms of x is that we know both *where* the ball is and *at a given time*, and that the equations are much easier to use. In many situations, particularly those involving motion, parametric equations are much easier to use than a single function relating two variables. When students take physics, they are presented with parametric motion equations similar to the ones above, yet the physics texts rarely use the term parametric. It is incumbent upon us as math teachers to help them see the connection, and the value of parametric equations in all contexts.