TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
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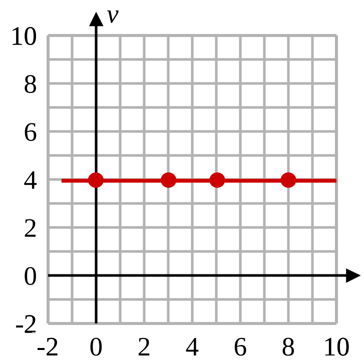
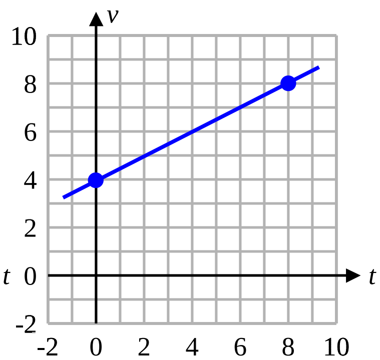
In the March 18, 2002 edition of *Take It to the MAT*, we looked at the concept of slope, how it relates to calculus, and how we can have students do calculus without actually *doing* calculus. In this newsletter, we will look at another big calculus concept: area under a curve. We will again approach it from a “non-calculus” angle.

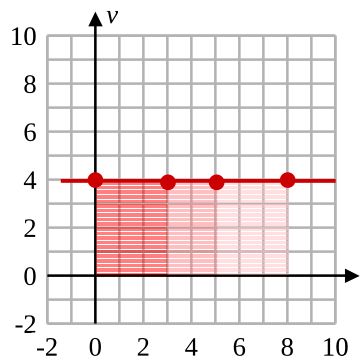
Let us consider, as we did last time, the idea of an object moving along a number line. We will not concern ourselves with its location on the number line over time to analyze its velocity as we did before. Now we are interested in its velocity as a function of time to determine how far it moved, or its displacement.

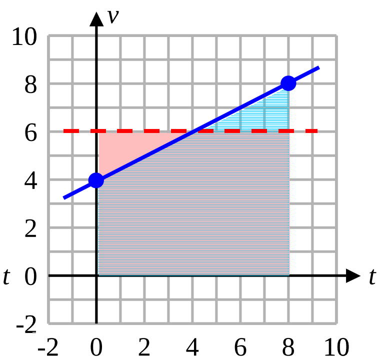
Imagine an object moving along the number line at a constant velocity of 4 cm/s. How far would it have traveled in 3 seconds? 5 seconds? 8 seconds? That’s pretty easy since we all know that *displacement* = *average velocity* $\times$ *elapsed time*. The graph at right shows just such a situation. But what if the velocity were changing? For example, the particle starts at $t = 0$ s traveling 4 cm/s and ends at $t = 8$ s moving 8 cm/s, accelerating at a constant rate all along the way. (This is shown in the second graph.) How far did it travel?



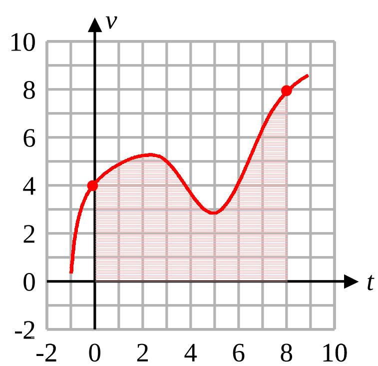


Is there anything about the graph we could use to determine displacement? Since velocity is represented on the vertical axis and time on the horizontal, then displacement might be the product of those two values. Looking first at the graph where velocity is constant, we can see that the *area* between the curve and the *t*-axis is equal to the displacement of the object. In the second situation, we should realize that if something accelerates *constantly* from 4 cm/s to 8 cm/s, its average velocity over time would be 6 cm/s. Thus it would travel 48 cm in 8 s. That also happens to be the area under the curve, which is in the shape of a trapezoid.





What if acceleration is not constant, like in the next graph? Velocity is not changing in a discernable manner and definitely not constantly. We seem to have found that displacement is equal to area, so we’ll find the area between the curve and the *t*-axis. How far did the object move in 8 seconds—what is the area? Since we don’t have a nice shape, it might be easiest to count squares. Each square is 1 cm/s $\times$ 1 s, or 1 cm of displacement. There are about 39 squares, so the object finished 39 cm from where it started. It’s also nice to know that we can work backward and find the average velocity from the displacement.



In a sense the students just did calculus by finding the area under the curve, but in a fairly unsophisticated way. It is especially helpful when considering things that do not move in predictable ways—like freeway traffic. But they don’t need formal techniques of integration to do it.