TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

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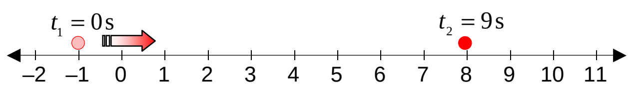
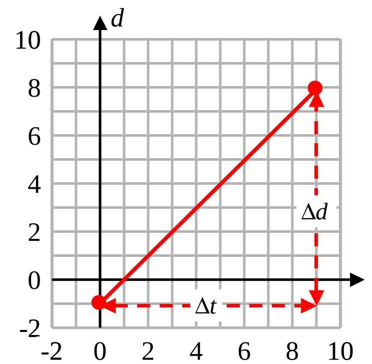
Calculus is often called “the jewel in the crown” of the high school math curriculum and is reserved for the best of our young mathematicians. In this and the next issue of *Take It to the MAT*, we’ll look at how students can do a little calculus in earlier courses—such as precalculus and second-year algebra—without *doing* calculus.

Slope is a big idea in calculus and students begin studying this concept in first-year algebra courses. However, its relevance to real-world situations, as well as other areas of study, is often ignored. One connection is to physics.

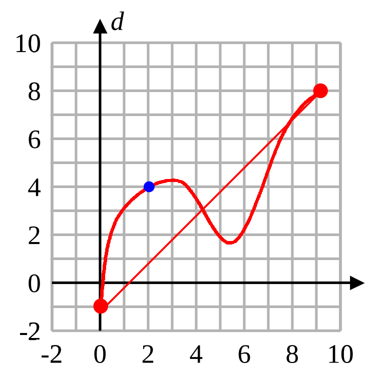
Early in physics, students learn the relationship between distance, time, and speed. Students already have some inherent notion of the associations among these quantities from practical experience. Later, in physics, the connections are formalized. There, we define the relationship as between *displacement*, time, and *average velocity*: $\bar{v} = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$, where $\bar{v}$ is average velocity and d_n is the *position* of an object at time t_n.



Think of an object moving along a centimeter number line. Let’s say its initial position is at –1 cm and it moves to its final position of +8 cm nine seconds later. Then, $\bar{v} = \frac{\Delta d}{\Delta t} = \frac{9\,\text{cm}}{9\,\text{s}} = 1\,\frac{\text{cm}}{\text{s}}$. Note that 1 cm/s is the *average* velocity. We do not know how the object moved in the five-second period between $t_1 = 0$ s and $t_2 = 9$ s. We only know where and when it started and finished, and thus can only discuss what happened *on average* during its travels.

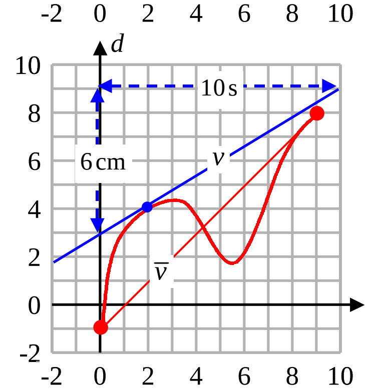


Suppose we plot our number line data on a coordinate plane. We’ll have the horizontal axis represent time, the vertical axis represent position. If we consider the average velocity as $\bar{v} = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1}$, then it becomes clear that average velocity is equivalent to slope. This is a crucial concept and a connection that is often ignored.



The really big idea that we want kids to know is *instantaneous* velocity and how it relates to slope. Once they have the idea that *slope of a line segment* equals *average velocity*, then we can move on to how an object is traveling at a particular point in time—and the key word is *point*.

Consider the second graph that shows where the object in our example was during that nine seconds. How fast was the object moving at the *point* in time $t = 2$ s? If student have an understanding that the steepness of the curve indicates speed, they should see that it is not quite as steep at the point $t = 2$ as our average velocity segment. Thus the *instantaneous* velocity is less than 1 cm/s.



To quantify it, students can draw a line through the point that has the same steepness—a tangent line. The slope of this tangent line is the instantaneous velocity at $t = 2$ s. The intent here is not to be exact, but to get close. Thirty students will probably get thirty different slopes, but all within a reasonable range. In our case, the tangent line we drew looks like it goes (approximately) through the points (0, 3) and (10, 9). The slope and thus the instantaneous velocity at $t = 2$ s is about 0.6 cm/s.

In a sense the students just did calculus by finding the slope of a tangent line. This technique helps make connections when students consider projectile motion, or anything else for that matter where the rate at which a quantity changes—in this case position—is not a constant. But they don’t need derivatives to do it, just a contextual understanding of slope.