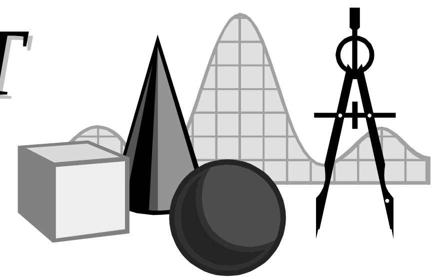
TAKE IT TO THE MAT

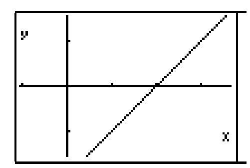
A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



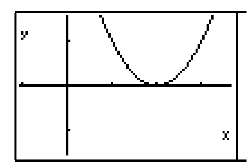
Math Audit Team
Regional Professional Development Program
February 14, 2000 — High School Edition

One of the major topics in the high school curriculum is sketching polynomial functions and finding their zeros. We have our students use many techniques to locate a curve's *x*-intercepts. Most of those intercepts are single zeros, but we have them practice solving equations with double, triple, or even higher-degree roots. Then they explore the nature of the curve at these points. In this issue, we will examine the function's behavior at and near these points more closely.

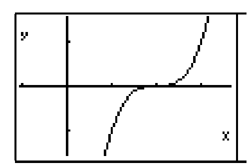
Consider the four functions and their graphs shown at right. The functions all have zeros of 2, but in varying degrees. As we trace the graphs from left to right, we see the typical patterns produced by zeros of different degrees. In the first graph, the curve is linear, intersects the *x*-axis at $x = 2$, and has a constant slope. In the second, it "bends" as it approaches the intercept of (2, 0), but does not cross the *x*-axis—it turns back into the first quadrant. The third degree function shows characteristics of both of the first two—it crosses through, but bends before doing so, and does so a little more than the degree-two function. The last function, of fourth degree, does the "touch and go" like the quadratic, but seems to be flatter near the intercept.



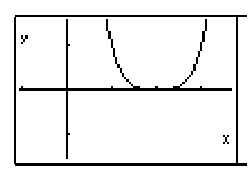
$y = (x - 2)$



$y = (x - 2)^2$



$y = (x - 2)^3$



$y = (x - 2)^4$

Why does the curve for odd zeros "go through," but for even ones doesn't? Why does the curve "flatten" more near the intercept as the degree of the zero increases?

Let's address these questions by choosing two values on either side of the zero $x = 2$, equal distances away. The values 1.9 and 2.1 look good.

	$y = (x - 2)$	$y = (x - 2)^2$	$y = (x - 2)^3$	$y = (x - 2)^4$
$x = 1.9$	$y = -0.1$	$y = +0.01$	$y = -0.001$	$y = +0.0001$
$x = 2.1$	$y = +0.1$	$y = +0.01$	$y = +0.001$	$y = +0.0001$

As we examine the table, the odd/even power issue is easy to address. For $x < 2$, values of $(x - 2)$ are negative, and for $x > 2$, they are positive. So, for odd powers of $(x - 2)$, y will be negative if $x < 2$, and positive if $x > 2$. But for even powers, y is always positive when $x \neq 2$. This explains the "touch and go" nature of even-degree zeros.

Something else jumps out at us from the table. As the degree of the function increases, the values of y decrease for these particular x's. We often think that functions with higher degree are steeper, but that's end behavior. When a number between 0 and 1 (or –1 and 0) is raised to higher and higher powers, its magnitude approaches 0. This explains the "flattening" effect near the intercept.

Exploration of this help's students understand the nature of graphs and their zeros. As in other topics, patterns abound. Finding the *why* behind these patterns enriches students' appreciation of them and deepens students' understanding of the underlying mathematics.