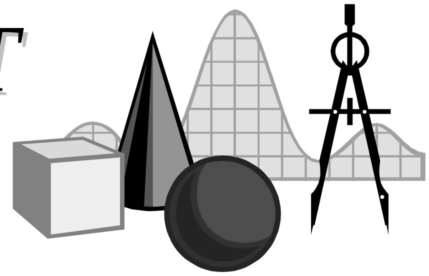
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION



Math Audit Team
Regional Professional Development Program
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Let's start this issue of *Take It to the MAT* with a few quick exercises.

(1) Simplify $\sqrt{x^2}$

(2) Solve $|x| = 4$.

(3) Solve $x^2 = 16$.

We'll get to the answers later.

Very often we present rules, formulas, and equations to students to learn. Students memorize the rules, we test them, they do well, they forget, we wonder. The next year's teacher wonders if the previous year's teacher taught them anything. Much of the problem stems from the fact that the rules, formulas, and equations are taught as a series of disjointed facts. Connections are rarely made, so students end up trying to recall a plethora of different facts rather than one general concept.

The three exercises above are all connected, yet are frequently taught at three different times as three different concepts.

In exercise 1, students would most likely respond, "*x*." After all, the square root and squaring are inverse operations that "undo" each other. Unfortunately, the students would be wrong; the correct answer is $|x|$. In the expression $\sqrt{x^2}$, if we were to let $x = 4$, then $\sqrt{4^2} = \sqrt{16} = 4$. This jibes with the student's answer. But what if we let $x = -4$? We now have $\sqrt{(-4)^2} = \sqrt{16} = 4$. Thus the answer is, in this case, $-x$ (the *opposite* of *x*).

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

While there are several good definitions of $|x|$, note the more formal definition of $|x|$ as shown in the box at right.

In exercise 2, we can use the definition of absolute value to solve this equation. If $|x| = 4$, then *x* could be 4 or –4. We can shorten this to $x = \pm 4$.

In exercise 3, some students will again respond, "4." They forget that $(-4)^2 = 16$. One method to solve the equation is by subtracting 16 from both sides, factor the left side, then apply the zero product property. An alternate method, and one that is typically taught, is to take the square root of both sides. The process is shown in the box at right. (We must remind ourselves that even though *every* real number, except zero, has *two* square roots, one positive and one negative, the symbol $\sqrt{\ }$ requires only the *principal* root—the positive one—be given.)

$$\begin{aligned} x^2 &= 16 \\ \sqrt{x^2} &= \sqrt{16} \\ |x| &= 4 \\ x &= \pm 4 \end{aligned}$$

Notice how all three exercises link together. Students need not consider squares, square roots, and absolute value as three unrelated topics. There are connections among mathematical concepts; they need to be made.