TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
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With this issue of *Take It to the MAT*, the Regional Professional Development Program begins it third year of providing teachers with a periodical addressing mathematics instruction. All editions of *Take It to the MAT*—elementary, middle school, and high school—for 2001–2002 will be published tri-weekly. We hope you find the topics interesting and useful. —Eds.

Solving systems of equations is an important skill in the fields of algebra and analysis. It provides a tool for solving applications such as linear programming and particle motion. There are many methods to solve systems of equations—substitution, graphing, elimination. This issue focuses on the use of matrices in solving linear systems and the various techniques involving matrices.

Let us begin with a discussion of the possible systems of simultaneous equations. First, a system may be *consistent*, that is it has **at least one** set of values that satisfies all equations. If a unique solution exists, the system is deemed *independent*. If there are multiple solutions, the system is referred to as *dependent*. If there is no set of values that satisfies all equations simultaneously, the system is *inconsistent*.

The techniques described below are in most texts at the algebra level and above. They are not presented in detail here in the interest of brevity.

The method to solve systems commonly introduced first is that of *Cramer's Rule*. Developed by Swiss mathematician Gabriel Cramer (1704–52), Cramer's Rule uses *determinants* to solve a system of linear equations. Solutions using Cramer's Rule are problematic in that if the determinant of the coefficients of the system is zero, the system is either dependent or inconsistent. Further examination of the determinants created by substituting the system's constants for columns of coefficients can reveal whether the system is dependent, but the solutions are still unknown.

A second method involves matrix algebra and inverses. Namely, if A, B, and X, are matrices and $AX = B$, then $X = A^{-1}B$. The problem with this method is that if A does not have an inverse, because its determinant is zero, we can't get a consistent, independent solution.

Finally, we will examine *Gaussian elimination*, which uses row operations to reduce an augmented matrix to a triangular form, such as $\left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 3 & 1 & -1 & 2 \\ 2 & 3 & 2 & 7 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -2 & 1 & 7 \\ 0 & 7 & -4 & -19 \\ 0 & 0 & 4 & 12 \end{array}\right]$. Further reduction to row-echelon, or reduced row-echelon, form—which can be done by most graphing calculators—leads to a quick solution. If one row is all zeros, then the system is dependent and other rows can be used to determine the set of solutions. If the first three columns of a row are zero with the fourth non-zero, the system is inconsistent.

Students need to be aware of the various methods to solve systems of equations with matrices and without. However, the teaching of inefficient methods, particularly Cramer's Rule, should be approached judiciously and in favor of better techniques.