TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
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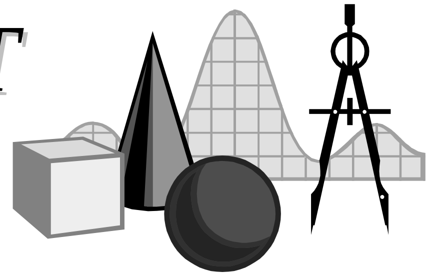

In the November 6, 2000 issue of *Take It to the MAT* we examined scientific notation and some ideas on building conceptual understanding. In this edition we will look at algorithms related to scientific notation and a variety of calculator concerns.

Sometimes students hear recipes concerning scientific notation.

> "To convert a numeral in standard notation to scientific notation, move the decimal point of the numeral to the left or right until you get a numeral greater than or equal to one but less than ten. Count how many places you moved the decimal point and that will be the power of ten, positive if you moved to the left, negative if you moved to the right."

$$146.9 = 1.469\times10^{2}$$

A similar recipe exists for converting scientific notation to standard notation.

> "If the exponent of the numeral is positive, move the decimal point to the right a number of places equal to the exponent. If the exponent is negative, move the decimal point to the left."

$$2.35\times10^{4} = 23500$$
$$\Rightarrow 2.3500.$$

Unfortunately, when these recipes are presented without conceptual understanding, they are easily forgotten and do not build number sense. Such algorithms must be developed *after* the concepts involved have been mastered. When this does not occur, students make obvious errors. For example, students may be given the algorithm,

> "The exponent tells you how many zeros you place behind the number or in front of the number."

If the mantissa is one, this statement is partially true. It is the case that 1×10^{6} is the digit one followed by six zeros (1000000). In the case of 1×10^{-6}, we could imagine placing six zeros in front of the number (0000001), but where do we place the decimal point? If we place it immediately in front (.0000001), we are wrong as this is 1×10^{-7}. If we place it after the first zero (0.000001)—since we formally write decimal numerals less than one with the preceding zero—we do have a correct numeral. But, now we have added another step to the algorithm, one that is counterintuitive. It is certainly not the case that 4.25×10^{2} is the numeral 425 followed by two zeros as in 42500. While a recipe may work for certain cases, it does not work for all, and students can become easily confused. It is better that algorithms incorporating counterintuitive steps or exceptions are avoided.

Lastly, calculators can cause problems because of their varied displays. When some calculators display `2.357 11`, it is meant to be interpreted as 2.357×10^{11}. It is very important that students do not misinterpret the result as 2.357^{11}. Others may show the value as `2.357E11`; again, students must know how to interpret this. (The "E" may correspond to *exponential notation*, another name for scientific notation.)

Scientific notation is an important tool in a student's repertoire, particularly because of its use in science and higher mathematics. The key to its instruction is developing number sense along with, and preferably before, the provision of recipes.