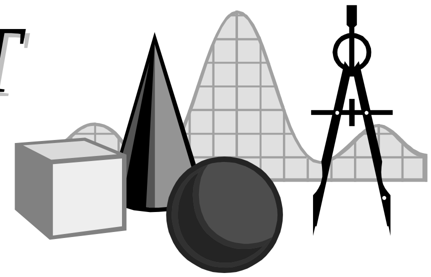
TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Math Audit Team
Regional Professional Development Program
October 23, 2000 — High School Edition



Relations, functions, and their inverses are important topics in the high school curriculum, particularly in higher mathematics. In the next few issues of *Take It to the MAT*, we will examine the definitions of relation, function, and inverse, and discuss both mathematical and non-mathematical examples of them.

A relation is defined as *an association between, or property of, two or more objects*. “Is equal to” is a relation as is “lies between.” For example, $y = x$ describes an association between two objects, x and y; b lies between a and c is a relationship between three objects. More complex mathematical relationships can be defined, such as “is four more than twice the value of,” or $y = 2x + 4$.

The key here is that there must be multiple objects. “Is even” is not a relation because it describes a property of only *one* value.

Some non-mathematical examples might be “is a brother of” or “owns a dog.” “Is a brother of” can describe a relation, but “owns a dog” cannot. For example, “Fred is the brother of Steve,” shows an association between two objects—Fred and Steve—as to “brotherliness.” Since Fred and Steve are two objects, it is by definition a relation. However, “Steve owns a dog,” is not a relation because owning a dog is a property of only *one* object.

Very often we write relations as sets of ordered pairs. The relation described above, “is four more than twice the value of,” could be written as $\{(0, 4), (-1, 2), (\pi, 2\pi + 4), \dots\}$. If a family consisted of the children {Steve, Fred, Jane}, then “is a brother of” would be written as {(Steve, Fred), (Fred, Steve), (Steve, Jane), (Fred, Jane)}. In that example, we have listed every possible ordered pair.

Relations may also have certain properties: *reflexive*, *symmetric*, *transitive*. A relation is *reflexive* if every element is related to itself. For example, “is equal to” is reflexive for the set of complex numbers since every element would equal itself. “Is a brother of” is not reflexive; one is not a brother to oneself.

A relation is *symmetric* if a relationship in one direction is also true when reversed. We defined the relation “is four more than twice the value of.” This is not symmetric. **Fourteen** is four more than twice **five**, but **five** is not four more than twice **fourteen**. Neither is “is a brother of” symmetric. It does hold for some cases—Steve is a brother of Fred and Fred is a brother of Steve—but, while Steve is a brother of Jane, Jane is not a brother of Steve. “Is the opposite of” would be symmetric; –3 is the opposite of 3, 3 is the opposite of –3.

Finally, a relation is transitive if both a relates to b and b relates to c imply a relates to c. “Is greater than” is a good example. Five is greater than three, three is greater than two, and thus five is greater than two. “Is equal to” also is transitive, but “is the opposite of” is not. Is “is a brother of” transitive?

More about relations—and their cousins, functions—next time.