TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
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One of the topics students learn in algebra is *slope*. Slope is a foundational concept for higher-level mathematics, science, economics, business, statistics, and many other fields. That is because slope is one way to express change or how one variable is affected by another. The groundwork for understanding slope is laid in the middle grades through the topics of *ratio* and *rate*. This issue of *Take It to the MAT* will look at the connections between ratio, rate, and slope.

We have all been to the grocery store and have seen that fruits and vegetables are typically priced by weight. Sometimes there is a price per item, but almost all produce is charged at a per pound (or some other unit of weight) *rate*. That is, there is a special *ratio* between different units where the value of the denominator is 1 unit.

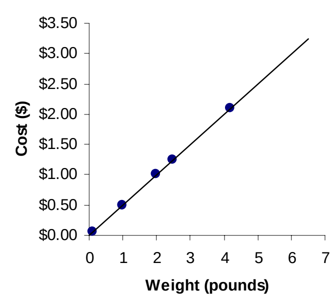
Let's consider a 50-cent per pound *rate* for bananas. Actually, the *rate* of 50 cents per pound is the *ratio* $\frac{\text{50 cents}}{\text{1 pound}}$ or 50 cents:1 pound. The table below right shows how the weight of bananas and their cost are related. Since *ratios* compare the relative sizes of two numbers, we could write several ratios using information from this table: $\frac{\$0.05}{\text{0.1 pound}}$, $\frac{\$1.00}{\text{2 pounds}}$, $\frac{\$2.10}{\text{4.2 pounds}}$, etc. If we write each ratio with one in the denominator we end up with $\frac{\$0.50}{\text{1 pound}}$.

It is conceptually important that students see that "1" in the denominator. We too often get in the habit of simplifying the ratio and writing $\frac{\$0.50}{\text{pound}}$ or \$0.50/pound. While this is the end goal—we don't say "miles per *one* hour" or "gallons per *one* square foot"—students need that conceptual bridge between writing any ratio and writing it simplified with only a unit as a denominator. It is also important that students write and say ratios and rates in a variety of ways, other than "per." The rate above can not only be read "50 cents *per* pound" but also as "50 cents *for each* pound," "50 cents *for every* pound," or the less desirable, "50 cents *a* pound."

Cost of Bananas, 50¢/lb.

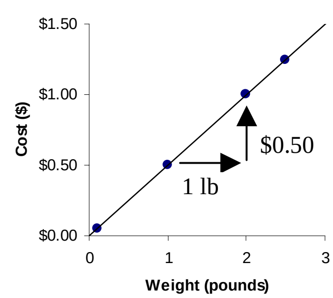
Weight (lb.)	Cost ($)
1	0.50
2	1.00
2.5	1.25
4.2	2.10
0.1	0.05
⋮	⋮
w	0.50*w*

Students should be able to use the rate to write an equation expressing cost as a function of time, as in the last line of the table. It can be in words, *Cost* = (\$0.50)(*weight in pounds*); or in symbols, C = \$0.50w. They should also graph the function as shown to the right, with our points from the table.



One thing is immediately apparent about the graph at the right that all students should see. It is that the points all seem to fall on a line. That is, as one "runs" the graph from left to right, the points "rise" at a constant *rate*. And that rate seems to be 50 cents for every pound.

The rate of 50 cents per pound is equivalent to $\frac{\$0.50}{\text{1 pound}}$, which is the ratio between the vertical and horizontal changes between any two points on the graph. That ratio of change is *slope*. Or in other words, *slope is a ratio*, or more precisely, *a rate of change*. And, slopes do have units. In this case, the slope is not 0.50 as one would infer from the equation C = \$0.50w, but $\frac{\$0.50}{\text{1 pound}}$! The slope tells us how much one variable changes with respect to the change in another, and that is very much a ratio or a rate.



One last point. Students should, as much as possible, learn mathematics from three perspectives: Numerical, Analytical, and Graphical (NAG). In our example we experienced ratio, rate, and slope numerically through a table, analytically through an equation relating cost and weight, and graphically through our graph of the function on the coordinate plane.