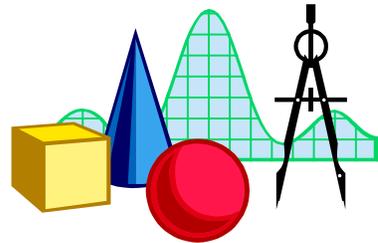# TAKE IT TO THE MAT

A NEWSLETTER ADDRESSING THE FINER POINTS OF MATHEMATICS INSTRUCTION

Regional Professional Development Program
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What is the answer to $3 + 4 \times 6$? Is it 42? 27? Something else? Do we add or multiply first? Do we move from left-to-right, or can we perform the operations in any order we choose? Herein lies the whole notion of *order of operations*. This *Take It to the MAT* will address *order of operations* and issues related to it.

The answer to the problem above is 27, because we do the multiplication first. (See an expanded solution to the right.) The question is, "Why do we do multiplication first? Why not addition—or subtraction?" Mathematicians have decided on the convention of doing multiplication before addition because of the nature of the operations.

$$3 + \underbrace{4 \times 6}$$
$$= 3 + 24$$
$$= 27$$

Number is about quantity, and the most basic mathematical activity to determine quantity is counting. Addition is merely repeated counting. Think about one of the first addition strategies students learn—counting on. After that, multiplication is simply repeated addition of the same value. Multiplication is therefore an even higher form of counting. To extend this even further, exponentiation (powers) is repeated multiplication. That is, $4^3 = 4 \times 4 \times 4$. Thus, there is a hierarchy of operations that determines what we do first.

"What about subtraction and division?" you ask. Mathematicians would probably tell you not to worry because subtraction and division are just inverse operations of addition and multiplication, respectively. Therefore, they are, in a sense, just addition and multiplication. Subtraction is addition of an opposite and division is multiplication of a reciprocal. (See the expressions to the right.) Thus, when evaluating expressions with mixed operations, multiplication and division are done together, then addition and subtraction are done together. **We do *not* do all multiplication before all divisions, or all additions before subtractions.**

$$10 - 5$$
$$= 10 + (-5)$$
$$10 \div 5$$
$$= 10 \times \frac{1}{5}$$

If given the expression $5 + 6 - 3 + 4$, what do we do first? We know we are supposed to do addition and subtraction together, but with three operations something must be first. In this case, we move left-to-right, doing each operation in turn. See the solution to the right. Notice, we did not add 5 and 6 to get 11, add 3 and 4 to get 7, then subtract 7 from 11.

$$\underbrace{5 + 6} - 3 + 4$$
$$= \underbrace{11 - 3} + 4$$
$$= \quad 8 + 4$$
$$= \quad 12$$

Grouping symbols, such as parentheses and brackets, are a special case of the order of operations. If we place operations inside grouping symbols, we are making a statement to ignore the accepted convention and do what's inside first. In a sense, we place a priority on certain operations. For example, in $(3 + 4) \times 6$, we will add 3 and 4 first, because the grouping symbols place priority in the addition, before the multiplication.

$$\underbrace{5 + 6} - \underbrace{3 + 4}$$
$$= 11 - 7$$
$$= 4$$

The order of mathematical operations is done as follows:

- Do what is inside grouping symbols (parentheses, brackets, etc.) first, following the order of the steps below.
- Exponents and roots.
- Multiplication and division, as encountered, from *left to right*.
- Addition and subtraction, as encountered, from *left to right*.

Your final challenge is to evaluate the following expression: $90 - (6 + 9 \div 3) \times 3^2$.